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DEVOTED TO THE INTERESTS OF ENGINEERING AND ARCHITECTURE AT HARVARD UNIVERSITY

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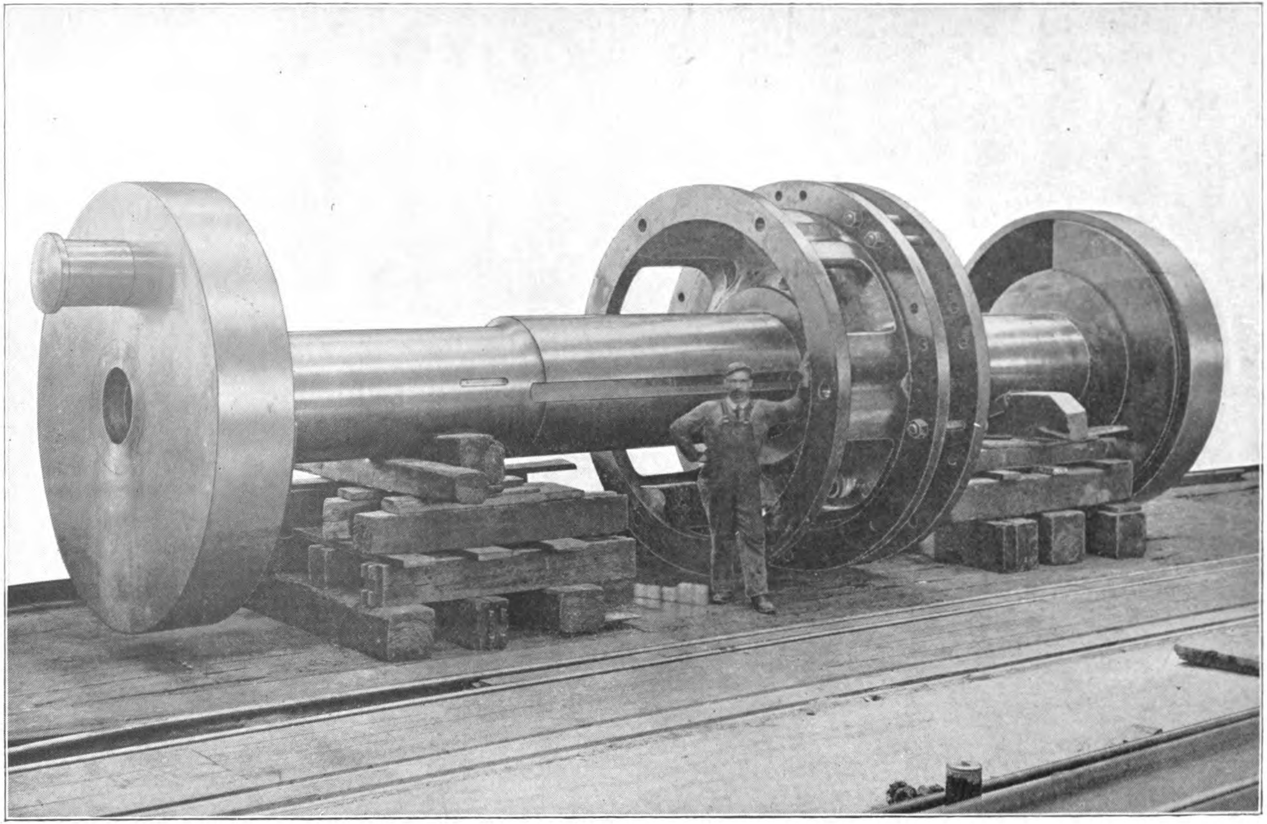

FIG. 1.— SHAFT OF FLUID-COMPRESSED NICKEL STEEL, HYDRAULICALLY FORGED ON A MANDREL, OIL-TEMPERED AND ANNEALED, FOR AN 8,000 H. P. ENGINE FOR THE BOSTON ELEVATED RAILWAY. DIAMETER OF FLY-WHEEL FIT, 37 INCHES; DIAMETER OF JOURNALS, 34 INCHES; DIAMETER OF CRANK DISC FIT, 32 INCHES; DIAMETER OF AXIAL HOLE, 17½ INCHES; LENGTH, 27 FEET 10 INCHES; WEIGHT OF SHAFT, 65,410 POUNDS; WEIGHT OF CRANK DISCS, 8 FEET DIAMETER, 58,004 POUNDS; WEIGHT OF FLY-WHEEL HUB, 46,986 POUNDS; TOTAL WEIGHT, 170,400 POUNDS. CRANK DISCS AND FLY-WHEEL HUB FURNISHED BY CORLISS STEAM ENGINE COMPANY, AND WERE ASSEMBLED AND FITTED TO SHAFT COMPLETE BY THE BETHLEHEM STEEL COMPANY.

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NICKEL STEEL.

CLIFFORD TAFT HANSON, L.S.S. 1900.

In the history of iron and steel there have been many special steels which on their first introduction gave promise of great usefulness, but a more extended acquaintance led to disappointment. Quite the reverse, however, has it been with nickel steel, which in point of tonnage, as well as in the importance and variety of purposes to which it had been successfully applied, occupies the first place among special steels.

For a number of years nickel steel acquired celebrity because of its extended use for military purposes. Introduced to public notice in 1889 by Mr. Riley in a paper read before the Iron and Steel Institute of Great Britain, its field of usefulness has been rapidly enlarged until at the present day the use of nickel steel is as thoroughly recognized as that of carbon steel.

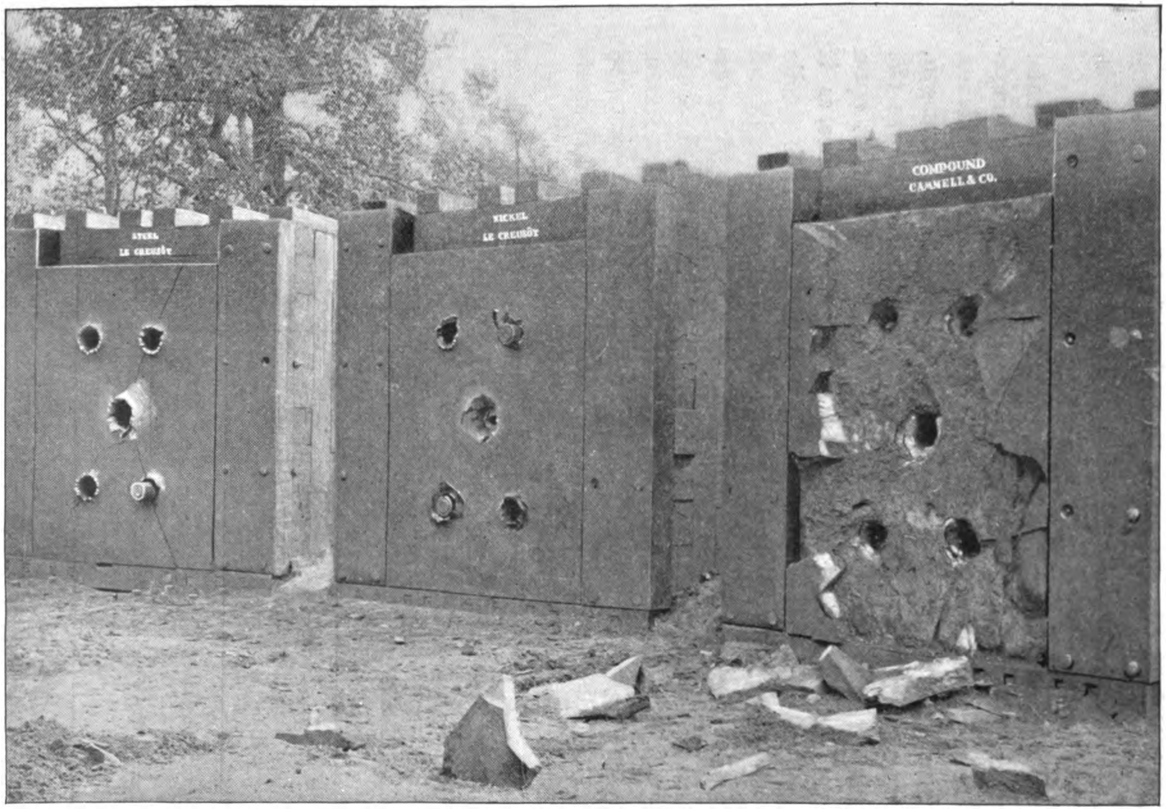
Soon after the appearance of Mr. Riley's paper, experiments in the use of nickel steel for armor-plates were undertaken by the Creusot Works in France, and were so successful that in 1890 a plate of nickel steel was ready to be submitted for public trial. Up to that time "compound" plates made by Sir John Brown & Co. and Charles Cammell & Co., of Sheffield, England, and "all-steel" plates by Vicker & Co., were competing for supremacy as a protective sheathing for war vessels. The "compound" plate consisted of a face of hard steel welded

to a tough wrought iron back. The steel face was about one-third the thickness of the whole plate; and its effect was to offer a resistance to penetration of the shot, while the tough iron back prevented the plate from cracking to fragments. The "all-steel" plate claimed, by means of its hardness and toughness, to break and stop the projectile and yet to remain in good shape upon its backing.

The probable superiority of nickel steel for armor had been early brought to the attention of the Navy Department, and in 1890, in response to the invitation of this department to foreign armor makers, a competitive trial of armor-plates was held at Annapolis, Md. The plates were all of the same size, 6 feet x 10½ inches in thickness. One was a "compound" plate, and the other two were "all-steel" and "nickel-steel" plates respectively. Each plate was submitted to four rounds from a 6-inch gun firing a Holtzer steel shell, with a striking velocity of 2,075 feet per second, and then to one round from an 8-inch gun firing a Firth steel shell, with a striking velocity of 1,850 feet per second. These velocities were calculated to drive the shell just through a good homogeneous steel plate.

The "compound" plate was completely penetrated by all projectiles fired at it and its steel face destroyed. The "all-steel" plate kept out all shots fired against it, but was badly cracked at its centre by the last round of the 8-inch gun.

The resistance obtained from the nickel-steel plate far exceeded all expectations. At the fifth shot, with a striking energy of 4,988 foot-tons, the 8-inch armor piercing projectile broke into many pieces without developing in the plate a sign of a crack. This experiment demonstrated that a moderate amount of nickel — about 3¼ per cent. — so increased the toughness of steel of a given hardness or tensile strength as to greatly add to its resistance to cracking from the shock of projectile impact. As a result of this trial, nickel steel was adopted for the armor of the Navy. The production of nickel steel was undertaken by the Bethlehem Steel Company and the Carnegie Steel Company, Limited, and within a year after the above-mentioned trial successful results were obtained. Since then



All Steel. Nickel Steel. Compound Plate.

FIG. 2.— VIEW OF ARMOR-PLATES AT THE ANNAPOLIS TRIAL AFTER FIVE SHOTS AT EACH PLATE. THE FRAGMENTS OF THE COMPOUND PLATE ARE SHOWN IN THE POSITION IN WHICH THEY FELL.

nickel-steel armor-plate has been regularly produced in large quantities, and at the present time it is used in part or exclusively as a protective coating on nearly all modern war vessels, wherever constructed.

An examination of the physical characteristics of this metal shows it to possess valuable qualities which explain its toughness and resistance to shock. The nickel acts as a hardener, replacing a part of the carbon and with a given tensile strength, increases somewhat the elongation and to a greater degree the contraction of area at the point of fracture. Its effect upon the elastic limit is, however, of the greatest importance, as it raises this property in a marked degree relatively to the tensile strength, and thus insures a combination of elastic strength and ductility, or toughness, unknown in any other metal. The presence of the nickel also renders the steel sensitive to the good effects of tempering, and the desirable qualities above mentioned are accentuated by this treatment. When compared with simple steels of the same tensile strength, nickel steels show an increase in elongation and contraction of area.

The Bethlehem Steel Company gives the following table of comparative tests of nickel steels and simple steels in oil-tempered, annealed forgings: —

COMPARATIVE TESTS OF NICKEL STEEL AND SIMPLE STEEL FORGINGS.

Composition of Forgings.	Elastic Limit. Lbs. per Sq. In.	Tensile Strength. Lbs. per Sq. In.	Elongation. Per cent.	Contraction of Area. Per cent.
No nickel, 0.20 carbon	28,000	55,000	34	60
3.5 per cent. nickel, 0.20 carbon .	48,000	85,000	26	55
Influence of 3.5 per cent. nickel .	+20,000	+30,000	—8	—5
" " 1 per cent. nickel .	+5,714	+8,571		
No nickel, 0.30 carbon	37,000	75,000	30	50
3.5 per cent. nickel, 0.30 carbon .	60,000	95,000	22	48
Influence of 3.5 per cent. nickel .	+23,000	+20,000	—8	—2
" " 1 per cent. nickel .	+6,571	+5,714		

Carbon Steel
Tensile and Bending Test Pieces.

Nickel Steel
Tensile and Bending Test Pieces.

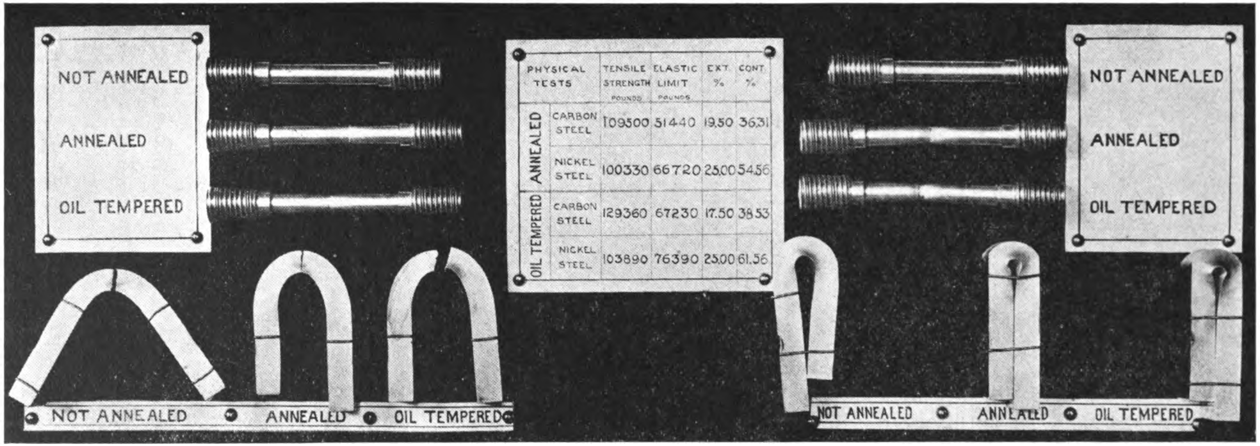


FIG. 3.— COMPARISON OF CARBON STEEL AND NICKEL STEEL FORGINGS.

COMPARATIVE TESTS OF NICKEL STEEL AND SIMPLE STEEL FORGINGS — *Concluded.*

No nickel, 0.40 carbon	43,000	85,000	25	45
3.5 per cent. nickel, 0.40 carbon .	72,000	110,000	18	40
Influence of 3.5 per cent. nickel .	+29,000	+25,000	—7	—5
" " 1 per cent. nickel .	+8,285	+7,142		
No nickel, 0.50 carbon	48,000	95,000	21	40
3.5 per cent. nickel, 0.50 carbon .	85,000	125,000	13	32
Influence of 3.5 per cent. nickel .	+37,000	+30,000	—7	—8
" " 1 per cent. nickel .	+10,570	+8,571		

This table is interesting, as showing that the effect of nickel on the elastic limit increases as the carbon increases. A nickel steel having 0.20 per cent. carbon and 3.5 per cent. nickel is equivalent to a 0.40 per cent. carbon steel in tensile strength, while the elastic limit and elongation are higher. In carbon steels of less than 0.50 per cent. carbon the elastic limit is about 50 per cent. of the ultimate strength, and usually less than this when properly annealed. Nickel raises the proportion about 5 per cent. for each 1 per cent. of nickel added.

Another characteristic of great value in nickel steel is what may be called its high vibrating strength; that is, the power of resisting a large number of small stresses, each well within the actual breaking strength. Under a fibre stress of 40,000 pounds per square inch, where a piece of 25 per cent. carbon steel will stand 429,000 alternations of stress before rupture occurs, a similar bar of nickel steel, with 3¼ per cent. nickel and .25 per cent. carbon, will stand about 1,850,000, and a 5½ per cent. nickel steel with 25 per cent. carbon, will stand about 4,370,000 alternations of stress before breaking.

This metal, offering a marked increase of elastic limit, toughness, and resistance to alternating stresses, has caused the United States Bureau of Steam Engineering to recognize its peculiar fitness for the severe service required of high-grade marine engine shafts, crank-pins, piston- and connecting-rods, and specifications have been drawn calling for nickel steel of such high physical qualities as can be obtained only under the

most approved methods of fluid-compression, hydraulic forging hollow on a mandrel, oil-tempering and annealing. These specifications direct that all principal forgings must be made of nickel steel, and must have a tensile strength of not less than 95,000 pounds, elastic limit not less than 65,000 pounds, elongation not less than 21 per cent. in two inches, oil-tempered and annealed. These requirements are met, and in many cases exceeded, at the works of the Bethlehem Steel Company.

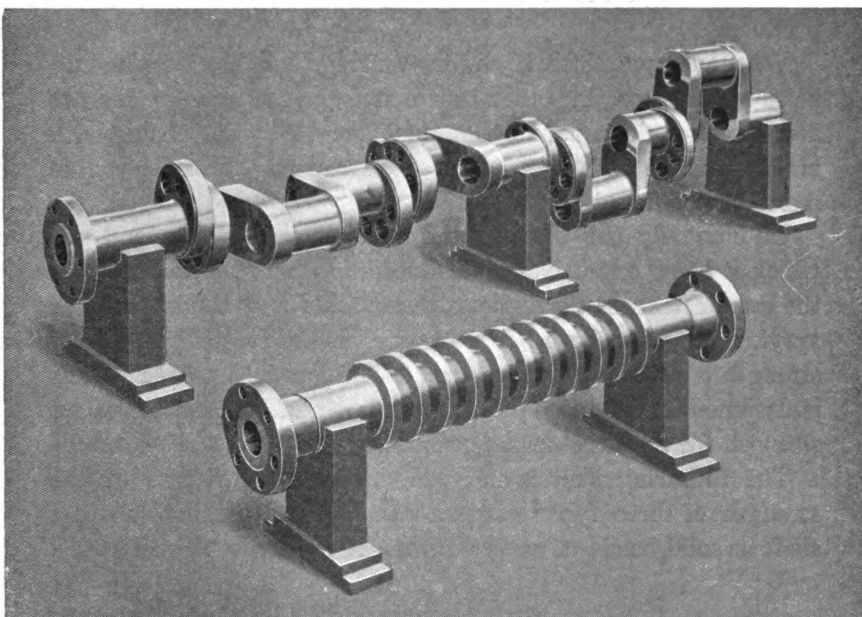


FIG. 4.—CRANK AND THRUST SHAFTS FOR THE U.S. TORPEDO BOAT "SHUBRICK," MADE OF OIL-TEMPERED NICKEL STEEL; 6¾ INCHES OUTSIDE DIAMETER; 4 INCHES AXIAL HOLE, EACH FORGED IN ONE PIECE; CRANK SHAFT TURNED ON 13 CENTRES. WEIGHT OF CRANK SHAFT, 1,650 LBS.; WEIGHT OF THRUST SHAFT, 1,800 LBS.

The successful manufacture of homogeneous nickel steel demands careful furnace practice. It is, however, in the forging or rolling and especially in the final heat treatment that greater care must be used than when manipulating simple steel of the same carbon content. Care should be taken to avoid

chilling the metal, as it is extremely sensitive to sudden even though slight changes of temperature and to careless handling. One of the main conditions of forging is a sufficiently powerful steam hammer or hydraulic press, otherwise the texture of the material remains crystalline and the forging is crude and liable to crack diagonally. The effect of the blows must penetrate to the heart of the mass and produce a blending of the metal; and the pressure must be continued for a sufficient length of time. Through the use of the hydraulic press this effect is produced.

Whenever practicable, engine forgings and shafting should be made hollow, in order to insure against and to remove hidden axial defects, and to reduce weight by dispensing with material which adds but little to the capacity of the piece to resist torsional and bending strain.

Undoubtedly the best type of hollow forged shafts, and one which is being extensively introduced, is where the walls are of the same thickness throughout, the outside and inside diameter varying together, both being greatest at the centre when most strength is required, and smallest at the journals. Such a shaft is designed on the principle of a girder, and offers the greatest strength for the least amount of metal.

The following sketch and table give a comparison of the strength of three types of stern-wheel shaft. Taking the first shaft as solid, made of wrought iron, 14 inches in diameter and

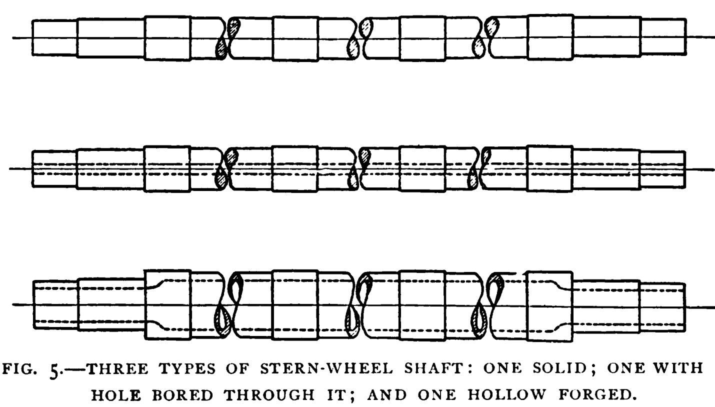


FIG. 5.—THREE TYPES OF STERN-WHEEL SHAFT: ONE SOLID; ONE WITH A HOLE BORED THROUGH IT; AND ONE HOLLOW FORGED.

30 feet long, and representing its strength by unity, the strength of the other shafts is given in terms of this value. The second shaft has the same diameter, is bored with a $3\frac{1}{2}$-inch hole and is 7 per cent. lighter than the solid shaft. The third shaft is hollow forged on a mandrel, with the ends closed down. The outside diameter is 22 inches, and the inside diameter 17 inches. It has the same weight as the solid shaft.

		Relative Strength.
1st.	Shaft Solid Wrought Iron	1
2d.	Shaft Bored Carbon Steel, Oil-tempered	2
	Shaft Bored Nickel Steel, Oil-tempered	3
3d.	Shaft Hollow Forged Carbon Steel	4
	Shaft Hollow Forged Carbon Steel, Oil-tempered	5
	Shaft Hollow Forged Nickel Steel	6
	Shaft Hollow Forged Nickel Steel, Oil-tempered	7

Many of the largest engine-builders are now using nickel-steel forgings where special strength is required, such as n engines for electric power plants, street and elevated railway power plants, pumping engines, rolling mill and factory engines, etc. The Boston Elevated Railway, as an example, has an engine of 8,000 H.P. with a Bethlehem nickel-steel shaft weighing 65,410 pounds, 37 inches outside diameter, with a $17\frac{1}{2}$-inch hole through its axis (figure 1).

The successful introduction of nickel steel for engine forgings has led many railroads to take up the use of this metal for locomotive forgings generally, as there is no doubt that nickel steel is the best material now available for crank-pins, cross-head pins, piston-rods, locomotive driving-wheel axles, pedestal cap bolts, and other parts subjected to severe alternating stress and wearing action. Given a material of an elastic limit far higher than the rapidly alternating stress occurring in service, and you at once secure a longer life, and, what is usually more important, safety from sudden shocks.

In Figure 3 are given results of the series of tensile tests and bending tests taken from the nickel steel forgings recently furnished by the Bethlehem Steel Company for a locomotive

which is being built for them by the American Locomotive Company at their Scranton Works. This illustration shows that when subjected to simple annealing or to oil-tempering, nickel steel gives better elastic limits with, at the same time, better percentages of extension and contraction of area than carbon steel of the same tensile strength. The increased toughness of the nickel steel is strongly illustrated by the bending test.

When it is considered that nickel steel will outlast simple steels in the general wear and tear, it is not surprising that the material should be tested experimentally for rails. In 1899 the Pennsylvania Railroad ordered a quantity of nickel steel rails to be laid on the west track of the famous "Horseshoe Curve," near Altoona, Pa. These rails, with 3.22 per cent. nickel and .504 per cent. carbon, showed great rigidity under the straightening presses, and exhibited such hardness during drilling that in some cases five twist drills of ordinary steel were used up in drilling one hole. These rails were put in the up track; and, although the service is not as heavy as it would be on down tracks, they have given perfect satisfaction, with no perceptible wear on them.

The satisfactory results of a three years' trial of these experimental nickel steel rails has just caused the Pennsylvania Railroad to place an additional order with the Carnegie Company of Pittsburg for 5,000 tons of nickel steel rails with the necessary angle splice bars. The Baltimore & Ohio Railroad has also just placed an order for 1,000 tons, and the Pennsylvania Railroad West of Pittsburg an order for 3,000 tons with the same company. The cost of nickel steel rails and splice bars is estimated at about twice as much as the ordinary steel rails; but the expectations are that they will last from three to four times longer, fully justifying the increased expenditure. Furthermore, the steel company agrees to buy back the worn-out rails, and credit the railroads for the nickel contained therein.

It is certain that the experience of the Pennsylvania Railroad, with the increased wear of nickel-steel rails on curves, will cause their adoption on elevated railroads and in subways.

As a material for structural steel for buildings and bridges, the high elastic limit of nickel steel makes it far more advantageous than ordinary steel. This point has been shown by tests made upon thin plates, angles, and rods.

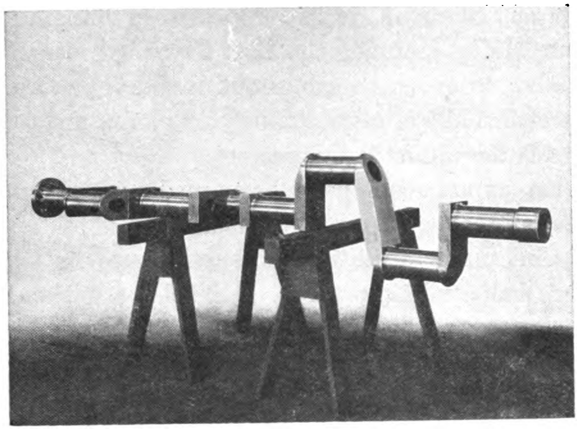


FIG. 6.— A FOUR-THROW, OIL-TEMPERED, NICKEL STEEL CRANK SHAFT FOR A HIGH-SPEED YACHT ENGINE OF 4,000 H. P. DIAMETER OF SHAFT AND PINS, 5 INCHES; DIAMETER OF BORE, 3 FEET $3\frac{1}{2}$ INCHES X 4 INCHES; LENGTH, 9 FEET 5 INCHES; WEIGHT 579 LBS.

A 0.25-carbon steel Z bar, annealed, tested by J. G. Eaton, showed: —

Carbon, Per cent.	Nickel, Per cent.	Elastic limit, Lbs. per sq. in.	Ultimate strength, Lbs. per sq. in.	Elongation, Per cent.	Reduction of Area, per cent.
0.25	none	36,420	62,410	26.25	58.5
0.25	3.3	59,500	84,640	24.50	54.0

A glance at these figures makes it plain that a gain to the working load has been increased in the ratio 36 to 59, or practically 64 per cent., whilst the elongation is sufficient for all ordinary purposes.

The resistance of nickel steel to jarring shock renders its use in eye-bars for bridge construction particularly desirable. Such bars are subject to repeated jarring strain, and the failure of an eye-bar means the collapse of a bridge. The necessity thus aris-

ing for a steel with high elastic limit has led to the use, in some cases, of a 0.40 per cent. carbon simple steel. The same high elastic limit can be obtained by using an 0.18 or 0.20 carbon steel, with 3 to 3.5 per cent. of nickel, which steel is much less liable to damage in its manufacture by over-heating than the simple steel of higher carbon. It is proposed, in building the new Manhattan Bridge spanning the East River between New York and Brooklyn, to make the cables out of short eye-bars of nickel steel, somewhat like a magnified bicycle chain, instead of wire strands, as is the ordinary practice.

Corrosion experiments have been made with specimens of nickel steels in competition with specimens of soft steel and wrought iron, under conditions which indicate its adaptability for boiler and condenser tubes, ship plates, piston-rods, sea-water pumps, wire cables, wire ropes, etc. The corrosive action of the atmosphere on nickel steel containing 30 per cent. nickel is, even in the presence of moisture, very slight. In fact, the action is so slight after two years' exposure as to warrant a 30 per cent. nickel steel, so far as its exposure to air is concerned, to be practically non-corrodible. An instance is cited of a graduated rule made of 36 per cent. nickel steel which was left for several months in a moist atmosphere without showing the least sign of rust.

Comparatively little is known in this country regarding nickel steels of high percentage of nickel. In France, however, investigators have been working on them for some years, and have attained surprising results.

Some of the high nickel steels show, with proper heat treatment, remarkably high elastic limit, in some cases over 100,000 pounds per square inch. The manufacture of this steel on a commercial basis will meet many requirements in the trade, especially in the construction of automobiles and other mechanical devices where lightness and strength combined are a desideratum. An alloy with about 25 per cent. nickel has been found to be non-magnetic at ordinary temperatures, but to become magnetic when cooled to 0° C. (32° F.). It retains this magnetism until heated to about 580° C. (1,076° F.), when it

again becomes non-magnetic. This alloy can, therefore, exist in two states, magnetic and non-magnetic, according to the previous heat treatments. The magnetic condition of alloys of more than 25 per cent. nickel changes with the temperature, but is always the same for any given temperature. These variations in magnetism of nickel steel alloys may be used as magnetic indicators of temperature changes, such as circuit breakers, automatic fire alarms, and other instruments in which an electric current is broken by a rise in temperature, produced either by the heating effect of a current or by an external source of heat.

An alloy of high electrical resistance is largely used for resistance coils in rheostats to regulate electrical currents and in electrical heaters to generate heat by electrical resistance. A rheostat demands a wire of a certain diameter in order to insure rigidity; and, unless this material is made of high electrical resistance, the current will exceed a certain desired volume. A resistance wire must also resist oxidation and be free from brittleness after repeated heating and cooling. The high electrical resistance of 25 to 30 per cent. nickel steel, has led to its use for the above purposes instead of German silver. It has about 48 times the resistance of copper, while German silver has only 18 times and ordinary steel wire about 8 times the resistance of a copper wire of the same diameter. Nickel-steel wires, owing to this high percentage of nickel, are nearly incorrodible, and on account of their high tensile strength are much less liable to break than German silver. After repeated heating and cooling, German silver becomes brittle, while nickel-iron alloys appear to retain their original elasticity and strength.

Nickel steel costs more than ordinary steel, but in many cases its superior qualities make the increased first cost of the material a matter of secondary consideration. Fine shafting, connecting-rods, and crank pins of nickel steel may increase the cost 15 per cent. to 25 per cent. over the figures for simple steel forgings; but the increase in efficiency, which varies with the amount of carbon present, runs from 40 per cent. in soft steel to over 60

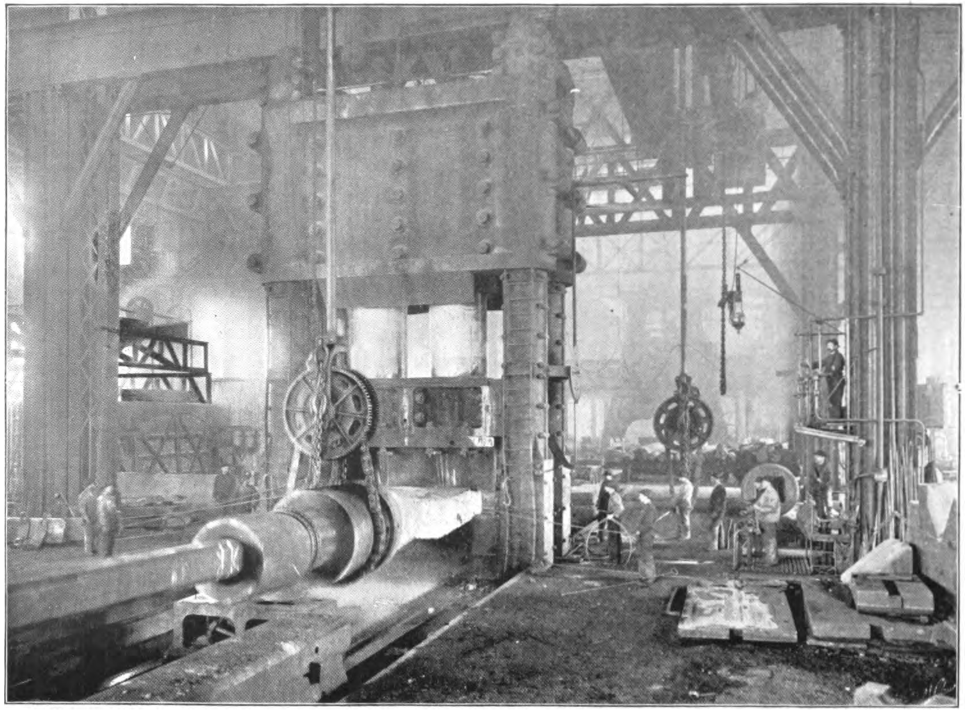


FIG. 7.— A NICKEL STEEL ARMOR-PLATE IN THE BIG HYDRAULIC FORGING PRESS AT THE BETHLEHEM STEEL COMPANY'S WORKS.

per cent in hard steels. As this gain in efficiency may be utilized either to decrease the weight of the part or to increase the amount of work done, the net saving by the use of nickel steel is very evident.

No material in this age of progress and development can long remain exclusively or even principally used. When a better material is known, it will be employed. The field for the use of nickel steel is one of magnitude; and, as time goes on, the ability of this metal to meet every demand in the industrial field will be more fully recognized by engineers.

THE MOULD-LOFT.

STANLEY CUNNINGHAM, JR., '00.

THE mould-loft, in a shipyard, is an intermediate step between the drafting-room and the steel shop, forge shop, foundry, or wherever else the necessary materials and parts are prepared for the hull of a ship. There the plans of the hull, especially of those parts which form the skeleton of the structure, are drawn full-size, and all irregularities disclosed by enlarging the small drawings (usually ¼ inch to one foot) are corrected. The surface of the hull must be smooth and continuous; and, to that end, all intersections of planes with the hull must be curves without abrupt changes. The lines are therefore faired by altering the frames to produce continuous curves.

The loft must have a very large plane surface. It should be located in a steady building that is not likely to change its shape by racking or settling. Therefore, it is best that no heavy machinery be placed in or near it. The floor is laid with clear white pine boards, and covered with a dull paint without much oil, so that the surface may not be glossed. Along one side of the floor is run a hard pine batten about 1 inch thick and 3 inches or 4 inches wide. This should run true and straight throughout its length, as from this "base batten" all work is laid off; and it is as essential for it to be straight as it is for the edge of a drawing board to be straight.

When the lines of a ship are sent to the loft from the drafting-room, the first work to be done is to transfer the lines to the floor, at the same time enlarging them to their full size. The loft is seldom long enough to take the entire length of the ship. So the hull is divided into two or three lengths, and these are laid down, one over the other, with enough overlapping parts to enable the draftsman to fair each length into the other. The sheer and half-breadth plans (the longitudinal elevation and plans respectively) are laid down, one over the other, from the same base line; and the body plan (the cross section

showing the frames) is placed at any convenient point on the floor, generally where it interferes least with lines in the other plans running in the same direction as the frame lines.

The operations of laying down a ship on the floor are precisely the same as for drawing it out on a drawing board. The arrangement of plans, however, varies as noted; and the means employed, as well as the scale of the work, render the operation more laborious. Distances between points cannot be taken off by dividers, but must be marked on wooden poles, or battens, of hard or soft pine, about 1¼ inches square. It is also more difficult to project points from one view to another by means of a square or straight edge, and straight lines of any length are therefore made with a chalked cord. Curves are formed by means of wooden battens bent to pass through the given points. They are drawn in with French chalk or soapstone. The battens for sheer lines and similar long flat curves, like water lines in the body plan of the ship, are flat, about ¾ inches thick and 3 inches to 4 inches wide. Lighter battens are used for sharper curves; and, where the curve changes from an easy sweep to a sharp turn, the batten is tapered. The battens must be chosen with care, to be free from any permanent set, and should take a fair curve, when bent. They are held in place on the floor by awls, driven in beside them, or through holes previously bored.

Starting from the base batten, a line is laid off parallel to it to form the base from which all points are measured. The water lines are laid off parallel to it; and, if the keel does not pitch either way, its upper edge is placed on the base line. Very often a vessel is intended to draw more water astern than in the bow, and the keel has then a decided pitch towards the water lines. In such a case it is located as any other line would be, by measurement from the base line. The first line is squared off the base at the point chosen for the body plan, and parallel to it the frame lines in the sheer and half-breadth plans are drawn in. A spacing pole, having the frame spacing marked on it, is used to lay out the frame points along the base line, and also along one of the water lines at some distance above the base. Between these points

the frame lines are snapped in and prolonged to reach to the main deck height. Great care must be taken to get the frame lines located accurately. A small error in the length of the spacing pole multiplies rapidly when the pole contains but a few frame spaces and has to be applied a number of times to give points enough for the length of the ship to be shown. The distance apart of extreme points may be checked with steel tape, or the frames made laid out from the start with a tape. The permanent spots for frame lines and water lines in the sheer and half-breadth plans are marked in black or colored pencil. For other points chalk is used; and all lines are drawn in chalk, except where they are very important and must be kept for some time for reference.

With the lines, offsets are generally sent to the loft from the drafting office. These include the length over all; the moulded beam; the depth; the sheer heights at bow, stern, and lowest point; the dead rise of the floor; the tumble home, if any; the fall in at the bilge; and the lengths of ordinates on the water lines, and sometimes on diagonals and buttock lines; the location and spacing of frame lines, water lines, bow and buttock lines; the beam camber; the shape of the stem and stern forgings or castings; and, in general, all points relating to the shape of the hull. Blue-prints of the construction and details of various parts should be supplied to the loft as soon as they can be decided upon, though these are not necessary for the fairing. The offsets should be scaled from the original drawing, if possible, to avoid any mistakes likely to occur in tracing or in the stretching and shrinking of the blue-print paper after printing. However, the loftsman is often left to take his own offsets by scaling the lines as they are sent him. They are not, as a rule, taken for every frame. It is sufficient to take them at intervals, depending on the quickness of the curves, being spaced closer as the curve is sharper near the bow and stern. The points, as determined, are spotted in on the floor; and then the work of fairing proceeds exactly as on a drawing board, by finding the intersections of planes with the lines laid down. The most useful lines in fairing are the diagonals, the buttock,

and the water lines; in general, those lines that cross the frames most nearly at a right angle. The work is done by the system of giving and taking till the lines in all plans form true curves, especially in the body plan. A few intermediate frame lines may be run in, in the body, as a test; but, if the others were well chosen, they should show no unfairness. A final test is to project some of the diagonals into the half-breadth or sheer plan. If these show fair and all the other lines are true, the ship may be considered a continuous curved surface without breaks.

In case the water lines are long, easy curves, they, with the diagonal and buttock lines, may be faired on a contracted frame spacing. The same half-breadths and offsets are used; but the frame spacing is contracted to one-half or one-third the true spacing, which brings the points nearer together and gives a sharper curve, to which the batten will be more likely to spring fair. This mode of fairing is useful only for the middle body: the ends must be faired full size.

The sheer curve for the deck at the side is given by arbitrary rules, and may be faired in accordance with these. The registration companies, as Lloyd's, require a certain beam camber or curve, to be determined by the breadth of the ship. Since the deck is then higher at the centre than at the side, its intersection with the hull will not run parallel to the longitudinal line along the centre; for the two meet at the bow and stern. This follows from the fact that all deck beams have the same curve. In the stern, abaft the transom frame, which is the frame over the stern post, cant frames are worked, where the stern rounds in rapidly. These are sometimes used in the bow, especially in bluff bowed ships. They are shown in their projected form in the half-breadth, transferred to the sheer and finally to the body plan. In general, the chief usefulness of the two longitudinal plans is found in fairing. This is by far the most important plan, as with it, together with the frame spacing and the sheer outline of the bow and stern, the lines of the whole ship may be constructed.

When this work has been done, the ship being laid down and

faired, the offsets of the diagonals, the bow and buttock lines, the water lines, the half-breadths at the main deck and other decks, and the heights of the decks at the side are marked off on poles for transferring to the board in the shop. These poles contain a full record of the shape of the ship, so that the lines may then be dispensed with.

Where the ship has an inner bottom, it may be faired by diagonals, in the same way as the outside shell. Floors may be faired on contracted frame, spacing by diagonals and buttock lines.

The poles prepared with the offset points of the water lines, etc., are used in making the scrieve board. This is simply a large plane surface placed near the furnace and bending slabs; and it is used to show the shape to which the frames, floors, and other parts must be bent. It is made of pine boards, large enough to take the full midship section, the bow and the stern. If put down in the mould loft, it is made in sections that may be taken apart for convenient removal to the shop. Lines are marked on it with a scrieve knife, which cuts a fine groove which cannot be rubbed out and lost in the course of testing the completed pieces. Some lines are painted in to make them distinct and to avoid confusion. On the board is scrieved the body plan of the ship, with its water lines and section lines, every frame line, reverse frame and floor, the line of the keelsons, the stringer angles, the bilge keels, the plate edges, the rib-bands, the decks, and the web-frames. Sometimes two boards are used: one to show the whole forebody, starboard and port, and one for the whole afterbody; or the two bodies may be placed on one board, with their base lines on opposite ends. Owing to the confusion of lines that exists with this method, often but one side of the ship is shown, placing the forebody and the afterbody on the same centre line, but with their base lines on opposite ends. This, of course, shows different sides of the ship in each half; but it is more convenient to work with than it would be if both had the same base, for the two sides may be used at once, without the work on one side interfering with that on the other. The frame lines and sheer of decks

are transferred directly to the board from the body plan. The frame points, offsets on the water lines, diagonals, and bow and buttock lines are put down, and a fair curve run through them and scrieved in where the intermediate frames run close. Points may be fixed by graduating up the space, or they may be put in from the lines in the sheer and half-breadth plans. The reverse frames are put in in the same way, and the ends of the floors shown. The location of the keelsons and the stringer angles is usually roughly located in the drawings: they must, however, be faired by the loftsman. Set off from the centre line are the half sidings for the stem and stern post, which may be constant or varying, depending on the type of construction, and the heels or landings of the frames are marked on them. The stringer angles are run on the inside of the frames, so that their frame points are set in the depth of the frame perpendicularly from the outside.

Rib-bands of pine or spruce, 4 inches to 6 inches siding, are used to keep the frames in their designed position while erecting on the stocks, until the outside plating is put on and some of the keelsons or stringers in place. Therefore, they are run in the way of the outside strakes of plating, so that the inside strakes which are put in place first may go on without removing the rib-band. Some rib-bands follow the deck lines, others run around through the bilge or under the bottom. Their position is nicked on the frames before setting up, and the position of the frames is also marked on the rib-bands. In order to get the proper spacing for the frames on the rib-bands where the ship runs in at the bow and stern, the rib-bands are "expanded" from the scrieve board. A pole is laid along the run or girth of the rib-band, and the frame points marked upon it. These are laid out from the base on the regular frame spacing, and a batten or a series of battens bent around them to take the curve. On the battens are marked the frame crossings; and they are used to lay off the rib-bands, ready to hoist into position on the frames. The mark on the frame and that on the rib-band are made to agree, which insures the right position of the frames.

The plate edges are put on the scrieve board, either from offsets supplied by the draftsman or by measurement and trial of the plates by the loftsman. If the shell plating was ordered from a model, the plate lines must be taken to agree with the plates as ordered. The lines or edges are most easily laid out on the model by means of pine battens pinned in place. Points scaled off it may then be faired on the floor and checked, to see that the plates are ordered large enough. The plate lines above the bilge may be faired in the sheer, and those below in the half-breadth. The width of the strakes on the midship section is shown by painting in position a section of the plate laps: from these the "sight edges" (edges of the outside plate) are run toward the stem and the stern. It should be noted that the true width of the plate can only be measured by girthing on the frame on the scrieve board. Where the frames are straight and parallel to the centre plane of the ship, the true shape of the plates will appear in the sheer. Where the plates run up to the bow and stern, there is a certain amount of curve up, or "sny," which must be allowed for in ordering or laying out the plates.

To bring the plating into the stem and stern, since the girth of the plating amidships is greater than the length of the stem, the plates must be made to taper. In order that they may not become too narrow, one strake of the plate is stopped some distance from the stem, and the next below or above it widened to cover the end and fill its place. Such a strake that does not land on the stem or stern post is called a "stealer." The sheer strake is that which follows the upper deck and shows the line of the sheer outside of the hull: the other strakes are arranged to give graceful and easy curves in the sheer. When the sight edges of the plates are scrieved in place, the inside edges are marked out on the scrieve board by setting off the width of the lap along the frame, and running a line through the points. The outside edges are pointed in with a narrow stripe of a distinctive color, and the inside edges indicated by a broken stripe in the same color. The rib-bands and decks, stringer angles, keelsons, etc., are also painted in some distinguishing color.

The web-frames are marked in place on the scrieve board. They are the deep frames, with brackets or plates between the frame and reverse bar, worked where extra transverse strength must be supplied, as where the deck beams are omitted, in the way of cargo and engine-room hatches.

In order that the frames may be worked to the proper bevel, bevel boards for the frames and also for the reverse bars are made from the lines on the scrieve board. Since the position of the plate laps is marked on the frames, this is a good place at which to give the bevels. They are obtained by measuring the perpendicular distance along a plate lap between one frame scrieve and the scrieve next outside it. Knowing the frame spacing, this will give the "fall in" in that space from which the angle of the bevel may be determined and marked on a board, by means of a convenient rig arranged for the purpose. The boards are about four feet long by four feet wide, and are a half-inch thick and are made to show the bevel along the run of the frame at each plate lap. The reverse bar levels are taken off in the same way, being given at the keelson and stringer points, since these are marked on the bars.

The principal moulds that are made for the hull are the stem, stern post, beam camber, and midship section mould. The last is especially useful where there is a long dead flat, where the frames all take the same curve, the only change being in their length, due to the sheer of the upper deck. This sheer, marked on the mould, gives the length at which to cut each frame. A mould, made to give the beam camber for a full length beam, gives also the length of each deck beam. A bevel board is marked from the scrieve board to show the angle between the deck and the shell plating at every frame, which is used in making the bracket or welded knees for the frames. A list of moulds to be made would include those for nearly all the plates, angle bars, and various iron shapes to be put into the ship, since they are used to mark off all the work for punching and cutting. Those used but little are made of one-fourth inch poplar batten stock, while the more important are of white pine. In some ships, where there is a long dead flat amidships, and

short bows and stems, ingenious methods have been employed for using one mould for marking a number of pieces, making it adjustable for slight variations in shape. This is called "universal work." Mould-making is an important part of the loftsman's duties, distinct from laying off, but must be merely referred to here.

The practice of laying off is given by Thomas Watson in his "Naval Architecture" in a practical form. Some of his methods are departed from in the ordinary work in the loft. Each shipyard has its own special way of doing the work, which varies necessarily with the type and size of the ship to be built. As a rule, methods of laying off depend in the end on descriptive geometry, which is the basis for all such work.

GRAPHIC METHOD OF DETERMINING THE TURNING MOMENT ON A CRANK SHAFT.

IRA N. HOLLIS.

THE study of the moment on a crank shaft is important from two points of view: —

1. The design of the parts for strength.
2. The reduction of variations in the angular velocity.

That part of the problem involving the reciprocating parts has also a direct bearing upon balancing an engine and the prevention of vibration. Those familiar with the steam-engine will understand at once that the turning moment on the shaft for a single double-acting cylinder passes through two zero points and two maxima during every revolution. When the

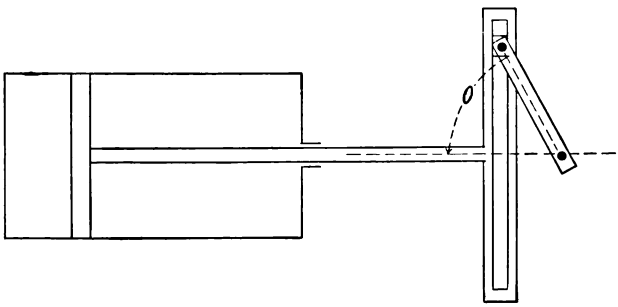


FIGURE I.

crank is on the dead points, the moment arm is zero, whatever the pressure on the piston may be; and it follows that the moment is zero. When the crank is nearly at right angles to the centre line of the engine, the moment arm is a maximum; and the moment is correspondingly large. Its values for all positions of the crank may be represented as a curve by referring it to rectangular axes. The crank angles are usually laid off along the horizontal axis, and the moments parallel to the vertical axis, both to an appropriate scale. In its simplest

form, with a uniform pressure on the piston and a straight yoke connection, as in figure 1, the curve of crank-shaft moments is an ordinary sine curve, modified only by the effect of the reciprocating parts.

The graphical method of finding the moment is so evident that it is not necessary here to do more than indicate the algebraic process of expressing it.

Let p = pressure per square inch on the piston.
A = area of the piston in square inches.
r = length of the crank in feet = ½ stroke.
θ = angle between the crank and the centre line of the engine.
W = weight of reciprocating parts.
M = turning moment on the shaft in foot pounds.
N = number of revolutions per minute.
s = distance of piston from outer end of stroke.

Then, neglecting the frictional resistances and the weight of the reciprocating parts,

$$M = p\ A\ r\ \sin\theta,$$

which is the ordinary sine curve.

If the reciprocating parts of the engine be taken into account, their acceleration parallel to the centre line must be found.

$$s = r\ (1 - \cos\theta)$$

$$\frac{ds}{dt} = r \sin\theta \frac{d\theta}{dt} = \frac{r\pi N}{30} \sin\theta$$

$$\frac{d^2s}{dt^2} = \frac{r\pi N}{30} \cos\theta \frac{d\theta}{dt} = \frac{r^2\pi^2 N^2}{900} \cos\theta.$$

The pressure required to produce this acceleration must be subtracted from the steam pressure available for transmission to the crank pin. Indicating its value by F, we have

$$F = \frac{W}{g} \cdot \frac{r\pi^2 N^2}{900} \cos\theta.$$

Note.—That the value of F is simply the component parallel to the axis of the cylinder of the centrifugal force of the reciprocating parts considered as concentrated at the crank pin centre.

Subtracting the value of F from $p\ A$, we get the turning moment

$$M=(p\ A-\frac{W}{g}\frac{r^2\pi N^2}{900}\cos\theta)\ r\sin\theta$$

$$=p\ A\ r\sin\theta-\frac{W}{g}\frac{r^2\pi^2N^2}{1800}\sin 2\theta$$

The last term represents the departure from the sine curve in case of uniform steam pressure. It is simply a sine curve of double frequency.

The straight yoke engine is not much used, and it may be dismissed at once. Passing to the engine with a connecting rod, let $l=$ the length of the rod, and $W=$ the weight of the reciprocating parts, including the piston, piston rod, cross-head, and one-third of the connecting rod.

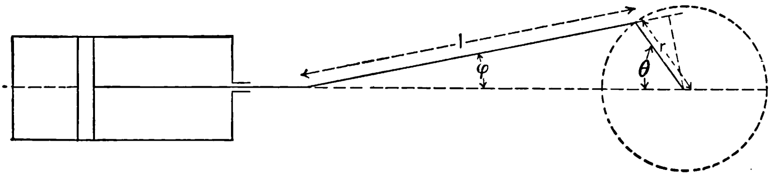


FIGURE 2.

Neglecting the weight of the reciprocating parts, we have

$p\ A=$ pressure transmitted to the crank pin.

$\frac{p\ A}{\cos\phi}=$ pressure in line of the connecting rod.

$$M=\frac{p\ A}{\cos\phi}r\sin(\theta+\phi)$$

$$M=p\ A\ r\frac{\sin\theta\cos\phi+\cos\theta\sin\phi}{\cos\phi}$$

$$M=p\ A\ r\sin\theta+p\ A\ r\cos\theta\tan\phi$$

This is again a sine curve plus a modification, if the value of p be uniform. We can replace ϕ by its value in terms of θ

$$l\sin\phi=r\sin\theta,\ \sin\phi=\frac{r}{l}\sin\theta$$

$$tan\ \phi = \frac{\frac{r}{l} sin\ \theta}{\left(1 - \frac{r^2}{l^2} sin^2\ \theta\right)^{\frac{1}{2}}}$$

$$\text{and, } M = p\ A\ r\ sin\ \theta + \frac{p\ A \frac{r^2}{l} sin\ \theta\ cos\ \theta}{\left(1 - \frac{r^2}{l^2} sin^2\ \theta\right)^{\frac{1}{2}}},$$

$$\text{or, } M = p\ A \left\{ r\ sin\ \theta + \frac{\frac{r^2}{l} sin\ \theta\ cos\ \theta}{\left(1 - \frac{r^2}{l^2} sin^2\ \theta\right)^{\frac{1}{2}}} \right\}$$

The part in the brackets may be called the equivalent moment arm. It is not necessary that p should be constant, as the formula holds good so long as p is measured on the indicator card for the position of the piston corresponding to the crank angle θ.

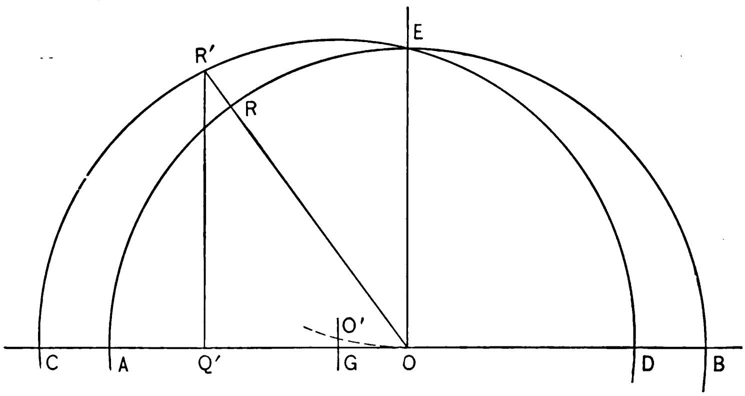


FIGURE 3.

For the purpose of representing this formula graphically, we may neglect the part under the radical. The resulting discrepancy is very small, easily within the error of measuring pressures on an indicator card.

$$\text{Then, } M = p\ A \left(r\ sin\ \theta + \frac{r^2}{2l} sin\ 2\theta\right)$$

or, $M = p\,A\,r\,sin\,\theta\left(1+\frac{r}{l}\,cos\,\theta\right)$

The moment arm $r\,sin\,\theta\left(1+\frac{r}{l}\,cos\,\theta\right)$ may now be obtained by the method shown in figure 3.

Let O A E B be a crank pin circle to any scale, so that O A $= r$. Draw O E perpendicular to O A and G O′ parallel to O E and at a distance from it $= \frac{r^2}{l}$. With E as a centre and r as a radius, describe a short arc cutting G O′ in O′; then, with r as a radius, and O′ as a centre, draw a second circle O′ C E D. For any crank angle A O R, the perpendicular R′ Q′ is the equivalent crank arm, and we need only multiply the value of $p\,A$ by it to get the moment on the crank

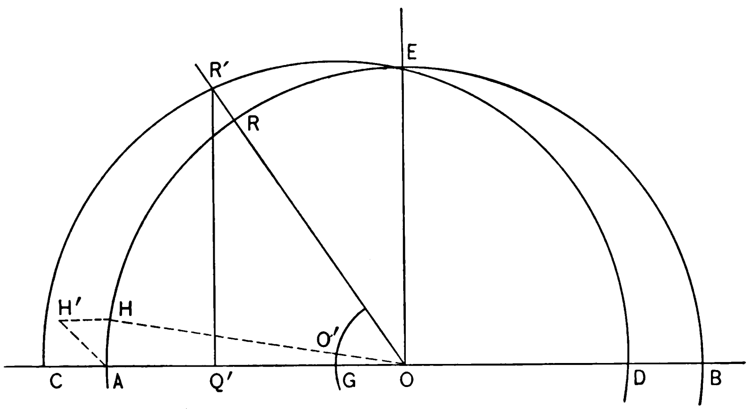


FIGURE 4.

shaft. The result will be found to be accurate enough for all practical purposes; but a closer approximation to the value

$r\,sin\,\theta\left(1+\dfrac{\frac{r}{l}\,cos\,\theta}{\left(1-\frac{r^2}{l^2}\,sin^2\,\theta\right)^{\frac{1}{2}}}\right)$ may be obtained as follows:—

In figure 4, describe as before the crank pin circle O A E B and the small arc G O′ with radius $= \frac{r^2}{l}$. Through A, draw

A H′, making 45° with O A produced, and lay off A H′ = O G; then draw H′ H parallel to A O, cutting the circle in H. The radius O H fixes the centre O′ of a second circle, described with O′ E as radius. As before, the equivalent crank arm for any crank position O R is R′ Q′. This is correct to within less than ½% of error for ordinary ratios of connecting rod to crank.

The distance of the piston from the outer end of the stroke is expressed by

$$S = r + l - r \cos \theta - l \cos \phi$$
$$S = r\,(1 - \cos \theta) + l\,(1 - \cos \phi)$$
$$S = r\,(1 - \cos \theta) + l \left(1 - \sqrt{1 - \frac{r^2}{l^2} \sin^2 \theta}\right)$$

Taking $\sqrt{1 - \frac{r^2}{l^2} \sin^2 \theta} = 1 - \frac{r^2}{2l^2} \sin^2 \theta$ (approximately)

$$S = r\,(1 - \cos \theta) + l \left(1 - 1 + \frac{r^2}{2l^2} \sin^2 \theta\right)$$
$$S = r\,(1 - \cos \theta) + \frac{r^2}{2l} \sin^2 \theta$$

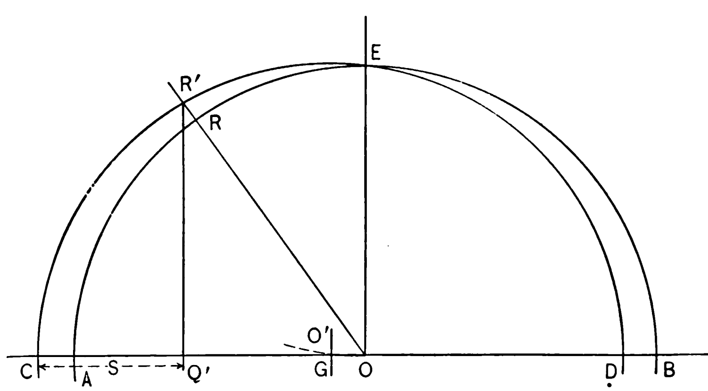


FIGURE 5.

This expression may be represented graphically with sufficient accuracy by means of an auxiliary circle similar to that used for the equivalent crank arm. In figure 5, describe the crank pin circle O A E B to any scale, and lay off $O\,G = \frac{r^2}{2l}$. Find the auxiliary centre as before by drawing the short arc

O O′ with E as a centre. Then describe the second circle with O′ as a centre and r as a radius. For any crank position O R′, project R′ upon O A at Q′; then C Q′ is the distance moved by the piston from the dead point. It will be observed that the first circle with O as centre is the crank pin circle, while the second circle determines the piston stroke C D. Only half the circle is required in all cases, as the return stroke is exactly like the outward stroke with the order of events reversed.

The pressure absorbed or given out by the acceleration of the reciprocating parts may be obtained by use of the equation

$$S = r\,(1 - \cos\,\theta) + \frac{r^2}{2l}\sin^2\theta$$

$$\frac{ds}{dt} = v = r\,\sin\,\theta\frac{d\theta}{dt} + \frac{r^2}{l}\sin\,\theta\,\cos\,\theta\frac{d\theta}{dt}$$

$$v = \frac{\pi N}{30}\left(r\,\sin\,\theta + \frac{r^2}{2l}\sin\,2\theta\right)$$

It may be noted in passing that this is the same expression as that for the moment with $\frac{\pi N}{30}$ substituted for $p\,A$. Hence the equivalent crank arm is also the velocity of the piston to some scale.

$$\frac{dv}{dt} = \frac{d^2s}{dt^2} = \frac{\pi N}{30}\left(r\,\cos\,\theta + \frac{r^2}{l}\cos\,2\theta\right)\frac{d\theta}{dt}$$

$$\frac{d^2s}{dt^2} = \frac{\pi^2N^2}{900}\left(r\,\cos\,\theta + \frac{r^2}{l}\cos\,2\theta\right)$$

The pressure required to produce this acceleration is, then,

$$F = \frac{W}{g}\cdot\frac{\pi^2N^2}{900}\left(r\,\cos\,\theta + \frac{r^2}{l}\cos\,2\theta\right)$$

If this be reduced to the pressure per square inch on the piston for more ready application to the indicator card, we get

$$\frac{F}{A} = \frac{W}{Ag}\,\frac{\pi^2N^2r}{900}\left(\cos\,\theta + \frac{r}{l}\cos\,2\theta\right)$$

This expression may be represented graphically precisely as in the diagram for finding the moment arm. In figure 6 take

the radius of the circle O A E B $= \frac{W}{Ag} \cdot \frac{\pi^2 N^2 r}{900}$ to any scale. This is simply the centrifugal force of $\frac{W}{A}$ when placed at the centre of the crank pin. O G must be laid off equal to $\frac{W}{Ag} \cdot \frac{\pi^2 N^2 r^2}{900 \; l}$.

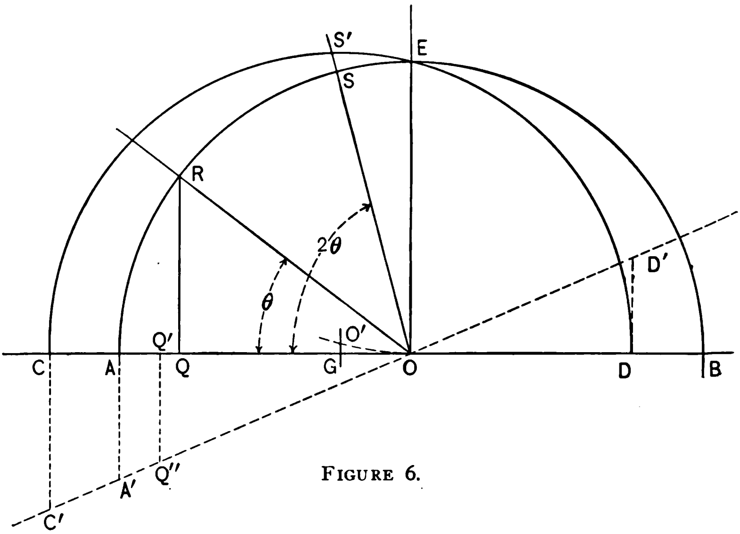


FIGURE 6.

If the circles be constructed as in the figure, the value of $\frac{F}{A}$ for any crank position O R is found as follows: Let A O R $= \theta$ and draw O S′, making the angle 2θ with O A. Project R upon O A, and lay off to the left of its projection, Q, the intercept S S′. We thus get O Q′ for the value of $\frac{F}{A}$ at the crank angle A O R. The intercept between the two circles becomes 0 when $2\theta = 90°$ or $270°$; that is, when the crank angle is 45° or 135°. Between these two angles its value is negative and is laid off to the right of the projection of R upon A B. When $\theta = 90°$, $2\theta = 180°$; and for greater values the angle simply returns on itself to save drawing the other half of the diagram.

In case we desire to use the circles already drawn for the equivalent crank arm, we can make the same construction for the acceleration pressure; but the scale may not be a convenient one. We therefore lay off A A′ perpendicular to O A and equal to $\frac{W}{Ag}\frac{\pi^2 N^2}{900} r$ to the same scale as the indicator card; then through A′ draw the line C′ A′ O D′. The value of $\frac{F}{A}$ for the point Q′ is found to the scale of the indicator card by drawing Q′ Q″ perpendicular to O A, cutting O C′ in Q″. Then is Q′ Q″ $= \frac{F}{A}$. This value must be subtracted from or added to the effective steam pressure for the corresponding position of the piston, and we thus obtain the pressure available for transmission to the crank pin. It must be remembered that the effective steam pressure is equal to the difference between the advance pressure on one side of the piston and the back pressure on the other side.

The crank shaft moment is now found by multiplying the available pressure by the area of the piston and by the equivalent crank arm as given in figures 3 and 4. Expressed algebraically, this is

$$M = A \left\{ p - \frac{W}{Ag}\frac{\pi^2 N^2}{900} r \left(\cos\theta + \frac{r}{l}\cos 2\theta \right) \right\} \left\{ r \sin\theta + \frac{r^2}{2l} \sin 2\theta \right\}$$

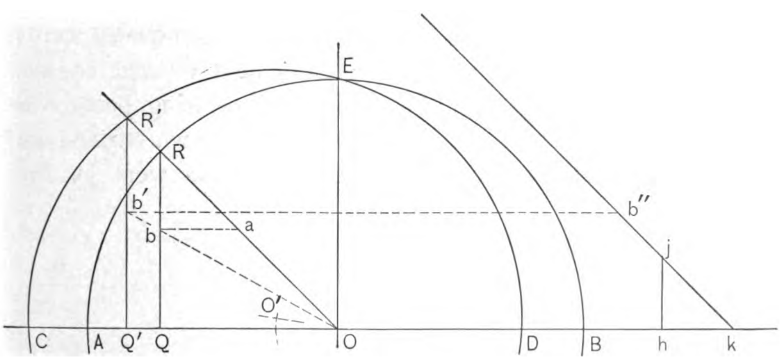


FIGURE 7.

We may obtain this moment graphically by means of the circles in either of the two figures mentioned. Let figure 7

be constructed exactly like figure 4. Lay off O a on the radius O R and draw R Q and R′ Q′. Project *a* horizontally upon R Q and draw O b produced to b′. Then will Q′ b′ represent the value of M to some scale for the pressure on one square inch of the piston. This may be reduced graphically to foot tons for the entire pressure on the piston by drawing a scale line at some convenient place outside of the circles. To the right of the figure draw j h perpendicular to O B produced and equal to q pounds to the same scale as the indicator card. Then find the value $\frac{qAr}{2240}$ and lay it off to any scale equal to j k with one end at j and the other end on O B produced. Project b′ horizontally upon k j produced, and we get k b″ for the turning moment in foot tons to the selected scale. It is now easy to get a number of values of the moment and to construct a curve referred to rectilinear axes or to the centre of the crank pin circle as a pole. In the above method only two quantities require arithmetic work, the expression $\frac{W}{Ag}\frac{\pi^2 N^2}{900}r$ for centrifugal force and the quantity $\frac{qAr}{2240}$ for the moment on the crank shaft when q is the pressure per square inch on the piston and r is the equivalent crank arm.

The graphic construction of a curve of crank-shaft moments for one revolution is shown in figure 8. The indicator cards were taken from the low pressure cylinder of a vertical engine. Consequently, the dead weight of the reciprocating parts was added to the effective steam pressure on the down stroke, and subtracted on the up stroke. The dimensions were as follows: —

Diameter of piston = 78″.
Mean area of piston = 4,752 square inches.
Length of crank = 2′.
Length of connecting rod = 9′.
Revolutions per minute = 74.3.
Weight of reciprocating parts = 13,168 lbs.

The centrifugal force of the reciprocating parts consid-

ered as concentrated at the centre of the crank pin is $\frac{W}{Ag}\frac{\pi^2N^2}{900}r = 10.40$ lbs.

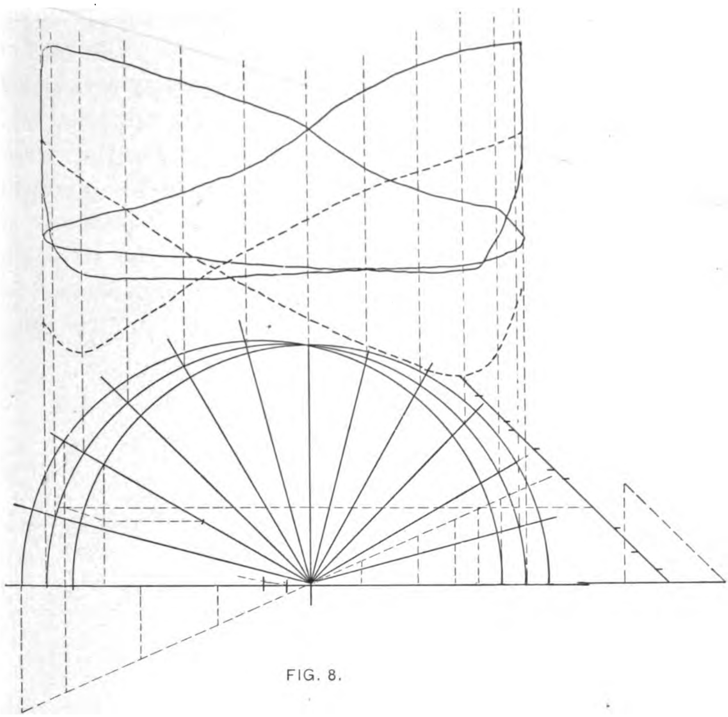


FIG. 8.

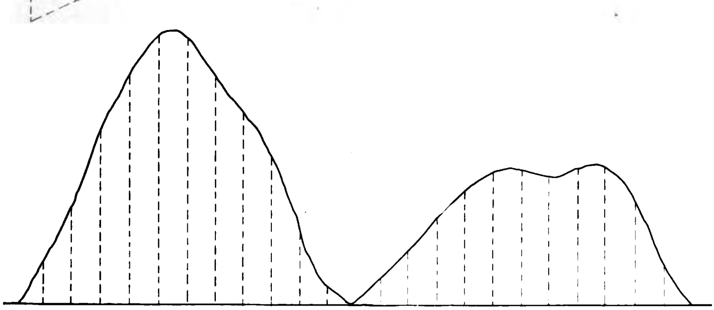


FIGURE 8.

The dead weight of the reciprocating parts per square inch is $\frac{W}{A} = 2.77$. This can be laid off at once on the card as a pressure, keeping in mind that it always acts downward.

The steps in the construction are self-evident, and no further explanation is given here.

The motion of the connecting rod differs from that of the other reciprocating parts in that it has oscillation through a considerable angle in addition to reciprocation. One end of the rod moves in a circle with an angular velocity assumed to be uniform, and the other end moves in a straight line with a variable velocity. An algebraic expression for the motion of any part of the rod, and the value of the forces producing the motion, may be found without difficulty. The forces are necessarily applied only at the crank pin and the wrist pin. They are represented in figure 9 by their components. Let P be the component at the crank pin parallel to the centre

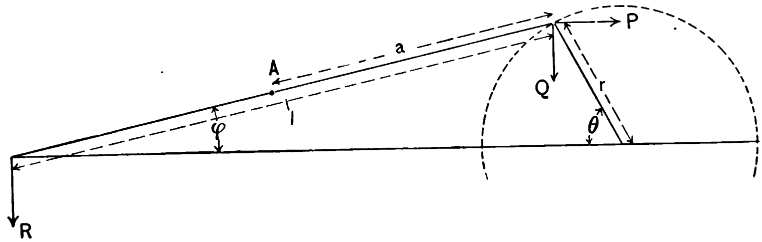


FIGURE 9.

line of the engine, and Q and R the components at the crank pin and the wrist pin respectively perpendicular to the centre line. The part of the rod acted upon by these forces is supposed to be the thin section at A, whose distance from the centre of the crank pin is a, and whose volume is unity. Let w be its weight. The forces P and Q have a resultant which meets the line of the force R at the point D (figure 10): hence the line D A must represent the direction of the resultant force applied to A, and it must also be the direction of its acceleration. If, then, the acceleration can be determined, all the forces may be found by the ordinary triangle of forces; or, if the components of the acceleration be taken, the values of P, Q, and R follow at once. Let O be the origin, and take the axes X and Y respectively in the centre line of the engine, and

at right angles to it. The co-ordinates of A with reference to X and Y are

$$x = -(r\ cos\ \theta + a\ cos\ \phi).$$

$$y = \frac{l-a}{l} r\ sin\ \theta.$$

As before, $r\ sin\ \theta = l\ sin\ \phi$, $sin\ \phi = \frac{r}{l} sin\ \theta$

and $cos\ \phi = \left(1 - \frac{r^2}{l^2} sin^2\ \theta\right)^{\frac{1}{2}} = 1 - \frac{r^2}{2l^2} sin^2\ \theta$, approximately.

$$x = -\left(r\ cos\ \theta + a\left(1 - \frac{r^2}{l^2} sin^2\ \theta\right)^{\frac{1}{2}}\right)$$

$$\frac{dx}{dt} = v_x = r\ sin\ \theta \frac{d\theta}{dt} + \frac{ar^2}{2l^2} \frac{sin\ 2\theta}{\left(1 - \frac{r^2}{l^2} sin^2\ \theta\right)^{\frac{1}{2}}} \frac{d\theta}{dt}$$

$$\frac{dx}{dt} = \frac{\pi N r}{30}\left(sin\ \theta + \frac{ar}{2l^2} \frac{sin\ 2\theta}{\left(1 - \frac{r^2}{l^2} sin^2\ \theta\right)^{\frac{1}{2}}}\right)$$

$$\frac{d^2x}{dt^2} = \frac{\pi^2 N^2 r}{900}\left(cos\ \theta + \frac{ar}{l^2} \frac{cos\ 2\theta + \frac{r^2}{l^2} sin^4\ \theta}{\left(1 - \frac{r^2}{l^2} sin^2\ \theta\right)^{\frac{3}{2}}}\right)$$

$$\frac{d^2x}{dt^2} = \frac{\pi^2 N^2 r}{900}\left(cos\ \theta + \frac{ar}{l^2} cos\ 2\theta\right), \text{ approximately.}$$

$$\frac{dy}{dt} = \frac{l-a}{l} \frac{\pi N r}{30} cos\ \theta.$$

$$\frac{d^2y}{dt^2} = -\frac{l-a}{l} \frac{\pi^2 N^2 r}{900} sin\ \theta.$$

From these equations the value of P and of $Q+R$ may be found.

$$P = \frac{w}{g} \cdot \frac{d^2x}{dt^2} = \frac{w}{g} \cdot \frac{\pi^2 N^2 r}{900}\left(cos\ \theta + \frac{ar}{l^2} cos\ 2\theta\right)$$

$$-(Q+R) = \frac{w}{g} \frac{d^2y}{dt^2} = -\frac{w}{g} \frac{l-a}{l} \frac{\pi^2 N^2 r}{900} sin\ \theta$$

$$Q+R = \frac{w}{g} \cdot \frac{l-a}{l} \frac{\pi^2 N^2 r}{900} sin\ \theta.$$

Since these forces are the components of a force applied at A to produce the acceleration, the force P applied horizontally at the crank pin will have the same value, and the force $Q+R$

will be the algebraic sum of the vertical forces applied at the crank pin and wrist pin.

Taking moments about C,

$$Pr \sin\theta + Ql \cos\phi = P\frac{l-a}{l} r \sin\theta + (Q+R)(l-a)\cos\phi$$

$$Q = (Q+R)\frac{l-a}{l} - P\frac{ar}{l^2}\frac{\sin\theta}{\cos\phi}$$

$$Q = (Q+R)\frac{l-a}{l} - P\frac{ar}{l^2}\frac{\sin\theta}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}}$$

The value of R may be found by taking moments about B, and we thus obtain the pressure on the slides.

$$Rl\cos\phi = Pa\sin\phi + (Q+R)\,a\cos\phi$$

$$R = P\frac{a}{l}\tan\phi + (Q+R)\frac{a}{l}$$

$$R = P\frac{ar}{l^2}\frac{\sin\theta}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}} + (Q+R)\frac{a}{l}$$

The turning moment on the crank shaft to produce the motion of the part of the connecting rod under consideration may be found by taking moments about O.

$$M = Pr\sin\theta - Qr\cos\theta$$

Then, putting in the value of Q, we have

$$M = Pr\sin\theta\left(1+\frac{ar}{l^2}\frac{\cos\theta}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}}\right) - (Q+R)\frac{l-a}{l}r\cos\theta$$

$$M = Pr\sin\theta\left(1+\frac{ar}{l^2}\cos\theta\right) - (Q+R)\frac{l-a}{l}r\cos\theta, \text{ approximately.}$$

If the entire mass of the connecting rod could be concentrated at its centre of gravity, the values of P, Q, and R could be found by substituting for a the distance of the centre of gravity from the centre of the crank pin. Let it be a_1 and let W be the weight of the entire rod: then

$$P_1 = \frac{W}{g}\frac{\pi^2N^2r}{900}\left(\cos\theta + \frac{a_1r}{l^2}\cos 2\theta\right)$$

$$Q_1 + R_1 = \frac{W}{g}\frac{l-a_1}{l}\frac{\pi^2N^2r}{900}\sin\theta$$

The linear acceleration of the rod is determined by these two equations; but, inasmuch as the rod really has its mass distributed on each side of the centre of gravity, the separate values of Q_1 and R_1 can be found only by a consideration of the angular acceleration. The line of the resultant of P_1, Q_1, and R_1 will not pass through the centre of gravity, as shown in Figure 11. It will thus be equivalent to an equal force acting in the same direction through the centre of gravity, and a couple. The moment of this couple can be found by taking the moments of P_1, Q_1 and R_1, about the centre of gravity, of the rod.

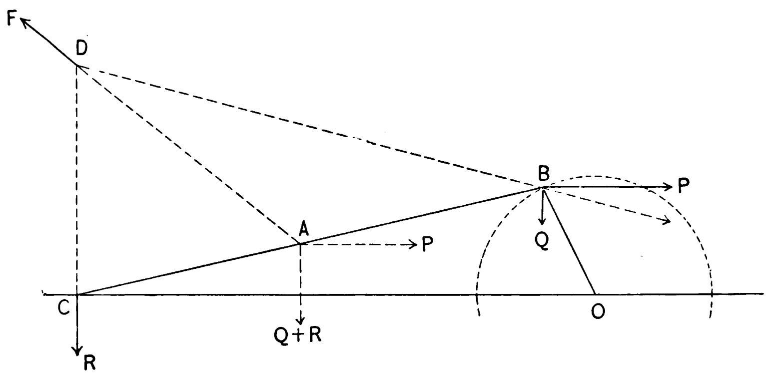


FIGURE 10.

Thus $M_1 = -P_1\, a_1 \sin\phi - Q_1\, a_1 \cos\phi + R_1\,(l - a_1)\cos\phi$

$$M_1 = -P_1\, a_1 \sin\phi - (Q_1 + R_1)\, a_1 \cos\phi + R_1\, l \cos\phi$$

Replacing $\sin\phi$ and $\cos\phi$ by their equivalents in terms of $\sin\theta$, and we have

$$M_1 = -P_1 \frac{a_1 r}{l} \sin\theta - (Q_1 + R_1)\, a_1 \left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}$$

$$+ R_1 l \left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}$$

To find an expression for the angular acceleration and thus a value of M_1, we take the value of

$$\sin\phi = \frac{r}{l}\sin\theta$$

$$\frac{d\phi}{dt}=\frac{r}{l}\frac{\cos\theta}{\cos\phi}\frac{d\theta}{dt}$$

$$\frac{d\phi}{dt}=\frac{\pi N r}{30 l}\frac{\cos\theta}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}}$$

and

$$\frac{d^2\phi}{dt^2}=-\frac{\pi^2 N^2 r}{900 l}\cdot\frac{\left(1-\frac{r^2}{l^2}\right)\sin\theta}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{\frac{3}{2}}}$$

From Mechanics, the value of $\frac{d^2\phi}{dt^2}$ is given by the moment of the force divided by the moment of inertia of the rod. Hence

$$\frac{d^2\phi}{dt^2}=\frac{M_1}{\frac{W}{g}k^2}$$

in which k is the principal radius of gyration. From this equation we have

$$M_1=\frac{W}{g}k^2\frac{d^2\phi}{dt^2}$$

and $$\frac{W}{g}k^2\frac{\pi^2 N^2 r}{900 l}\frac{\left(1-\frac{r^2}{l^2}\right)\sin\theta}{\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{\frac{3}{4}}}=$$

$$P_1\frac{a_1 r}{l}\sin\theta+(Q_1+R_1)\,a_1\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}-R_1 l\left(1-\frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}$$

The values of P_1, Q_1, and R_1 are now determinable from the three equations containing them. Having P_1 and Q_1, the moment on the crank shaft necessary to produce the motion of the connecting rod is determined.

A much easier method of finding this moment is obtained by considering the weight of the connecting rod concentrated at two points such that the moment of inertia remains the same as in the distributed mass. Let W_1 and W_2 be the weight of the two parts respectively concentrated at points b_1 and b_2 from the centre of gravity, then

$$W_1 + W_2 = W$$
$$b_1 W_1 - b_2 W_2 = O$$
$$b_1^2 W_1 + b_2^2 W_2 = Wk^2$$

From these equations,

$$W_1 = \frac{b_2 W}{b_1 + b_2} \text{ and } W_2 = \frac{b_1 W}{b_1 + b_2}$$

also $\quad b_1 b_2 = k^2$

If W_1 be placed at the crank pin centre, its acceleration is along the crank; and consequently it can exert no moment on the shaft. The accelerating or retarding moment is then wholly due to W_2. In this case $b_1 = a_1$ and $b_2 = \frac{k^2}{a_1}$.

Let $a_2 = a_1 + b_2 =$ the distance of W_2 from the centre of the crank pin. The forces acting at the crank pin are then obtainable by the equations already given.

$$P_2 = \frac{W_2}{g}\frac{\pi^2 N^2 r}{900}\left(\cos\theta + \frac{a_2 r}{l^2}\cos 2\theta\right)$$

$$Q_2 + R_2 = \frac{W_2}{g}\frac{l - a_2}{l}\frac{\pi^2 N^2 r}{900}\sin\theta$$

$$Q_2 = (Q_2 + R_2)\frac{l - a_2}{l} - P_2\frac{a_2 r}{l^2}\frac{\sin\theta}{\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}}$$

$$R_2 = (Q_2 + R_2)\frac{a_2}{l} + P_2\frac{a_2 r}{l^2}\frac{\sin\theta}{\left(1 - \frac{r^2}{l^2}\sin^2\theta\right)^{\frac{1}{2}}}$$

and the retarding or accelerating moment on the crank shaft due to the weight of the connecting rod is thus

$$M_2 = P_2 r \sin\theta - Q_2 r \cos\theta$$

or $\quad M_2 = P_2 r \sin\theta\left(1 + \frac{a_2 r}{l^2}\cos\theta\right) - (Q_2 + R_2)\frac{l - a_2}{l} r\cos\theta.$

These values of P_2, Q_2, and M_2 may be found graphically with the auxiliary circle used in determining the total effort on the crank shaft. The part of the rod W_2 is sufficiently near the cross-head to consider it without material error as one of the reciprocating weights. Its usual value is between $\frac{1}{4}$ and $\frac{1}{2}$ the total weight of the rod, and a fair average value

is $\frac{W}{3}$. The problem of the connecting rod thus resolves itself into finding the effect of a reciprocating weight, $\frac{W}{3}$, at the cross-head, upon the crank-shaft effort, and the effect of a centrifugal force due to $\frac{2W}{3}$ concentrated at the crank pin. The latter can be completely balanced by an equal and opposite force, the former cannot. An exact expression for the variable centrifugal force acting on the crank and due to the weight W_2 may be obtained from the equation

$$C = P_2 \cos \theta + Q_2 \sin \theta.$$

The centrifugal force for $W_1 = \frac{2W}{3}$ is constant and equal to $\frac{W_1}{g} \frac{\pi^2 N^2 r}{900}$.

CONCERNING VENICE.

WALTER DANA SWAN.

Instructor in Architecture, Harvard University.

It is a surprise to the average student visiting the city for the first time that Venice is so interesting architecturally. He is prepared somewhat for that picturesque quality made known to him by the traveller, the artist, and literature generally. The enthusiasm of Ruskin's many pages has probably seemed overwrought and therefore unreal to him, reading in the alcove of an American library. To be sure, in the face of the reality,— in St. Mark's Church, for instance,— that same ardent enthusiasm seems no more than a fitting tribute. The same sentiments were briefly expressed, by one who wasted no words, in the remark, "How those old Venetians did go in for beauty!"

But to find such interesting examples of the great periods of architectural history from the eighth century to the eighteenth makes each walk or gondola trip a delightful revelation. On the Grand Canal itself one may pass, so to speak, from the eleventh to the eighteenth centuries in the time which it takes a gondolier to ferry across from the Fondaco de' Turchi to the Church of San Geremia almost opposite.

One visitor to Venice during the last summer thought how interesting it would be to follow some historical sequence in an orderly fashion, with a sketch of a capital or two, an archivolt or so, and other details,— from each period. But Venice in reality, on the first visit at least, defies all thought of order or sequence, and calls one here for color, a picture, or mosaic, and there for a bit of carving or for a sunset; and it seems right and fitting to surrender.

So one's sketch book has merely notes of this and that: perhaps a Renaissance doorway under a Gothic arch of the Ducal Palace; possibly a gilded capital in St. Mark's, with an attempt to suggest as a background the earliest mosaics with their fine

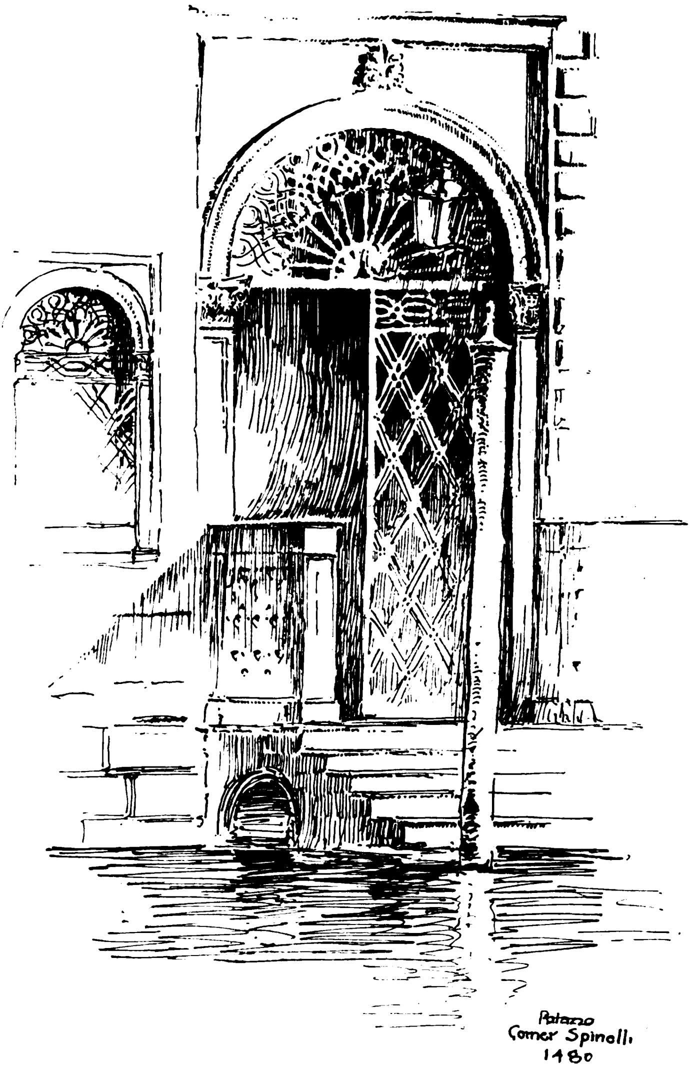


FIGURE I.

architectural lines; or, again, one of the attractive exterior niches on San Zaccaria,— the work of the earliest of the Lombardi; and so on. It is what appeals to one at the moment; and fortunate is he who can see, or rather feel, in what the real charm or beauty of the subject lies.

FIGURE 2.

Looking back upon it all, it appears that there were many interesting questions which suggested themselves,— for instance,

the connection of the Byzantine and Gothic styles with the work of the Lombardi, that famous guild, or family, who did so much to beautify the Venice of the Renaissance, if not to add to its constructive knowledge. Did the semicircular pediment of the Scuola di San Marco originate from the proximity to the pediments of St. Mark's, both suggested as in figure 5, or was it a natural step from the arched opening, as shown in the sketch of the doorway of the Palazzo Corner-Spinelli, built in 1480 (see figure 1), with the fronton of the portion of the old portal of 1481 of the Scuola di San Giovanni Evangelista intermediate?

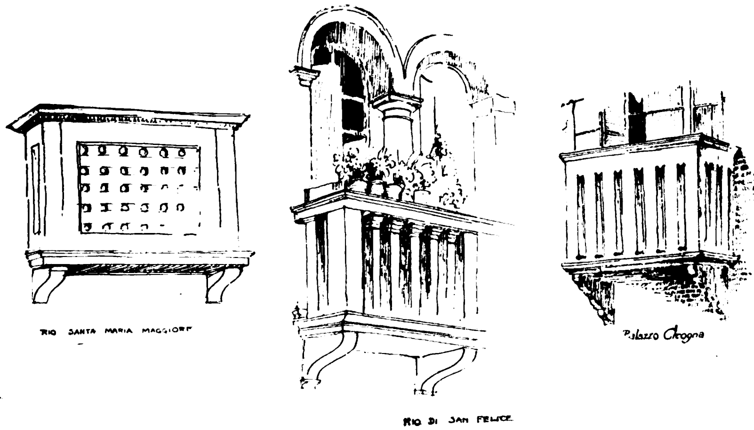


FIGURE 3.

(See figure 2.) And what relation was there between these two motives and that almost similar one of the Florentine, Benedetto da Majano, in his doorway,* to the Sala d' Udienza in the Palazzo Vecchio in Florence, also executed in 1481? We are told that Benedetto visited Hungary in 1488. Why not previously, since he seems to have had influential friends there? And possibly he stopped long enough in Venice on his way home to watch the Lombardi at work on their delicate arched open-

* There is a cast of this doorway in the large drawing-room in Robinson Hall.

ings and circular tympanums. Or was the motive originally a Florentine one, not derived, as Anderson supposes, from the Byzantine surroundings?

Not only to illustrate this train of thought were the accompanying sketches made, but also because they seemed characteristic subjects for broad pen and ink treatment; and the pleasure of doing them is their only excuse. They are drawn from sketches and photographs made during a two weeks' stay in Venice.

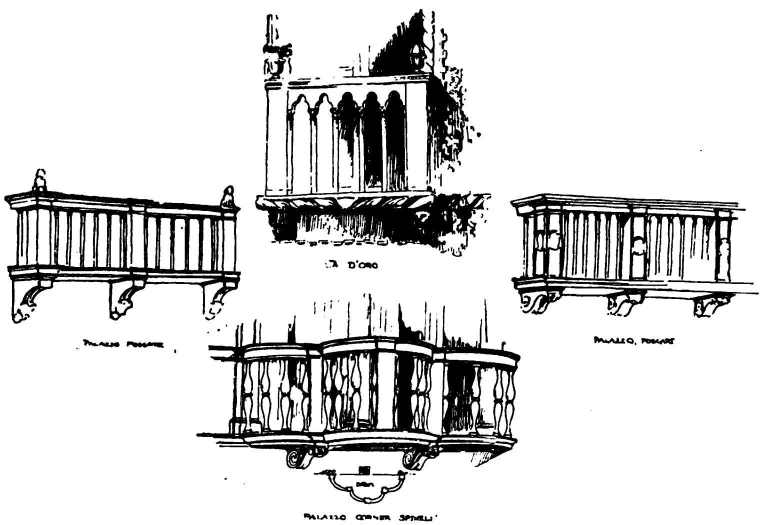


FIGURE 4.

The small campanile in the sketch of the portal of the Scuola di San Giovanni Evangelista is one of the many such in the city; and the greatest of them is — or was — the campanile of St. Mark's, which lay in ruins at the time of our visit. The tower fell on the morning of the 14th of July,— the anniversary so dear to French freedom,— the fall of the Bastile. At least one poem was published in the Italian papers, contrasting the fall of one nation's curse with that of the pride of a former republic.

It was some two weeks before any really sufficient reasons were given for the catastrophe. The Italian papers were full of non-technical surmises as to earthquakes, the rotting of piles, the results of former strokes of lightning, dredging the neighboring canals, etc.; but, until the report of Signor Boni, the architect now in charge of the rebuilding, who has been officially investigating the subject, is published, the cause of the fall will

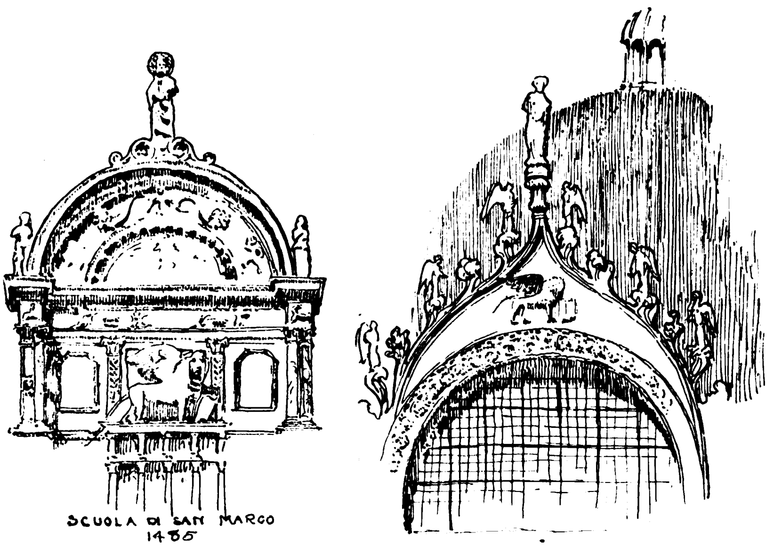


FIGURE 5.

continue to be largely conjectural.* It has seemed evident to certain American architects, however, who were in Venice at the time and immediately afterwards, or who had studied the campanile previously, that the brick walls of the original structure were not really adequate for the addition made to its height in the sixteenth century.

A careless cutting of the lower part of this old portion near the base a short time previous to the fall seemed to hasten the

***Since this article was written in the early fall, a fairly complete statement of the bibliography on this matter was published in the *Quarterly Bulletin of the American Institute of Architects*, Vol. III number 3.**

disintegration of the wall above. Another interesting theory is that this cutting affected the stability of the spiral series of arches which supported the staircase, or, rather, inclined plane, which extended from the bottom nearly to the top of the tower. One can imagine the effect of a corkscrew pressure upon the faulty masonry at the bottom, the great resulting cracks, and the final collapse.

The son of a well-known Boston architect was in the campanile an hour or so before its fall, climbed a short distance up, and saw St. Mark's through one of the great vertical cracks which had opened ominously during the night. A graduate of the Department of Architecture, now a holder of the Rotch Travelling Fellowship, had been examining these cracks at a safe distance fifteen minutes before the tower fell. He had left the Piazza, much to his later regret, and was in a neighboring square when the bearer of the evil tidings rushed into it, pale with excitement and covered with finely powdered yellow dust. There resulted somewhat of a panic in the narrow Merceria, or alley, leading to the Piazza. There all Venetians wished to see what they could not otherwise seem to realize,—that their cherished campanile could really fall.

The Boston architect referred to was that morning making a sketch of the Cà d' Oro, a mile or so away on the Grand Canal. The angel on the summit of the campanile was in his composition, I believe. Certainly, he saw it one moment, and the next it was gone. He thought that his gondola must have swung with the tide; but, soon after, the crowds rushing across the near bridge towards the Piazza gave him the clew to the strange disappearance of the angel. He went with all others to the scene, and describes vividly the changed Piazza, with its great mountain of powdered débris, and the beautiful colors of St. Mark's marbles covered with the strange pall of white dust.

One man only whom we met in Venice actually saw the campanile fall, although I have heard of others since. He was a young Italian naval officer, stationed on his ship on duty in the harbor of Venice. While at morning drill with his crew, his eye caught the tottering of the angel, and the next instant arose the

great cloud of mortar dust almost as high as the angel had stood. He immediately sent the crew to the spot, prepared for hospital duty. Wonderfully enough, no one was injured in the slightest by the disaster. Solemn mass in thanksgiving of this fact was afterwards impressively observed in St. Mark's Church, itself miraculously spared by the collapse rather than the pitching for

FIGURE 6.

ward of the tower. One child who saw the fall from the other end of the Piazza described it graphically as a "sitting down." The Venetians are extremely superstitious about the whole affair; and that the crowning angel should have fallen face downward, almost on the threshold of St. Mark's, seemed quite as important to them as the fact that most of Sansovino's carving on the Logetta buried beneath the ruins was quite unharmed. It seemed from appearances that most of the tower building in

Venice had been somewhat careless, for there is hardly one of the many campanile which does not lean.

There was much excitement about this matter during our two weeks' visit; and once we were called to see the tower of San Stefano, which was expected to fall before we could reach the spot, but which up to the latest accounts is still standing, propped and guarded, to be sure.

FIGURE 7.

Some one should make in Venice a careful study of certain interesting architectural features very closely connected with Venetian life, the entrances and the balconies. The former always seem hospitable, appropriate, and usually charming in detail; and the latter are so absolutely essential and "intimate," as the French say, whether on a stately palace or on simple houses on the side canals. They are always suggestive to one who cares for design, and it is surprising to find even the simpler ones of marble.

One cannot think of Venice, after being there, without the rich beauty of St. Mark's. It is the first and last Venetian glory that one goes to see. It must have influenced every artist, architect, and designer who has entered it, from the eleventh century to the present day, from the builder of the French church of St. Front in Perigeux, who copied its plan, to the late William Morris, who said, I believe, that the Byzantine art seemed to him that of the real Renaissance. With that thought in mind it is interesting to compare the cold, severe formality of a fine work of the later Renaissance, Palladio's San Giorgio Maggiore across the Basin, with the harmonious warmth of the Venetian Byzantine church. One feels the distinct difference between the love of correctness and precision of ideas, with that "going in for beauty" which seems the keynote of the outward expression of the earlier Venice. It is the latter which the modern student has often heard described, but never experienced so full elsewhere in Italy as here in Venice.

ARMATURE DROP AND REGULATION OF ALTERNATORS.

COMFORT AVERY ADAMS,

Assistant Professor of Electrical Engineering.

(Continued from Vol. I. page 275.)

There are two much-used modifications of the above-described general method, which will now be considered.

Both of these modifications involve further approximations which introduce considerable errors except when applied to machines of extreme proportions. The approximations in both cases are based upon the similarity between the effects of armature reaction and armature reactance, since the magnitudes of the armature m. m. f., A, and of the reactive drop Ix_a, are both proportional to I, and their effect upon the quantities of which they are components is in each case increased by a decrease in power-factor; *i.e.*, by an increase of the lag of I behind E, see figure 7.

E. M. F. Method.

In this method the above-mentioned similarity between the effects of armature reaction and armature reactance is taken advantage of by converting the armature reaction into its equivalent armature reactance, which is added to the true reactance, and thus reduces the whole problem to a reactance basis.

The reduction is made as follows: starting with the general diagram, figure 28, extend Ix_a to E_F, $\overline{OF}$ to ϕ_F, and $\overline{OA}$ to ϕ_A, so that $E_F \div E = F \div R = \phi_F \div \phi$, and $\phi_F \div F = \phi \div R = \phi_A \div A$. In other words, the triangles $\overline{OE_a E_F}$, $\overline{ORF}$, and $\overline{O\phi\phi_F}$ are similar. From these proportions and similarities we get the following interpretations: —

ϕ_F is the flux which would be produced by the m. m. f., F, in a magnetic circuit having a reluctance equal to that of the circuit in which the m. m. f., R, produces the flux ϕ. Similarly, ϕ_A is the flux which would be produced by the m. m. f., A, in a magnetic circuit of the same reluctance.

E_F is the e. m. f. which would be induced by the cutting of the flux ϕ_F, and Ix_A that which would be induced by the cutting of the flux — ϕ_A, under the same conditions under which the e. m. f., E_a, is induced by the cutting of the flux, ϕ.

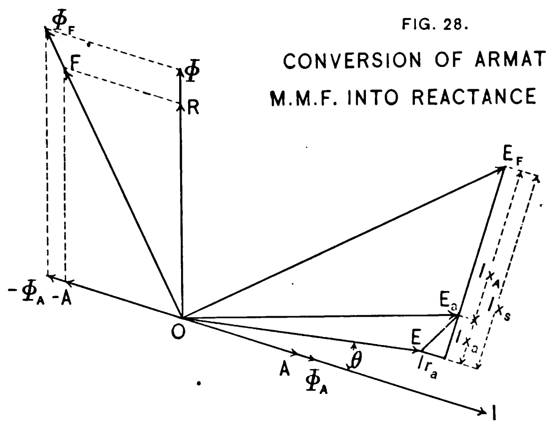


FIG. 28.
CONVERSION OF ARMATURE M.M.F. INTO REACTANCE E.M.F.

We may thus look upon the flux, ϕ_F, as made up of two components, one, ϕ, necessary to generate the e. m. f., E_a, and the other — ϕ_A, the cutting of which generates the e. m. f., Ix_A.

Similarly with the e. m. f.'s; E_F may be considered as the vector sum of E_a and Ix_A, or as the vector sum of E, Ir_a, Ix_a, and Ix_A. The last two components, being in the same direction, may be combined in one, Ix_s, where $x_s = x_a + x_A$. This total reactance x_s is called the "synchronous reactance," which is thus the sum of the leakage reactance, x_a, and a fictitious reactance, x_A, which represents the armature reaction; *i.e.*, the corresponding reactance e. m. f., Ix_A, is the e. m. f. which would be induced by the cutting of the fictitious flux — ϕ_A, which flux would be produced by the m. m. f. — A if acting in a magnetic

circuit of the same reluctance as that in which the m. m. f., R, produces the flux, ϕ.

In reality the m. m. f. — A, which is one of the components of the actual total m. m. f., F, is consumed in balancing the armature m. m. f., A.

The equivalence of the reactance x_A to the armature reaction is thus dependent upon the assumption that the reluctance of

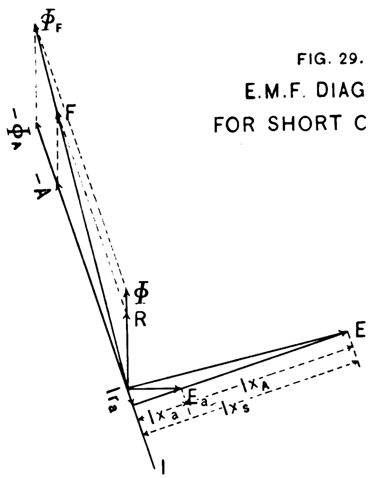


FIG. 29.
E.M.F. DIAGRAM
FOR SHORT CIRCUIT.

the magnetic circuit is constant at the value corresponding to the actual flux, ϕ. The greater the variation in reluctance, the greater will be the error involved in this method. It should be noted, however, that, in cases where the armature reaction is relatively small, the error involved will be correspondingly small, other things being equal.

Measurement of "Synchronous Reactance."

On short circuit the e. m. f. diagram degenerates into that of figure 29, the interpretation of which is similar to that of figure 28.

Each point on the short circuit characteristic gives the value of F corresponding to a particular short circuit current I, and each point on the saturation curve gives the open circuit e. m. f.

FIG. 30.
E.M.F. DIAGRAM
FOR SHORT CIRCUIT
#1.

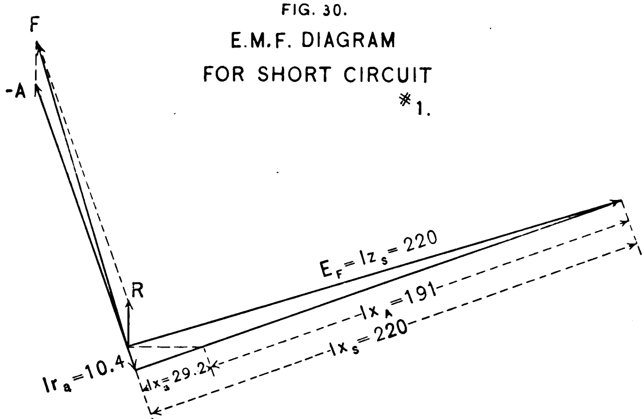


FIG. 31.
E.M.F. DIAGRAM
FOR SHORT CIRCUIT
#2.

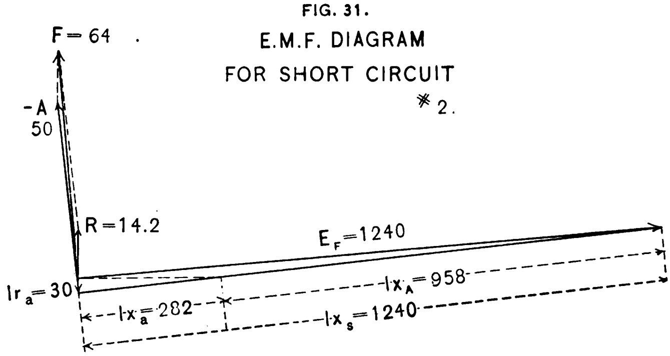
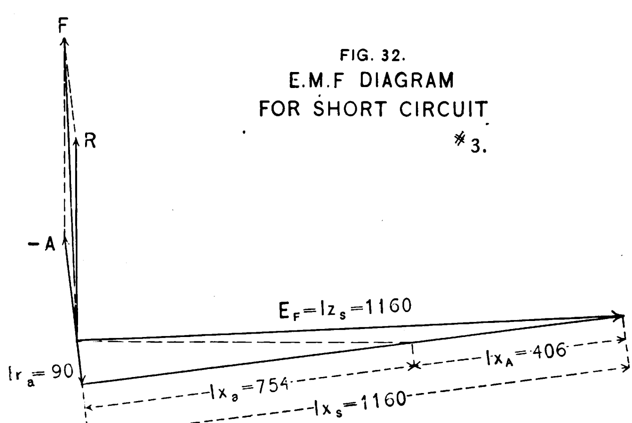


FIG. 32.
E.M.F DIAGRAM
FOR SHORT CIRCUIT
#3.



corresponding to a particular value of F; *i.e.*, the e. m. f. which would be induced if F were the only m. m. f. acting in the magnetic circuit.

From the short circuit characteristic take any value of F and its corresponding short circuit current, I: take from the saturation curve the e. m. f. corresponding to the chosen value of F. This will be $E_F = Iz_s$ on the diagram, where z_s is called the "Synchronous * Impedance."

Knowing I, z_s can be calculated, and, knowing r_a, x_s can be calculated. $z_s = \frac{E_F}{I}$, and $x_s = \sqrt{z_s^2 - r_a^2}$.

In the great majority of cases x_s is so large as compared to r_a that x_s is practically equal to z_s.

Examples of the Determination of z_s and x_s.

Consider the 50 K. W. Alternator of example #1, the saturation and short circuit curves for which are shown in figure 46.

Take the point on the short circuit curve corresponding to an armature current of 160 amperes; the corresponding field current is 13.5 amperes, and the open circuit e. m. f. corresponding to this field circuit is, from the saturation curve, 220 volts.

The synchronous impedance is, therefore, $z_s = 220 \div 160 =$ 1.38 ohms, and the synchronous reactance, $x_s = 1.38$ ohms.

The corresponding short circuit diagram is shown in figure 30. Values of z_s have been calculated for several points on the short circuit curve, and have been plotted in curve III., figure 46.

If the saturation and short circuit curves had been corrected for the remanent magnetism, the synchronous impedance would

* The adjective "synchronous" is applied to the impedance z_s and the reactance x_s because the measurements necessary for the determination of these quantities are made when the machine is running in synchronism with the current in the armature, as distinguished from the impedance and reactance measured by passing an alternating current from some outside source through the armature while the latter is stationary.

have been practically constant within the range of observed values. Without this correction it is nearly constant. But the range of observed values does not ordinarily reach the bend of the saturation curve, and therefore does not correspond to working conditions. As saturation increases, the synchronous impedance decreases, and within the working range it differs for each value of the load and e. m. f. This is one of the sources of error of the e. m. f. method, as will be explained later.

Figures 31 and 32 show the short circuit diagrams for machines #2 and #3, for which the synchronous reactances are 4.13 and 5.8 ohms respectively.

It will be observed in connection with these diagrams that E_F was taken from the saturation curve, and does not necessarily bear the same relation to F that E_a bears to R, except when the saturation curve is a straight line, which is nearly true on the lower part of the curve corresponding to any safe short circuit current.

The difference between the three machines is very marked. In the first the leakage reactance is small, and the synchronous reactance relatively very large, being 7.5 times as great: whereas in the second case the synchronous reactance is 4.4 times as great, and in the third case only 1.54 times as great as the leakage reactance.

It is thus evident that any error due to the inequivalence of Ix_A to the corresponding armature m. m. f. will be much smaller in the last case than in the first.

Examples of the Determination of Excitation and Regulation by Means of the Synchronous Reactance, or E. M. F. Method.

Machine #1. First for full non-inductive load, figure 33. Given: terminal volts, $E = 220$; amperes per phase, 76; $r_a = .065$; $x_s = 1.38$. Then: $Ir_a = 5$ volts; $Ix_s = 105$ volts; and $E_F = \sqrt{(225)^2\ (105)^2} = 248$. The corresponding F is, from the saturation curve, 15.65 amperes, and the regulation is

$$\frac{248 - 220}{220} = 12.7\%.$$

FIG. 33.
NON-INDUCTIVE LOAD.
MACHINE #1.

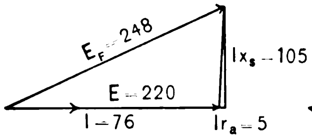


FIG. 34.
INDUCTIVE LOAD.
(P.F. = .8) MACHINE #1.

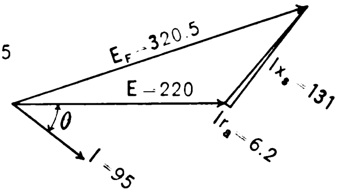


FIG. 35.
NON-INDUCTIVE LOAD.
MACHINE #2.

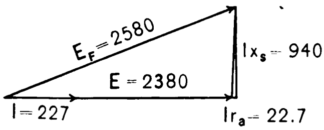


FIG. 36.
INDUCTIVE LOAD.
(P.F. = .8) MACHINE #2.

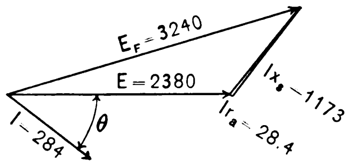


FIG. 37.
NON-INDUCTIVE LOAD.
MACHINE #3.

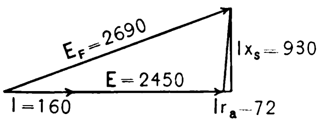
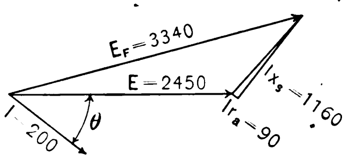


FIG. 38.
INDUCTIVE LOAD.
(P.F. = .8) MACHINE #3.



Inductive load, see figure 34.

Power factor = .8; $I = 95$; $Ir_a = 6.2$; $Ix_s = 131$;

$$E_F = \sqrt{(E \cos \theta_x Ir_a)^2 + (E \sin \theta + Ix_s)^2} = 320.5.$$

From the saturation curve, $F = 24.5$ amperes, and the regulation is $\dfrac{320.5 - 220}{220} = 45.7\%$.

Machine #2. Non-inductive load, see figure 35. Given:

$E = 2380$; $I = 227$; $r_a = .1$; $x_s = 4.13$. Then: $Ir_a = 22.7$; $Ix_s = 940$; $E_F = 2580$; $F = 171$ amperes; and the regulation is 8.4%.

Inductive load, see figure 36. Power factor $= .8$; $I = 284$; $Ir_a = 28.4$; $Ix_s = 1173$; $E_F = 3240$ volts.

It would take such a considerable prolongation of the saturation curve to reach 3240 volts that the approximation of F is very rough; it is probably about 350 amperes; the regulation is, then, 36.2%

Machine #3. Non-inductive load, see figure 37. Given: $E = 2450$; $I = 160$; $r_a = .45$; $x_s = 5.8$. Then: $Ir_a = 72$; $Ix_s = 930$; $E_F = 2690$; $F = 87$ amperes; and the regulation is 9.8%.

Inductive load, figure 38. Power factor $= .8$; $I = 200$; $Ir_a = 90$; $Ix_s = 1160$; $E_F = 3340$; this is again so far beyond the observed part of the saturation curve as to be merely a guess at $F = 250$ amperes; the regulation is, then, 35.6%.

These results are tabulated (table I, page 347) for purposes of comparison. A refers to observed values, B to those calculated by the general method, and C to those calculated by the e. m. f. method. No observed values for the inductive load are available for machines numbers 1 and 2, but the table contains the results of observation and calculation for two large modern European alternators.

Both of these machines are of the revolving field type: #4 has partly closed slots and high saturation; #5 has closed slots and moderate saturation.

Remembering that the magnitude of the error involved in this method is dependent upon the degree of saturation and upon the relative magnitude of the armature m. m. f. transformed into reactance e. m. f., the results given in table I are readily understood.

In machine #1 the degree of saturation is so low that, notwithstanding the large armature m. m. f., the results obtained by this method are fairly accurate for the non-inductive load; but in the case of the inductive load the saturation begins to count, and the discrepancy is considerable.

In machine #3 we have the reverse situation, high saturation and very low armature reaction. The saturation is so high that the results of the e. m. f. method are considerably in error even in the case of non-inductive load, and yet more with the inductive load, although the armature m. m. f. is unusually small.

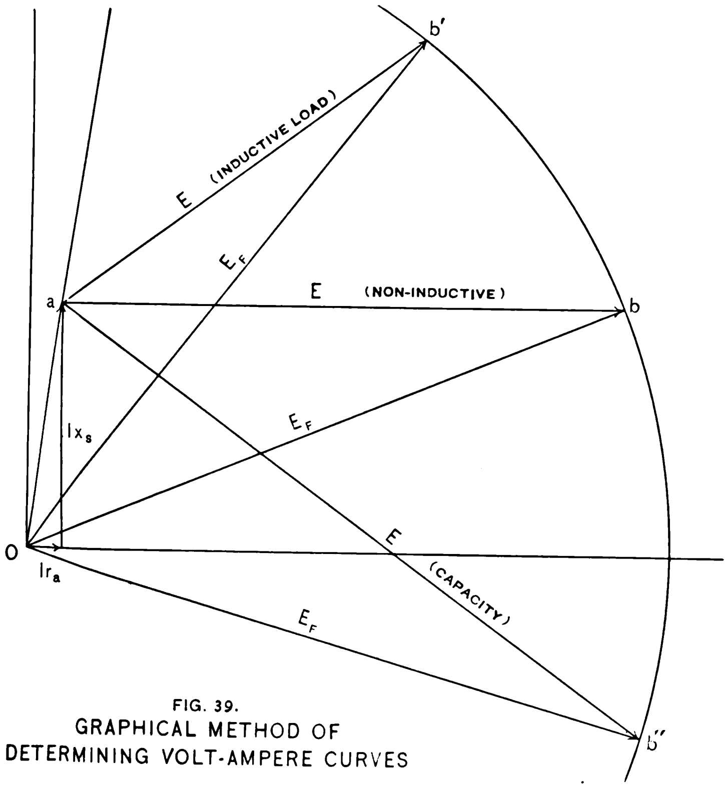


FIG. 39.
GRAPHICAL METHOD OF
DETERMINING VOLT-AMPERE CURVES

In machine #2 the saturation is fairly high, the armature reaction moderate, and the errors very great. The non-inductive regulation is 110% too large, and the inductive regulation 150% too large.

In every case the excitation and regulation determined by the e. m. f. method are too large as compared with the results obtained by the general method or by observation.

There are two reasons for this: first, the synchronous reactance which is usually measured at low reluctance, on the straight part of the saturation curve, is too large and gives too large a value of E_F; second, even though E_F were not too large, the corresponding value of F as taken from the saturation curve

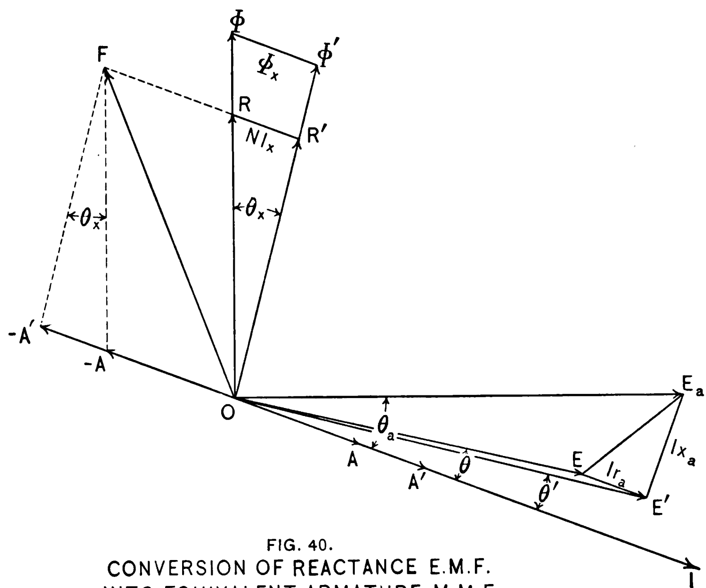


FIG. 40.
CONVERSION OF REACTANCE E.M.F.
INTO EQUIVALENT ARMATURE M.M.F.

would be too large unless the saturation curve were a straight line passing the point E_a and the origin. In the e. m. f. method, contact is made with the saturation curve at a point corresponding to E_F which is larger than the real induced e. m. f., E_a, and therefore corresponds to a higher saturation, higher reluctance, and a disproportionately large value of F.

Thus both of these factors tend to make too large the results obtained by the use of this method, and the error is very considerable except in machines with unusually low saturation and small armature reaction.

The advantage of this method is its great simplicity, which is a considerable advantage in the treatment of more complicated problems, such as synchronous motor operation.

As an example of this simplicity, the volt-ampere curves, such as those of figures 26 and 27, can be very easily obtained by this method, as follows: If F is constant, E_F will be constant. Strike a circle of radius E_F, figure 39; take the phase of the armature current in the horizontal; draw the line Iz_s in its proper direction with respect to I. For every value of I there will be a point a on the line $\overline{oa}$, and, if the load is non-inductive, the horizontal intercept $\overline{ab}$ will be the terminal e. m. f., E. If the current lags by an angle θ, the terminal e. m. f. will be given by the line $\overline{ab}'$ drawn at the proper angle above the horizontal.

If the current leads the e. m. f., E, will be given by the line $\overline{ab}''$ drawn at the proper angle below the horizontal.

Volt-ampere curves obtained in this way would have exactly the same shape as those of figure 48, obtained by the m. m. f. method.

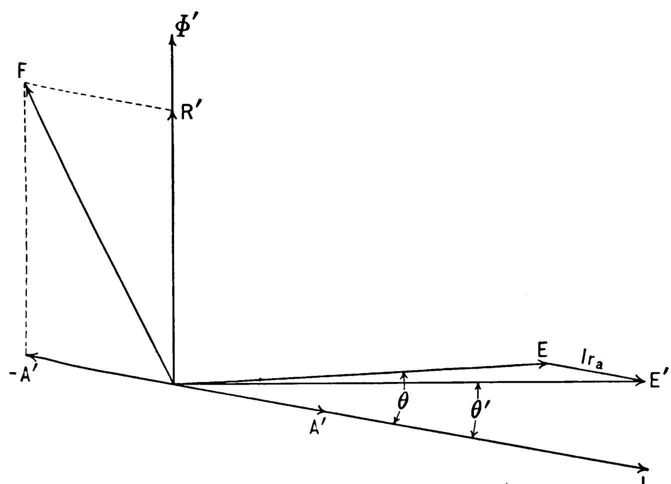


FIG. 41.
AMPERE-TURN OR M.M.F. DIAGRAM

These curves, although inaccurate for any real machine, show in a rough manner the relative variation of E and I. One interesting fact is made evident thereby, namely, that the decrease in p. f. from unity to .8 causes a greater drop in termi-

nal e. m. f. than the decrease from .8 to zero. Another point is the rapid drop of E toward the end. Until within a few years many machines were worked well out on the volt-ampere curve, so that only a moderate current overload would be obtained at short circuit, thus safeguarding the armature from burn-outs; but modern close-regulating machines have a relatively low synchronous impedance, and therefore a relatively high short circuit current, from two and one-half to four times the normal.

M. M. F. Method.

In this method the transformation described above for the e. m. f. method is reversed, and the problem reduced to an m. m. f. basis. The transformation is made as follows: —

Starting with the general diagram, figure 7, make the following changes, figure 40: Draw E′ equal to the vector sum of E and Ir_a; draw ϕ' perpendicular to E′ and of such magnitude that $\phi \div \phi' = E_a \div E'$, *i.e.* ϕ' is that flux which, if cut by the armature winding at normal frequency, would induce the e. m. f., E′; lay off R′ on ϕ' so that $R \div \phi' = R \div \phi$, *i.e.* R′ is the m. m. f. that would produce the flux ϕ' under the same conditions as to magnetic circuit under which R produces ϕ; draw $\phi\phi'$ and call it ϕ_x; draw RR′ and call it $\underline{Ni}_x$; complete the parallelogram OR′F by extending — A to — A′, the difference being $\underline{Ni}_x$.

The triangles OE_aE', or ORR′, and $O\phi\phi'$ are all similar, from which we get the following interpretations: —

ϕ_x is the flux which when cut by the armature winding at normal frequency will induce the e. m. f. $-Ix_a$; *i.e.*, ϕ_x is what we have defined as the armature leakage flux, and ϕ' is the vector sum of ϕ and ϕ_x, which are independently linked with the armature winding.

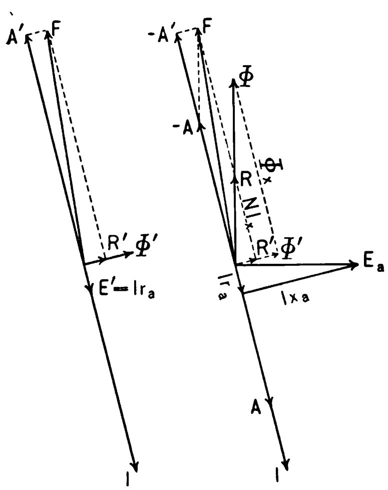
$\underline{Ni}_x$ is the m. m. f. which would produce the leakage flux, ϕ_x, if acting in a magnetic circuit of the same reluctance as that of the circuit in which R produces ϕ; but the path of the leakage flux is not in the main magnetic circuit, and A is the m. m. f.

which actually produces that flux in a magnetic circuit of much higher reluctance. Thus $\underline{Ni}_x$ is purely hypothetical,

A' ($= A + \underline{Ni}_x$) is the m. m. f. which, when subtracted vectorially from F, gives R′. In other words, an armature without leakage, but with an m. m. f. increased from A to A′, would act exactly as the one first considered, other things being equal. The diagram thus modified is shown more clearly in figure 41.

The difference between this diagram and that of figure 7 is briefly: The leakage reactance e. m. f., Ix_a, has been eliminated,

FIG. 42.
AMPERE-TURN OR M.M.F.
DIAGRAM FOR SHORT-CIRCUIT CASE



and for it has been substituted an m. m. f., $\underline{Ni}_x$, which, if acting in a magnetic circuit of reluctance $R \div \phi$, would produce a flux, ϕ_x, which, if cut by the armature winding, would induce the e. m. f., Ix_a; or we have transformed the leakage e. m. f., Ix_a, into an m. m. f. which is equivalent thereto only in the degree that the above assumption concerning the reluctance of the magnetic circuit is sustained by the facts of the case.

If the reluctance of the magnetic circuit is constant in the

region of normal operation, *i.e.* if the saturation curve is a straight line passing through the origin, $R' \div \phi'$ will be equal to $R \div \phi_2$, $\underline{Ni}_x$ will be the exact equivalent of Ix_a, and the two methods will give the same result. The error introduced by saturation is dependent upon the degree of saturation, and upon the magnitude of the reactance transformed.

The application of this method to the determination of the total excitation and regulation is very simple when the value of A' corresponding to the given armature current is known. This value of A' is given directly by the short circuit curve as follows: —

The m. m. f. diagram for short circuit is shown in figure 42, from which it is clear that, when x_a is large as compared with r_a, which is nearly always the case, F and A' are very nearly equal. But the short circuit F, and therefore A', for each value of the armature current, is given directly by the short circuit characteristic. As this characteristic is, for short circuit armature currents within the heating limit, nearly a straight line through the origin, A' will, within these limits, bear a constant relation to I and A. But it should be noted that this relation does not correspond to the conditions of normal operation, which usually takes place over the bend of the saturation curve, where an increase in induced e. m. f. means a more than proportional increase in m. m. f.

Examples of the Application of the M. M. F. Method.

Machine #1. Non-inductive load, figure 43. $E = 220$; $I = 76$; $r_a = .065$; $Ir_a = 5$ volts; $E' = 225$; from the saturation curve, $R' = 13.9$; from the short circuit curve, $A' = 6.2$; therefore, $F = \sqrt{R'^2 + A'^2} = 15.2$; the corresponding E from the saturation curve is 242.2, and the regulation 10.1%.

Inductive load, figure 44. Power factor $= .8$; $I = 95$; $Ir_a = 6$; $E' = \sqrt{(E \cos\theta + Ir_a)^2 + (E \sin\theta)^2} = 225$ (in most cases, as in this, E' is nearly enough $= E + Ir_a$); from the satura-

tion curve, $R' = 13.9$; from the short circuit curve, $A' = 7.9$; then F is the vector sum of R' and $-A'$ or

$$F = \sqrt{(R' + A' \sin\theta')^2 + (A' \cos\theta')^2}, \text{ where } \sin\theta' = \frac{E \sin\theta}{E'}$$

$= .586$, and $\cos\theta' = \dfrac{E \cos\theta + Ir_a}{E'} = .81$. Thus $F = 19.6$; $E_0 = 288$; and regulation 31%.

The same calculations are readily made for machines #2 and #3, and the results are set forth in table I for comparison. A refers to observed values, B to those calculated by the general method, C to those calculated by the e. m. f. method, and D to those calculated by the m. m. f. method.

TABLE I.

Machine.	Non-inductive Load.								Inductive Load.								Power Factor.
	Full Load Excitation.				Regulation %.				Full Load Excitation.				Regulation %.				
	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D	
#1	15.1	15.4	15.7	15.2	9.8	10 5	12.7	10.1		20	24.5	19.6		31.5	46	31	8
#2	150	149.5	171	147.3	4	3.9	8.4	3 4		200	350	181.3		14.5	36.2	10.1	.8
#3	80.8	79.5	87	72.6	7.4	7	9.8	3.9		146	250	84		24.5	36.3	8.60	.8
#3	—	—	—	—	—	—	—	—	108	107	127.5	76.5	19.3	18 8	23.8	8.4	.9
#4	132	139	197	131	4	5	12	3.8	200	206	400	158	14.8	15.5	49	9.1	.7
#5	141	139.5	146	134.5	5.5	5	7.3	3.9	168	169	220	146	13.6	14	23	6.8	.85

The results obtained by this method are too small, and for two reasons. First: the value of the m. m. f. $\underline{N}i_x = A' - A$, obtained from the short circuit curve, and which is to replace the leakage reactive drop, Ix_a, is too small because usually measured on the straight part of the saturation curve. Therefore, A' is also too small, as will be F obtained by the vector addition of R' and $-A'$. Second: the m. m. f. method makes contact with the saturation curve at a point corresponding to E', which, being always less than the real induced e. m. f., E_a, corresponds to a lower saturation, lower reluctance, and disproportionately low value of the m. m. f., R', unless the satura-

tion curve is a straight line through both of these points and the origin. Thus both A′ and R′, the two components of F, are too small.

This is shown clearly in figure 45, which shows the vector diagram for machine #3 under full non-inductive load by the

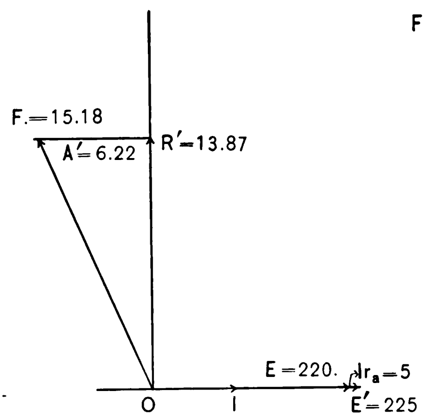


FIG. 43
MACHINE #1. FULL NON-INDUCTIVE LOAD. I—76

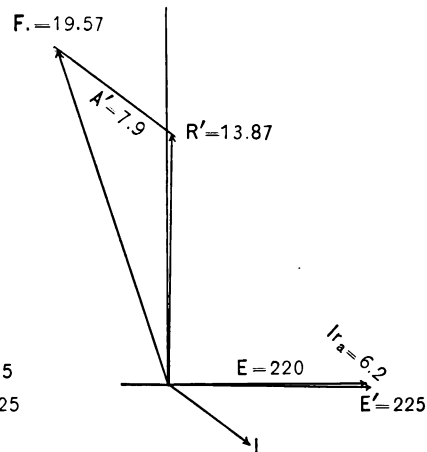


FIG. 44.
MACHINE #1. FULL INDUCTIVE LOAD. I—95

three methods. The diagram needs no explanation, the symbols having the same significance as above.

Thus in both the e. m. f. and the m. m. f. method, saturation enters at two points, introducing a double error; and the only case in which either method is strictly correct is that of a machine with a straight line saturation curve. With a given saturation curve, the error of the e. m. f. method increases with the armature m. m. f., A, and that of the m. m. f. method increases with the leakage reactance.

In machine #1, both saturation and leakage reactance are unusually small, which accounts for the good showing of the m. m. f. method in this case.

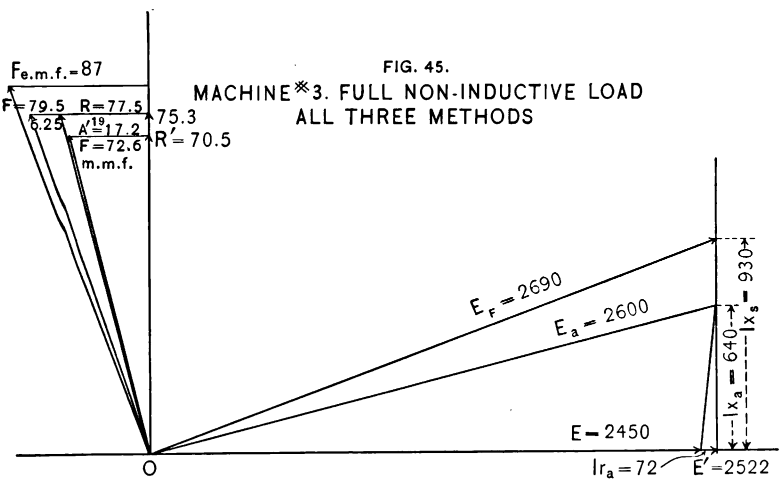
The low armature reaction of #3 would make it very suitable to the e. m. f. method, were it not for the high saturation.

On the other hand, the leakage reactance of this machine, coupled with its high saturation, makes it very ill adapted to the m. m. f. method, as is apparent from the results exhibited in table I. The great majority of modern close-regulating alternators resemble machine #2 rather than #3, and are therefore more suitable to the m. m. f. method, which is in very common use.

A very compact construction for this method is that of Arnold, shown in figure 46, for Machine #1. The figure is self-explanatory.

The ratio of A' to A is a very convenient constant in connection with the application of the m. m. f. method, and will be here designated by a. In a machine of familiar proportions a may be easily approximated. In Machines #1, #2, and #3, it has the values 1.15, 1.32, and 2.75 respectively. The latter is an extreme case, the average being from 1.2 to 1.4 for alter-

FIG. 45.
MACHINE #3. FULL NON-INDUCTIVE LOAD
ALL THREE METHODS



nate pole machines, and from 1.6 to 2 for inductor alternators. Even when a is no more than 1.2, the m. m. f. method will lead to very considerable errors for inductive loads unless the saturation curve be very straight.

If a is sufficiently small to warrant the use of the m. m. f. method, and if the saturation curve is fairly straight, the following approximations are very helpful in a first rough calculation.

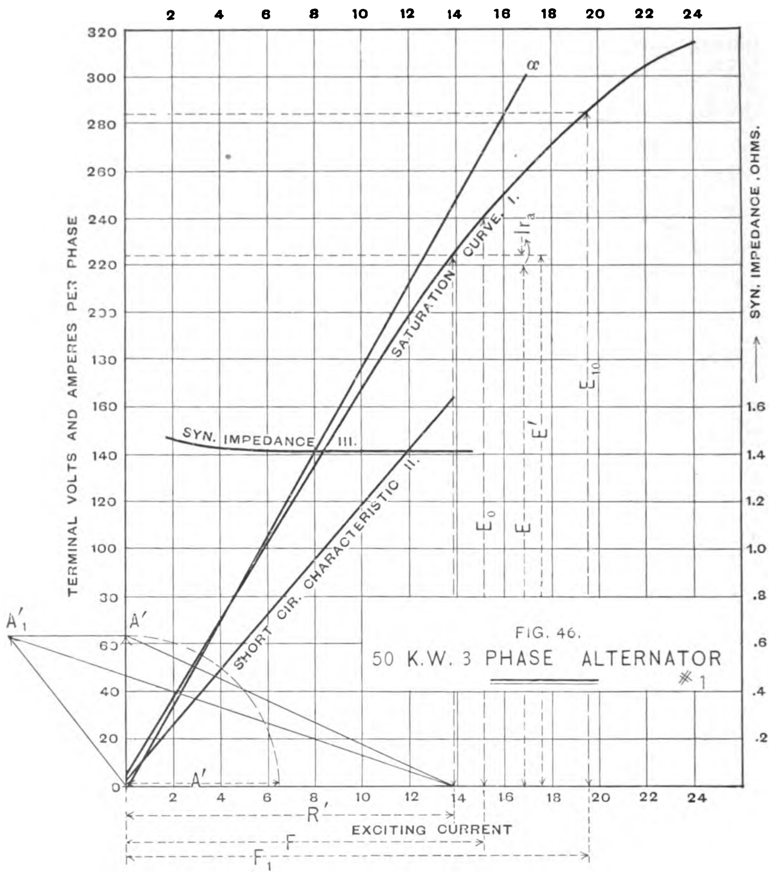


FIG. 46.

Neglect Ir_a, then $E = E'$, and $\theta' = \theta$, see figure 41. If it be desired to maintain E constant, it is only necessary to maintain R' constant. But $F = \sqrt{(R' + A' \sin\theta)^2 + (A' \cos\theta)^2}$ and $A' = aA$, where A can be calculated from the armature current and the conductors per pole, and a may be approximated

with a fair degree of accuracy if the type of machine is familiar. A rough approximation to F can thus be easily obtained.

In figure 47 are shown curves of F plotted against A′ (F being varied in such a manner as to keep R′ constant), for different values of the power factor, $\cos\theta$. F is plotted in per cent. of that necessary to produce normal voltage on open circuit, and A′ on the same scale; *i.e.*, when $A' = 100\%$, it has the same value in ampere turns as that of the no-load F, which would correspond to a considerable overload for a machine with good regulation.

If F be assumed constant and I assumed to vary, we can similarly plot R′ against A′, which will be an approximation to the volt-ampere curves, since with slight saturation E will be approximately proportional to R′. Figure 48 shows these curves for several power factors. The equation is $R' = \sqrt{F^2 - (A' \cos\theta)^2} - A' \sin\theta$. R′ and A′ are both plotted in per cent. of F, which is kept constant. $A' = 100\%$ corresponds, of course, to short circuit, which is from three to five times the normal full load current in modern machines of good regulation.

These same characteristics are exhibited in a more useful form in figure 49, where $(F - R') \div R'$ is plotted against $F \div A'$; but, if the saturation curve is approximately straight, $(F - R') \div R'$ will be approximately equal to the regulation.

As an example of the application of these curves, assume that it is desired to design an alternator which shall have an inherent regulation of 6% under non-inductive load; the lower curve shows that the corresponding value of $F \div A'$ is 3, *i.e.* that the full load m. m. f. must be three times the full load A′, or, if $A' = 1.3A$, the full load F must be 3.9A. As A is usually determined by other considerations, the magnetic circuit must be so designed as to require the above value of F.

The resistance drop in the armature, not taken account of in the curves of figure 49, would tend to increase the regulation, but saturation would tend to improve it, and the latter effect would usually predominate, so that the use of the curves will generally give safe results, unless *a* is large, *i.e.* unless the

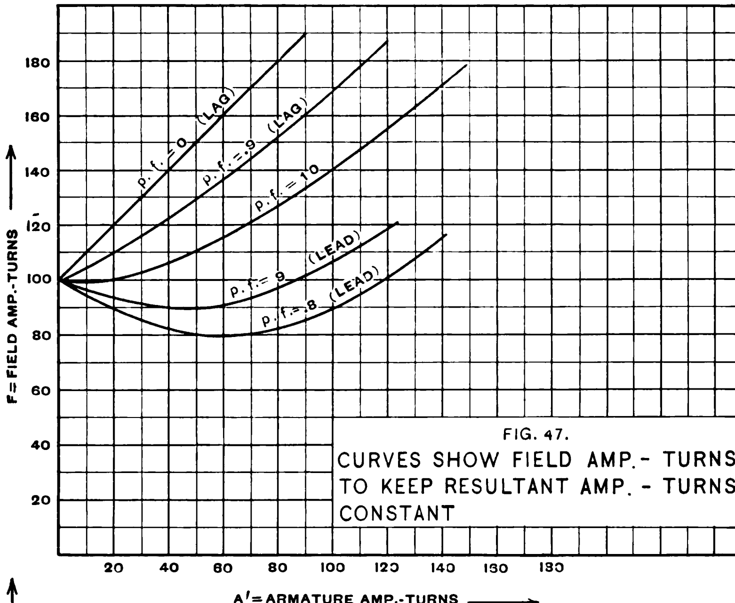


FIG. 47.
CURVES SHOW FIELD AMP.-TURNS TO KEEP RESULTANT AMP.-TURNS CONSTANT

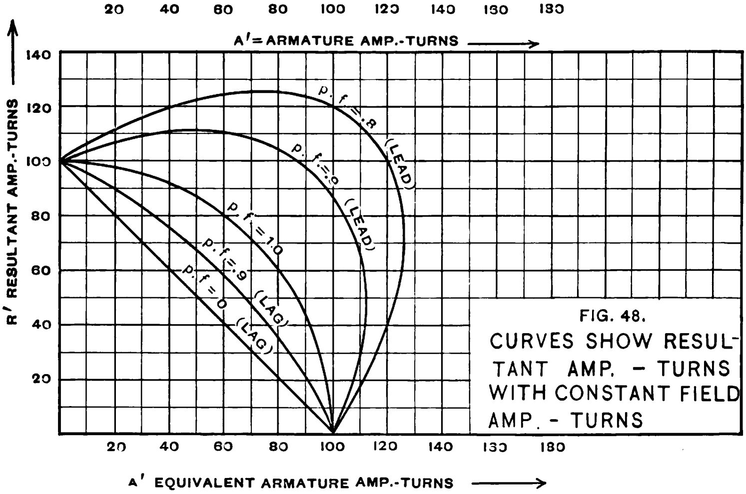
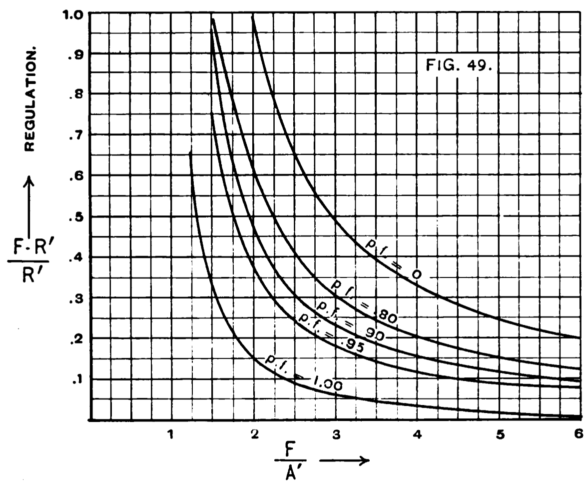


FIG. 48.
CURVES SHOW RESULTANT AMP.-TURNS WITH CONSTANT FIELD AMP.-TURNS

leakage reactance is large, when the error may be very large and on the unsafe side. Of course it is always possible to take so large a value of a that the result will be safe.

The curves of figure 49 show very clearly what a considerable effect upon the regulation is produced by a very slight reduction of the power factor from its unit value.

As bearing on the *Design of Alternators* the following analysis, based upon the first described method of calculating the leakage reactance (see page 148), may be of interest. It shows clearly the factors upon which depends the ratio of Ix_a (the leakage reactance e. m. f) to E (the terminal e. m. f.), this ratio being a direct measure of the importance of the leakage reactance.



Let ϕ' = flux per ampere inch of imbedded conductor;

l_a = length of armature core in inches;

N' = number of conductors per slot;

N_{sp} = number of slots per phase;

$N = N'N_{sp}$ = number of active conductors per phase;

$l = l_a N \div 12$ = number of feet of active conductor per phase;

L = inductance per phase in henries;

then $L = \phi' l_a N'^2 N_{sp} 10^{-8} = 12\phi' l N' 10^{-8}$

and $x_a = 2\pi n L = 24\pi n \phi' l N' 10^{-8}$ = reactance per phase.

Let E = terminal e. m. f.;

and e' = average volts generated per foot of active conductor;

then $l = \dfrac{E}{e'}$

$$x_a = 24\pi n\phi' EN' 10^{-8} \div e'$$

$$\frac{Ix_a}{E} = \frac{24\pi}{10^8} n \frac{\phi'}{e'} N'I \text{ where } I = \text{amperes per phase.}$$

Let $I' = N'I =$ amperes per slot

$$\text{then } \frac{Ix_a}{E} = \frac{24\pi}{10^8} n \frac{\phi' I'}{e'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{But } e' = 144 KkvB_g 10^{-8} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where B_g = mean gap density (per sq. inch.);
v = the peripheral velocity in feet per second;
k = the fraction of the armature surface covered by the poles;

and K = the product of the form factor and the differential factor, the latter taking account of the fact that a part of the conductors may be cutting flux in one direction while another part is cutting flux in the opposite direction. This differential action is considerable only in the case of the inductor alternator; in most other cases it is negligible and the differential factor unity.

Substituting (2) in (1) gives:—

$$\frac{Ix_a}{E} = \frac{24\pi}{144} \frac{n\phi' I'}{KkvB_g} = .524 \frac{n\phi' I'}{KkvB_g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) shows very clearly the factors which determine the ratio $Ix_a \div E$.

One interesting point is that this ratio is directly proportional to I', the amperes per slot, which shows the desirability of the distributed winding; but it should be remembered in this connection that, as I' increases and the slot increases in size, ϕ', will in general decrease, although not in the same degree, so that the product of $\phi' I'$ will increase as I' increases, though not so rapidly.

The frequency n is of course fixed for any given case, but in comparing the merits of different frequencies from the standpoint of the alternator equation (3) will serve.

In most alternate pole machines the product Kk varies only within narrow limits .6 to .8, whereas in the inductor alternator it varies from .25 to .45. In order to obtain the higher value

of this range for the inductor machine, it is necessary to employ the unislot winding, which means a relatively large value of I′ in equation (3).

Thus in the inductor alternator Kk is a little more than half as large as in the alternate-pole type, and $\phi' I'$ in general larger than for the distributed winding, other things being equal. On the other hand, B_g is usually considerably larger in the inductor, this and the consequent high tooth density being made possible by the non-reversing property of the latter in this type of machine. The peripheral velocity v has also a higher safe limit in the inductor type, which partly balances the low value of Kk and the large value of $\phi' I'$.

The ratios, $Ix_a \div E$, for the three machines #1, #2, and #3, for non-inductive load, are .07, .09, and .24, respectively. The large value for #3 is partly due to the type as above described, and partly to the low value of v, the peripheral velocity. With a suitable value of v, the ratio would have been about .15.

A similar measure of the importance of the armature reaction is the ratio $A \div R$, and may be obtained as follows: —

$$A = .707 \frac{N_a I_a}{2p'} = .707 \Delta\lambda$$

where $2p'$ is the number of poles; Δ, the peripheral current density (ampere-conductors per inch of armature periphery); and λ, the pole-pitch in inches.

But $\lambda = 6v \div n$ and

$$A = .707 \frac{6v\Delta}{n} \qquad (4)$$

If saturation be neglected, the following value of R is approximate: —

$$R = .626\, B_g l_g \qquad (5)$$

where l_g is the length of a single air gap in inches, and B_g the gap density. Then

$$\frac{A}{R} = 6.8 \frac{v\Delta}{nB_g l_g} \qquad (6)$$

It is interesting to note that $Ix_a \div E$ is proportional to $n \div v$, whereas $A \div R$ is proportional to $v \div n$: thus the effect upon

(3) of a change in either v or n will be at least partly balanced by its opposite effect upon (5).

Equations (3) and (6) show clearly most of the factors which determine the percentage effect produced upon the armature drop and regulation, by the armature reactance and reaction, respectively; these latter being the two most important causes of armature drop.

HARVARD ENGINEERING JOURNAL.

DEVOTED TO THE INTERESTS OF ENGINEERING AND ARCHITECTURE AT HARVARD UNIVERSITY.

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Entered at the Post-office, Boston, Mass., as second-class mail matter June 5, 1902.

Editorials.

THE 1904 Board of Editors takes charge of the JOURNAL with this issue. We feel greatly indebted to the last board for the time and trouble spent in organizing and systematizing the office work. With the JOURNAL now on a firm business basis we should be able to make it a still more valuable aid to graduate and undergraduate engineers.

We regret to say that, owing to stress of work, Mr. William R. Wade has been obliged to resign his position of Business Manager. Mr. Charles H. Fisher has been elected to fill the vacancy.

At the January meeting of the Harvard Engineering Society the constitution of the JOURNAL was unanimously approved.

At the annual meeting of the Harvard Engineering Society held in the Union on March 27, the following officers were elected for the ensuing year: President, Gilbert Simrall Meem, 1904, *Elec.*; Secretary, Charles Edwards Tirrell, 1904, *Mech.*; Treasurer, Thomas Coggeshall Eayrs, 1905, *Civil.* Professor Hollis was elected adviser, and Professor Love graduate secretary.

During the past year the following lectures were delivered under the auspices of the Society: —

March, 1902. Mr. John E. Cheney, "The West Boston Bridge."

April, 1902. Mr. A. H. Morse, '01, "Rope Transmission."

May, 1902. Mr. E. M. Blake, '99, "The Rapid Transit Subway in New York."

November, 1902. Professor Kennelley, "Cable-laying in the Gulf of Mexico."

December, 1902. Professor Sabine, "Sound as applied to the Acoustics of Building."

January, 1903. Mr. P. M. Blake, chief engineer Dam Commission, "The Proposed Charles River Dam."

February, 1903. Mr. H. A. Carson, chief engineer of the Boston Transit Commission, "The East Boston Tunnel."

The annual dinner was held May 14 at the Westminster Hotel. Professor Johnson acted as toastmaster.

Graduate Notes.

John W. Ames, '92, is a partner in the firm of Chase & Ames, Architects, Boston.

W. R. Copeland, '92, is at the Spring Garden Testing Station of the Philadelphia Water Works.

W. C. Fish, '86, is engineer of the General Electric Company at Lynn, Mass.

C. T. Hanson, '01, is engaged in work at the Bethlehem Steel Company, So. Bethlehem, Penn.

J. H. Page, '00, is in the House of Representatives of Arizona.

W. deV. Tassin, '92, is engaged in mineralogical work at the United States National Museum, Washington, D.C.

J. F. Vaughn, '95, is at present assistant engineer of the electrical department of the New York, New Haven & Hartford Railroad.

N. T. Weitzel, '01, is with the American Instrument Company of New York.

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DEVOTED TO THE INTERESTS OF ENGINEERING AND ARCHITECTURE AT HARVARD UNIVERSITY

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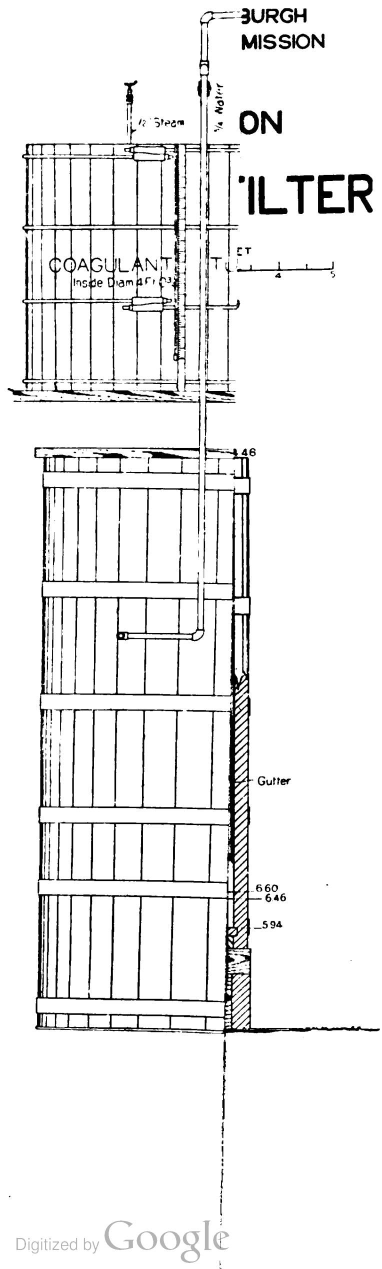
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HARVARD
ENGINEERING JOURNAL

Devoted to the interests of Engineering and Architecture at Harvard University

VOL. II | MAY, 1903 | NO. 2

WATER PURIFICATION.

WM. R. COPELAND, '92,

Bacteriologist of the Bureau of Filtration, Philadelphia.

PURE water is composed of hydrogen and oxygen, and nothing else. Such water as this has a great affinity for gases, and readily takes up dust or other foreign matter.

Chemically pure water, therefore, is a rare substance, and exists in a pure condition for a short time only.

The mineral impurities found in natural water consist of clay, loam or sand, and salts, such as the carbonates, chlorides, or sulphates of iron, lime, magnesium, soda, potash, etc.

The organic impurities consist of living organisms, such as bacteria and other micro-organisms; refuse from house-drains, barnyards, or factories, and the decaying bodies of animals or other dead organisms found in the air, forests, soil, and streams.

In view of the fact that greater volumes of storm water flow over the land in early spring than at any other season, the floods which occur in March and April generally carry greater volumes of clay, sand, and silt than floods at other times.

During the warm summer weather bacteria multiply rapidly, and as the droughts which occur during June, July, and August are followed by light fall rains, these storms are not heavy enough to tear the soil away, but flush great numbers of bacteria from the fields and sewers into the streams.

As a matter of fact, therefore, although the mineral impurities are generally greatest in the spring, the number of bacteria in surface water may be greater in the fall than at any other season of the year.

Self-purification of Streams.

We often hear a great deal of comment made upon what is known as "self-purification" of streams. The theory advanced is that, as a result of aeration, bacterial action, dilution, and sedimentation, impurities introduced at a given point into a river will be removed by the time the water has flowed for the distance of a few miles. There can be no doubt but that river waters do become partially purified through such causes, but the example of the spread of epidemics of typhoid fever from town to town on the Merrimack * is sufficient proof that self-purification cannot be relied upon to protect the consumers of river water from disease.

Purification by Filtration.

The process of purifying water by filtration may be divided into two parts: removal of organic matter by the action of living organisms, and the removal of suspended matter by the straining action of the filter bed.

Upon the grains of sand in a filter the bacteria congregate in great numbers. Certain species attack the organic matter carried in the water, breaking it up into such simple compounds as carbonic acid and ammonia. Other species carry on the process of purification by oxidizing the free ammonia into nitric acid. This acid, combined with salts of potash or soda, forms nitrates or saltpetre. In this way bacteria change dangerous organic substances into harmless mineral matter.

The oxidation of ammonia to nitric acid is known as nitrification, and the bacteria which carry the process through are called nitrifying bacteria. This reaction is of special importance in the purification of sewage by broad irrigation, in the

* Twenty-fourth Annual Report Mass. State Board of Health, 1892.

decomposition of dead vegetable tissues in forests or fields, and in the so-called "contact beds" of sewage-purification works. The process of nitrification goes on to some degree, however, in water filters at certain seasons of the year.

Water Filters.

Water filters consist, as a general thing, of a brick, concrete, earth, iron, or wooden tank which contains a bed of some material like charcoal, coke, gravel, or sand. The water to be purified is forced through the bed either by pressure or gravity.

In view of the fact that filters are built to run either at high rates of filtration, or else at low rates, they may be divided into two classes, American (or "mechanical filters") and English (or "slow sand filters").

Mechanical Filters.

In the first group we may include the so-called "household" filters, such as the Pasteur-Chamberland filters, and the true mechanical filters, such as are manufactured by the Continental Jewell Filtration Company.

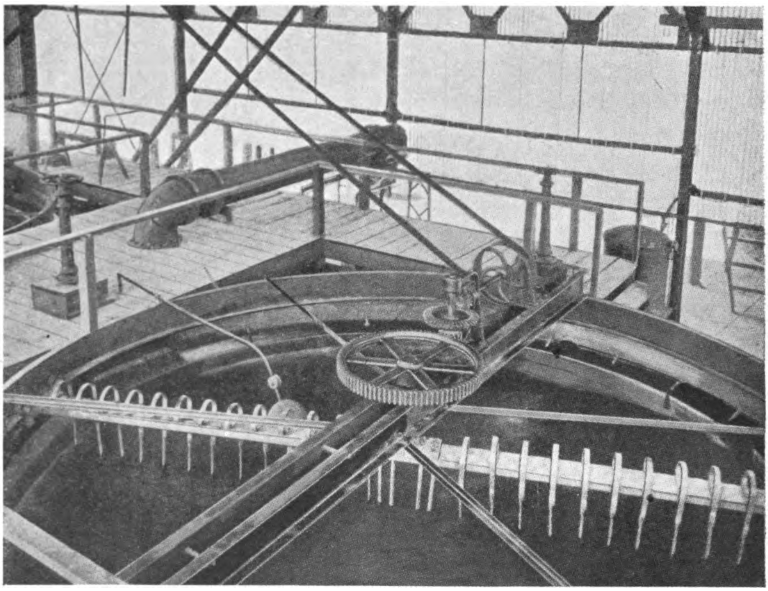
As household filters are of small capacity, it is sufficient to say that they often do a great deal of good work, but if left to the care of servants soon get out of order and the effluents become a menace to the health of the consumer.

In operating mechanical filters chemicals are employed to assist in the process of purifying the water, and power or elaborate mechanical devices must be used to operate the filter. The chemicals are added to remove either those mineral salts, such as lime and magnesium carbonates, sulphates or chlorides, which make the water "hard" and form "scale" in boilers, or else to remove suspended matter, such as bacteria, clay, etc.

In speaking of apparatus to remove hardness, or so-called "water-softening plants,"* it is sufficient to say that they depend for their purifying action on the fact that soluble salts

* "Purifying Water for Locomotive Boilers." G. M. Davidson, Engineering News, Vol. XLIX, No. 14, p. 296.

of lime and magnesium may be decomposed by the action of sodium carbonate, triphosphate of soda, etc., and precipitated as insoluble compounds, which can be removed by sedimentation or coarse strainers of cloth.



GRAVITY FILTER.

American Filters.

Mechanical filters of large capacity are often spoken of as American filters. American filters can be run at high rates, because the suspended matter has been removed by coagulants before the water is applied to the filter beds.

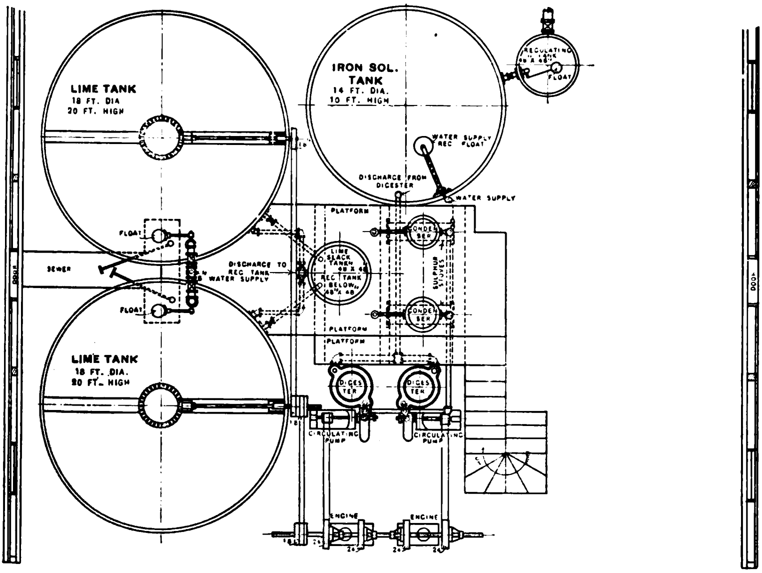
The particles contained in suspended matter vary in size from sticks, leaves, and grains of sand, to particles of clay and bacteria less than $\frac{1}{25000}$ of an inch long.

Coagulation.

The principle upon which coagulation depends is that soluble salts of alumina or iron, when brought into contact with alkalies

contained in river waters, are decomposed by the alkalies and precipitated as insoluble compounds.

For example, sulphate of alumina (soluble) combines with



COAGULATION PLANT.

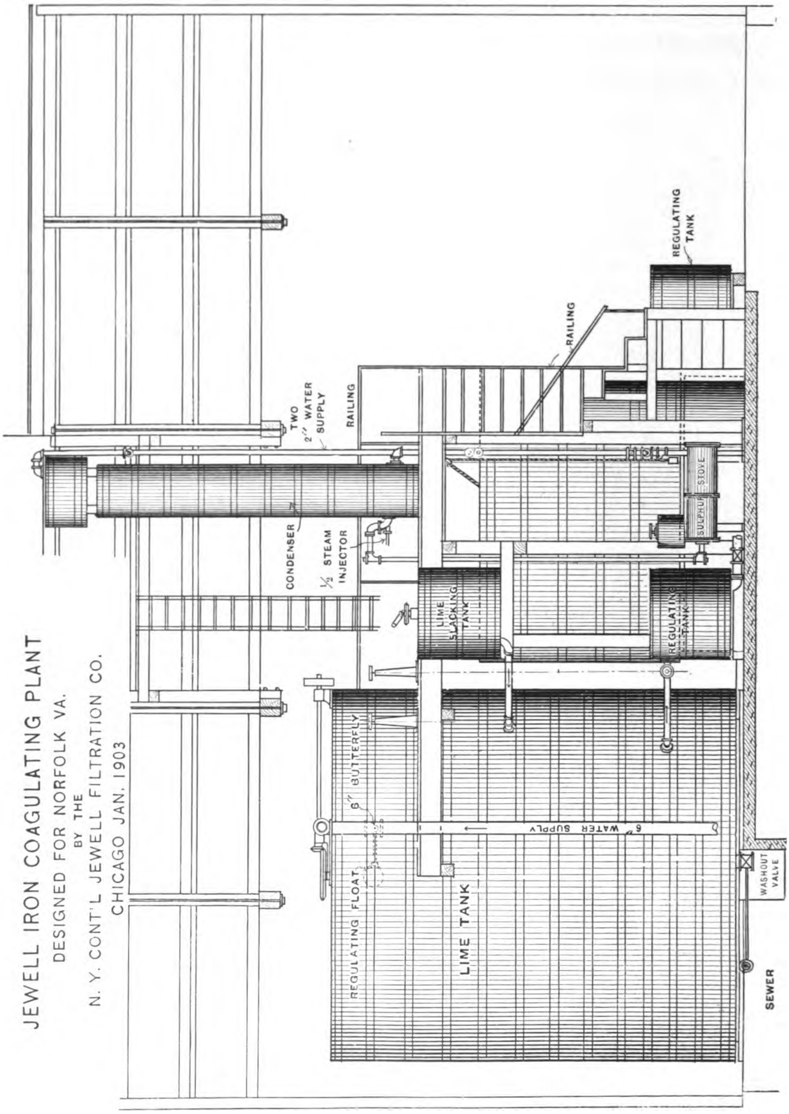
carbonate of lime and water to form insoluble aluminum hydrate. Thus:

$$\textit{Soluble}\ Al_2(So_4)_3 + 3CaCo_3 + 3H_2O =$$
$$\textit{Insoluble}\ Al_2O_6H_6 + 3CaSo_4 + 3Co_2.$$

The $Al_2O_6H_6$, or so-called "hydrate of alumina," formed is a flocculent gelatinous mass, which gathers the bacteria and particles of fine clay together, much as a sponge will take dirt out of water. The flocs of hydrate formed are often $\frac{1}{8}$ of an inch in diameter, and being insoluble in water may be strained out rapidly and efficiently by passing the water through a bed of sand.

Salts of Iron as Coagulants.

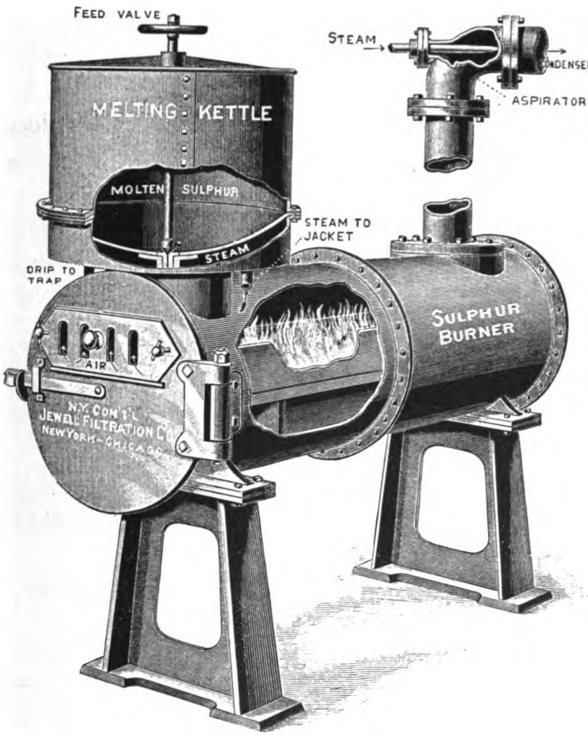
One of the salts of iron, $Fe_2O_6H_6$, forms a flocculent gelatinous precipitate in water. This hydrate of iron may be used as a coagulant in the same way that hydrate of alumina is



used. Iron forms two kinds of chemical compounds known as ferrous and ferric salts. The ferrous salts are formed by the action of strong acids on iron. As long as the solution formed remains slightly acid, the iron compounds remain in the ferrous

condition. But if they are exposed to the oxygen of the air, or brought into contact with alkaline compounds, the ferrous change into ferric compounds.

Alkalies such as carbonates or hydrate of soda, potash, and lime change soluble ferrous compounds into the ferric salt spoken of above as $Fe_2O_6H_6$.



DETAIL OF SULPHUR FURNACE.

When either sulphurous or sulphuric acid acts upon metallic iron, ferrous sulphateis formed. This compound separates from a cold saturated solution in green crystals, sold in commerce under the name of "copperous." This compound is readily soluble in water, but does not keep well, because, if it is exposed to air, it changes to ferric oxide. Some filter manufacturers * prefer to

* "Filtration at Lorrain, Ohio." W. M. Jewell, 1901.

make the ferrous sulphate by subjecting scrap-iron to the action of sulphurous acid in pickling-tanks.

In order to make sulphurous acid, "brimstone," or "flowers of sulphur," is burned in a furnace, and the fumes formed are blown by a jet of steam upward through a pile of coke or broken stone, over which water trickles in thin streams. The water absorbs the sulphur fumes (So_2) and forms sulphurous acid; the latter is poured into tanks filled with scrap-iron. The sulphurous acid attacks the iron, forming a solution of ferrous sulphate. From the "pickling-tank" the solution of iron runs into the water which is to be purified. As alkalies decompose iron solutions, the iron will be precipitated if the water to be treated contains sufficient carbonate of lime.

When the percentage of alkali carried by the raw water is too small to precipitate the iron, more alkali in the form of hydrate of lime or carbonate or hydrate of soda must be added.

Relative Cost of Coagulants.

The mechanical difficulties encountered in the process of adding iron and alkalies to water in proper proportions are very great. Nevertheless, the cost of ferrous sulphate and lime are so much less than sulphate of alumina that a great effort has been made to introduce iron salts as coagulants.

A report * made to the Jewell Filtration Company on the mechanical filters at Lorrain, Ohio, states that —

The comparative costs of operation are:

Sulphate of alumina ($2\frac{1}{2}$ grains per gal.)	=	$5.35	per million gals.
Sulphate of iron and lime	=	1.50	" " "

In a recent letter, Mr. Jewell states that by using brimstone "seconds" the cost of the iron process can be reduced further. It is only fair to say, however, that extra workmen and more skillful management are needed to run the sulphate of iron and lime than the alum coagulant. In order to make any process of coagulation successful, frequent chemical analyses must be made of the water before and after treatment, and the coagu-

* See preceding Report on "Filtration at Lorrain, Ohio."

lants added in amounts carefully adjusted to meet the varying conditions of turbidity and alkalinity in the raw water.

To accomplish this object the solutions are stirred constantly, and are added in measured volumes of known strength. Coagulants are generally run into the raw water before it enters the settling-basins.

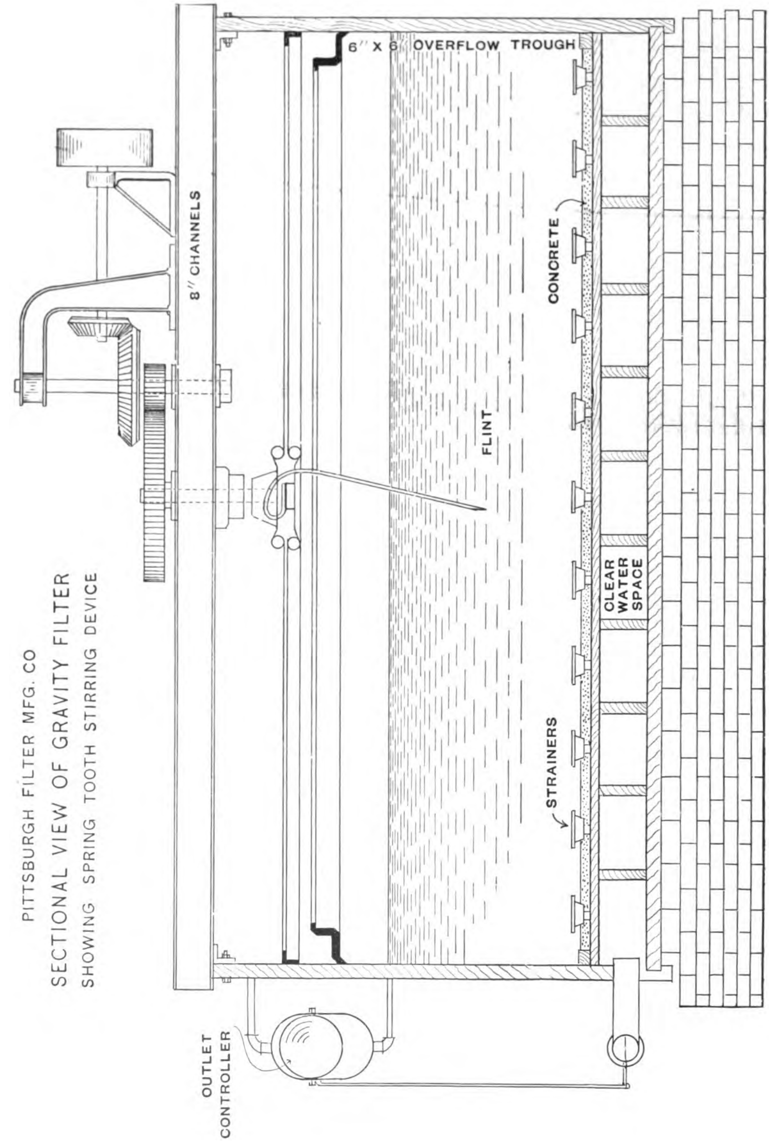
These chambers give time for the coagulant to become mixed with the water, for the reaction to take place, and for part of the suspended matter to settle out.

The filters * proper may be either round or rectangular, and are built either of concrete, iron, or wood. They contain a bed of sand from 2 to 4 feet thick, resting on a layer of fine gravel 3 or 4 inches thick. The gravel in turn rests upon a floor perforated by "strainers" for drawing off the filtered water. These strainers consist usually of a fine screen fastened in the top of a cup. The screen and cup are bound at the bottom to a series of pipes. They connect and form what is called a "manifold," which collects the filtered water and discharges it into the clear-water reservoirs. The sand used varies from 0.4 to 0.6 millimeters in diameter, and is very uniform in size.

Rates of Filtration.

The rate of filtration is usually regulated by some device such as a submerged weir or float-valve. Mechanical filters treat clarified water, and for that reason run at rates as high as 150 million gallons per acre in 24 hours. The rates usually range, however, from 90 to 125 millions. As these large volumes of water filter through a small area the downward pressure of the current makes the "schmütze decke" very dense and the bed of sand hard. In order to break up the compact mass and to flush the mud out of the sand, a stream of water is pumped into the bottom of the bed and forced upward through the sand. At the same time rakes composed of iron bars fastened to a beam are dragged through the sand, to stir the bed and cleanse the sand by knocking the dirt off the grains.

* Report of the Filtration Commission, Pittsburgh, 1899.

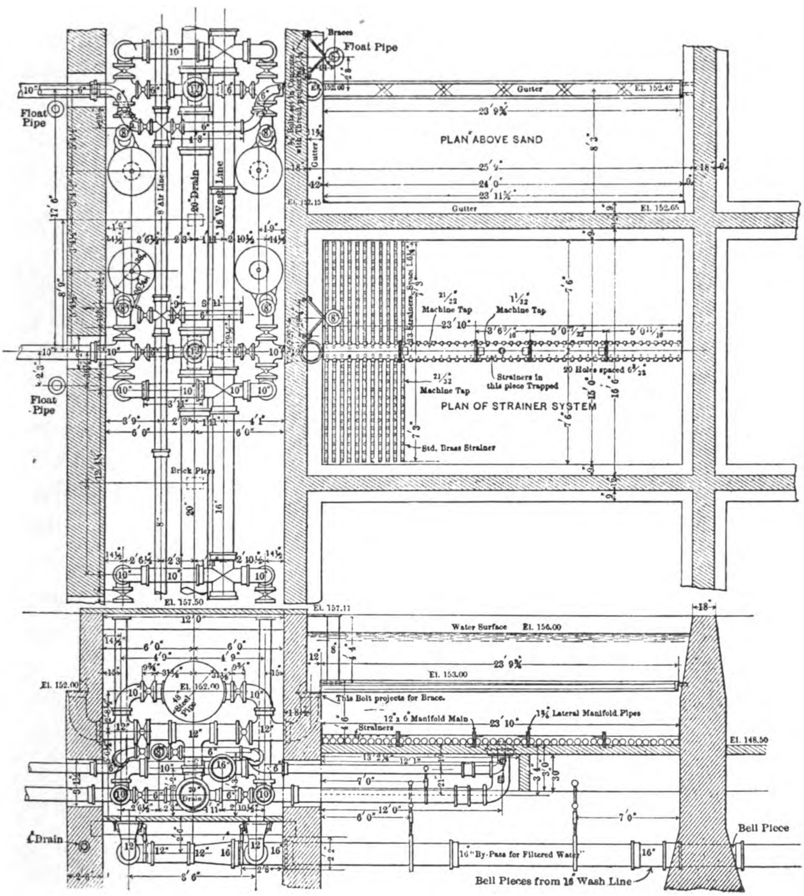


PITTSBURGH FILTER MFG. CO
SECTIONAL VIEW OF GRAVITY FILTER
SHOWING SPRING TOOTH STIRRING DEVICE

Instead of rakes blasts of air are used in the filters of Little Falls (N. J.) to stir the bed of sand.*

* "Filtration Works of the East Jersey Water Co., Little Falls, N. J." G. W. Fuller, Proceedings of the Am. Soc. of C. E., vol. xxix, No. 2, Feb. 1903.

Air is driven into the bed through the pipes at the bottom of the filter, and special precautions are taken, by using traps and a layer of gravel on top of the strainers, to distribute the air uniformly over the bed.



WATER PURIFICATION WORKS, LITTLE FALLS, N. J.*

Sand Filtration.

The so-called English or slow sand filters differ from mechanical filters in that coagulants are not used, as a general thing;

* This cut is taken from Proceedings of A. S. C. E., vol. xxix, No. 2, Feb. 1903.

the main portion of the bed is not disturbed except at long intervals; and the rates of filtration are comparatively low.

The filter consists of a basin built of concrete or similar material, having an inclined bottom, sloping sides, and generally an arched roof. Upon the inclined bottom tiled sewer-pipes are laid to collect the filtered water. Over the tiles coarse, medium, and fine layers of gravel are placed to a depth of from 10 to 15 inches, and upon the gravel a layer of sand lies, which is from 2 to 4 feet thick.

The water to be purified is poured upon the top of the bed of sand, percolates downward between the grains, through the gravel, and is collected by the underdrains at the bottom.

The bed of sand consists of grains which vary in size from 0.01 m.m. to 2 m.m. in diameter. As the small grains control the rates at which the water will pass through sand, it is desirable to find out what portion of the sand-grains are small. To determine the percentage of fine material, a known volume of sand is dried and passed through sieves having wire-screen bottoms. The sieves are placed one upon another, the top screens having a coarse mesh and the lower screens meshes with smaller holes.

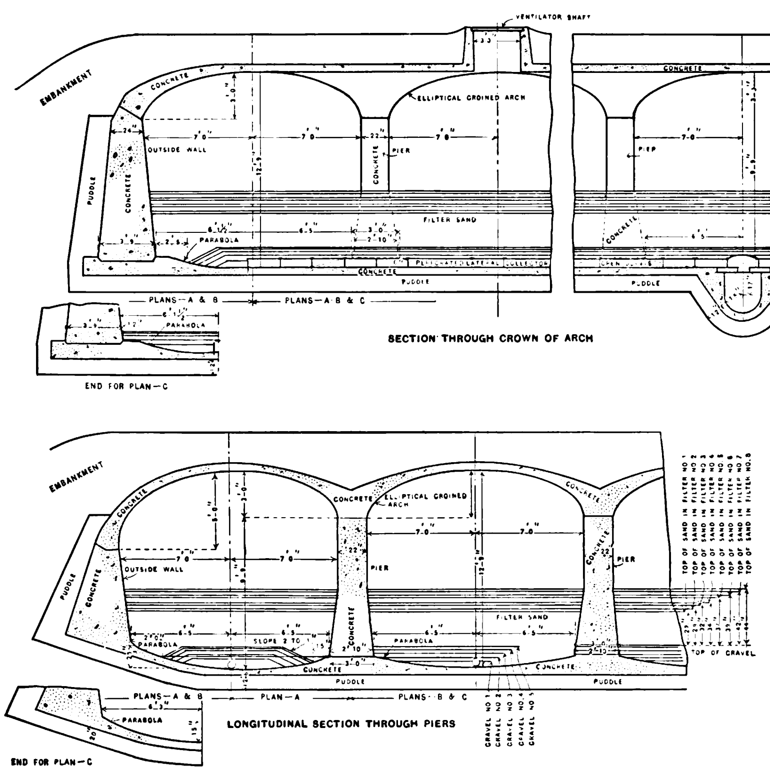
To determine the proportion of coarse and fine grains, the materials retained on each screen are weighed and the percentage by weight determined.

Sands are rated according to what is known as the "effective size." This is really the diameter of the coarsest particles in the portion of sand constituting the "finest 10 per cent. by weight" of the whole body of sand. Good filter-sands vary in effective size from 0.25 m.m. to 0.35 m.m.

The water on the surface of the sand creates a pressure on the water in the underdrains. This pressure is called the "head," and is the agent which forces the water through the sand. As the surface becomes clogged with mud, the pressure which the surface water exerts upon the water in the underdrains decreases. This decrease in pressure is termed "loss of head," and when the loss of head reaches a certain arbitrarily selected figure the filter is put out of service for cleaning.

Control of Rate.

The rates at which slow sand filters run vary from 1 to 10 million gallons per acre of area in 24 hours. Different methods are used for measuring and controlling the rate. In some cases the rate is controlled by simply opening or closing a valve on the outlet. More accurate methods have been introduced lately, however, which are based on measurements by a weir.

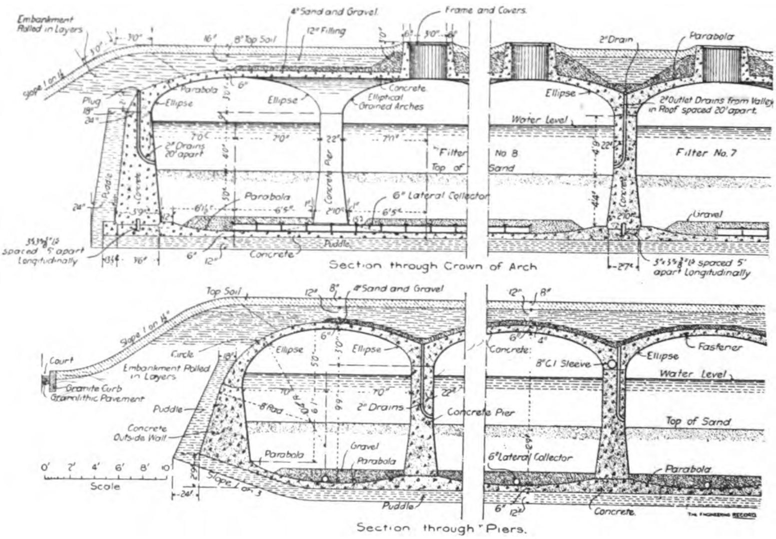


FILTRATION OF WATER SUPPLY, PHILADELPHIA.

The regulation of the rate of filtration is one of the most important factors in the operation of slow sand filters. As they are installed for the purpose of removing suspended matter, and as the process depends to a large extent upon having the water pass through all parts of the bed at a uniform rate, the efficiency

will be decreased by any changes which force the water through one part of the bed faster than through another.

The principal causes for variation in rate in different parts of the bed through the sand are "disturbance of the surface layers of the bed" and "fluctuation in rate of filtration." In regard to



UPPER ROXBOROUGH FILTERS, PHILADELPHIA.

the rate of filtration, it may be said that the rates at which a slow sand filter should run are governed by the work which the filter has to do. If used to filter waters which contain on an average more than 75 parts of turbidity per million, or more than 5,000 bacteria per cubic centimeter, the rate of filtration must be kept at 3 million gallons or less per acre in 24 hours. But if the water is very clear (25 parts of turbidity or less per million) and contains only a small number of bacteria (1,000 per c. cm. or less), the rate of filtration may be increased to 10 and perhaps to 15 million gallons per acre in 24 hours.

The manner in which the materials taken out of the applied water by the filter are distributed through the bed varies with the amount and character of the suspended impurities. To illustrate the conditions two examples may be cited, as follows:

First. When the applied water contains large amounts of suspended matter.

	Number of Bacteria per Gram.	Parts of Turbidity per Million.
Sample of schmutze decke . . .	5,000,000-30,000,000	500,000
Average of the layer of dirty sand . .	500,000- 3,000,000	100,000
Sand 1 inch below surface . . .	50,000- 300,000	25,000
Sand 3 inches below surface . . .	10,000- 50,000	5,000
Sand 6 inches below surface . . .	1,000- 10,000	1,000
Sand 12 inches below surface . . .	5,000	200
Sand 24 inches below surface . . .	1,000	
Sand 36 inches below surface . . .	1,000	

Second. When the applied water contains only a small amount of suspended matter:

	Number of Bacteria per Gram.	Parts of Turbidity per Million.
Schmutze decke	1,000,000-5,000,000	100,000
Average of the layer 1 inch below surface .	300,000- 500,000	50,000
Sand 1 inch below the surface . . .	100,000- 300,000	25,000
Sand 3 inches below the surface . .	100,000- 200,000	10,000
Sand 6 inches below the surface . .	25,000- 50,000	5,000
Sand 12 inches below the surface . .	5,000- 10,000	1,000
Sand 24 inches below the surface . .	5,000	200
Sand 36 inches below the surface . .	1,000	100

Scraping the Dirty Sand off the Filter.

As the schmutze decke increases in thickness and dirt gathers in the layer of sand at the surface of the bed, the flow of water through the filter decreases gradually.

In order to restore the yield from the bed to the normal volume, the layer of dirty material must be removed. The depth of dirty sand will vary from time to time as the volume and character of the suspended impurities change. For instance, the spring floods carry large volumes of mud, clay, and silt mixed. After such a freshet it is often necessary to scrape from 2 to 4 inches off of the surface of a filter. On the other hand, during hot, dry summer weather, the applied water may carry very little clay, but considerable numbers of algae and other micro-organisms. Such materials are coarse, and settle for the most part on top of the bed. Algae may take root and develop into masses of green slime during June, July, August, and September. If such growths clog the surface badly, covers must be built to shut out the sunlight, for algae will not grow in the dark.

As a general thing, the line between the layers of dirty and clean sand is sharply defined, especially when the suspended matter in the applied water is coarse. But experience has shown that it is advisable to take samples of sand from layers 1, 2, and 3 inches below the surface, and shake them up in bottles containing known volumes of clean water, in order to determine, by noting the turbidity, how far the suspended matter penetrates into the bed.

Scraping Sand Filters.

The process of scraping a filter may be divided into two parts — first, lifting off and piling up the dirty material; second,

WHEELING SCRAPED SAND FROM FILTER-BED TO SAND-WASHER.

carrying the dirty material away. The water on the surface must be drawn off before scraping, until the level of the water in the sand lies at least one foot below the dirty layer. Then, according to present custom, men wearing broad wooden-soled shoes walk out upon the surface of the bed and lift the layer of dirty material off, using long-handled shovels with flat blades carried parallel to the surface of the sand. The broad-soled shoes are worn so that the men shall not compact the surface of the clean sand

by walking on it with their boots, and the blades of the shovels are held flat in order to keep the surface of the bed as level as possible.

The material "scraped" off is thrown into piles, and is ready for the second step of "removing the dirty sand."

Two methods for removing the piles of dirty sand are used. By the first method men carry the sand out in wheelbarrows. In this process men load their barrows, wheel them a distance of 300 or 400 feet, dump, and return over the same distance with empty barrows. That is to say, during half of the time consumed by the wheeling process no sand is being moved. If one adds to this the fact that men coming out get in the way of men going in, etc., it is evident that the element of time consumed becomes an important factor, and delays are expensive.

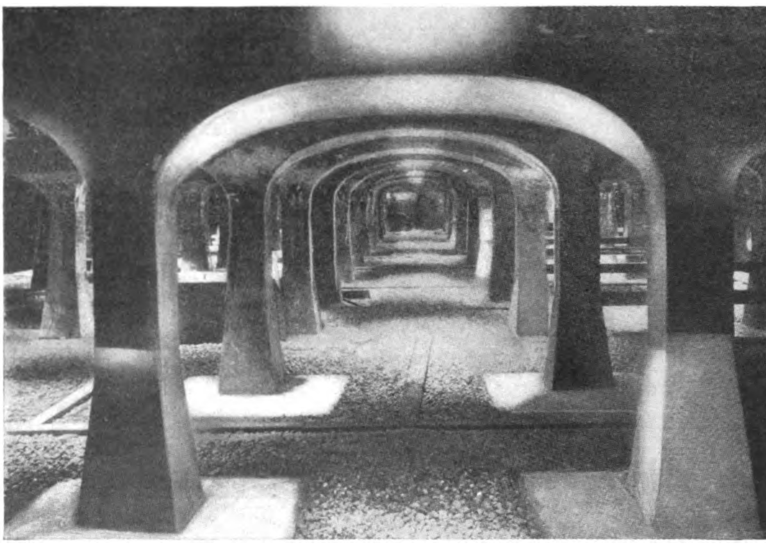
Removing Sand by Water.

Sand may be transported for 500 feet or more by a stream of water acting under a pressure. The sand is thrown into a hopper, falls to the bottom, and is struck by a jet of water, acting under pressure, which forces it through a pipe leading out of the filter. This method of moving sand is economical, for three reasons: First, the machine keeps sand moving all the time; second, the machine does as much work as six men can do working with wheelbarrows; third, the water used and the violent mechanical rubbing which the sand receives in passing through the pipe separate the dirt from the grains in a most thorough manner. The points of disadvantage are: First, the sand grains, striking against certain parts of the machine and pipe line, wear them out; second, a considerable sum of money must be spent to install the pipe-lines, purchase the hoppers, etc. That is to say, the cost for installation and repairs are large compared with the wheelbarrow method; but it is safe to say that the removal of dirty sand by water effects a saving of 30 to 40 per cent. over the wheelbarrow method.

Washing Sand.

The dirty sand removed from the bed must be cleaned by washing in water before it can be put back on to the filters.

To clean the sand, machines known as "sand-washers" are used. Sand-washers act in the same manner as the machine used to carry dirty sand out of the filter. This process of washing is continued in each of three hoppers placed in a row, or until the sand has been washed sufficiently clean. The jet in the last hopper forces the washed sand out through a pipe on to the clean sand pile.



LOWER ROXBOROUGH FILTERS, PHILADELPHIA.

Under these circumstances, it is probable that 4 men can wash and transport sand a distance of 400 feet at the rate of 8 cubic yards of sand per hour, using about 12 volumes of water to 1 volume of sand, for $1.00 per million gallons filtered.

Data * in regard to handling sand by wheelbarrows at Albany.

Cost per Million Gallons Filtered.

Scraping and wheeling out dirty sand	$0.69
Washing	.42
Refilling (wheeling back washed sand)	.32
Incidentals and lost time	.23
Total per million gallons of water filtered	$1.66

Note: The number of cubic yards of sand handled = 5,673.

* Transactions of the Am. Soc. of C. E., vol. xiii, 1901, p. 321.

Experience * has shown that the coarser particles of suspended matter can be removed from water by allowing it to stand in settling-basins for a period of 12 to 24 hours; but the finer particles will not settle in a period of less than 48 or 72 hours. Settling-basins which have a storage capacity of three or four days cost so much to build that the interest and sinking-fund charges increase the annual debt to a considerable degree. Add to this the fact that the efficiency of settling-basins is limited, and it becomes evident that settling-basins may be both unsatisfactory and expensive. They have one point in their favor, however, and that is that charges for maintenance and operation amount practically to nothing compared with the millions of gallons of water which they treat.

The efficiency of settling-basins may be increased by adding coagulants, and it is only fair to say that a large part of the finely divided clay can be removed in this manner. On the other hand, part of the coagulant may pass over to the filters, and by settling on the sand shorten the periods between scrapings.

The only other way to avoid the expense of constructing large basins, and to secure a plentiful supply of clarified water, is to pass the surface water through some sort of a preliminary filter.

Preliminary Filters.

Turbid waters may be partially clarified by passing them through beds of fine gravel, coarse sand, asbestos sheets, or sponges, etc. As in the case of other filters, the channels through preliminary filters must be small to remove clay. On the other hand, the principal object of a preliminary filter is to secure a large yield of water which carries less than 50 parts of suspended matter per million. Therefore the rate must be high, the filters must have comparatively small areas, and simple but efficient appliances for washing, so that when the filter has to be cleaned the process may be completed in the shortest possible time.

* Report on the Investigations into the Purification of the Ohio River Water, 1899, p. 114.

It is a difficult matter to build a preliminary filter which will combine all of these factors. Gravel filters are not efficient without coagulants; asbestos soon becomes matted and cannot be cleaned for use a second time; the supply of sponge clippings is limited, moreover, they must be imported, and when placed in a filter they absorb mud to a certain point and then

LOWER ROXBOROUGH FILTERS, PHILADELPHIA.

begin to give up what they have absorbed. Therefore, sand filters, built on the lines of a mechanical filter, with proper rakes or air-blasts, etc., for washing the sand as it lies in the bed, seem to offer the most effective, simple, and economical appliances for the preliminary treatment of water.

In regard to the question of the efficiency of preliminary filters, it may be said that a bed of sand properly constructed will remove 70 to 80 per cent. of turbidity from waters which contain 150 parts or less per million, at rates of filtration ranging from 100 to 125 million gallons per acre in 24 hours. They will remove 50 per cent. of turbidity from waters which contain as much as 300 parts, and from 30 to 40 per cent. from waters which contain more than 500 parts of turbidity per million. In order to secure a satisfactory effluent from a sponge filter, the water applied should never contain more than 100 parts of turbidity per million. If the water contains only 50 or 60 parts

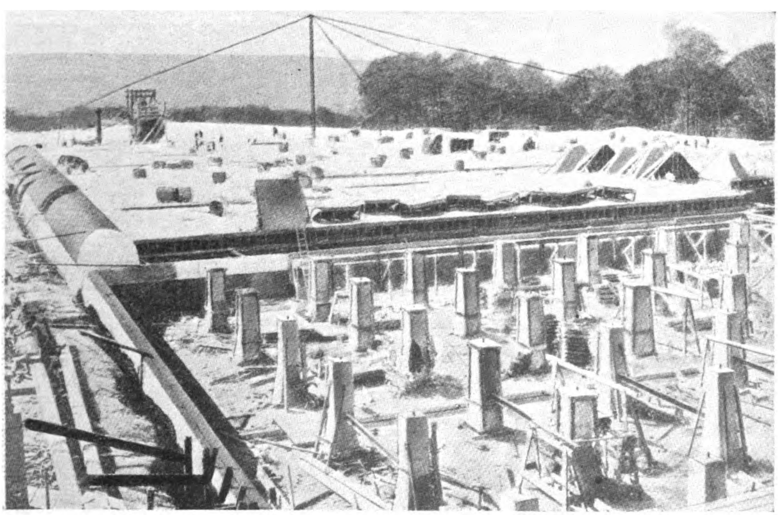
sponges will remove about 60 per cent. of turbidity, at a rate of filtration yielding 60 million gallons per acre in 24 hours.

Summary.

Water purification includes the removal of bacteria, chemical impurities, mineral matter, and organic compounds.

Bacteria may be destroyed by boiling, and salts in solution may be decomposed by the addition of "boiler compounds"; but mineral matter and organic compounds can be removed more economically and efficiently by filtration than by any other method. Mechanical filters, on the one hand, have advantages in that they occupy small areas, yield large volumes of water, and the methods of operation can be varied quickly and efficiently to meet the changing conditions of surface waters. On the other hand, sand filters cost but little for maintenance, operation, and repairs.

The process of water purification, in order to be thoroughly successful, must accomplish three objects: it must remove the germs of disease contained by the water, reduce the turbidity to a point where it is acceptable for household use, and it must be carried out on a strictly economical basis.



LOWER ROXBOROUGH FILTERS, PHILADELPHIA.

EAST RIVER BORINGS FOR BROOKLYN EXTENSION OF THE NEW YORK RAPID TRANSIT.

J. C. MEEM.

It is believed that a description of the East River borings for the Brooklyn extension of the Rapid Transit Subway of New York will form an interesting chapter in the history of that great work, and the purpose of this paper is to record the principal events in connection therewith, together with the methods and results of the work. The plans for this extension cover that portion of the proposed subway between Park Row and Broadway, in Manhattan, and Flatbush and Atlantic avenues, in Brooklyn, and the general line of the route is down Broadway to State street, through State street and across Battery Park, going under East River approximately at the foot of Whitehall street, and crossing the bulkhead line in Brooklyn at the foot of Joralemon street; thence up Joralemon street and across Court Square to Fulton street; up Fulton street to Flatbush avenue, and through Flatbush avenue to Atlantic avenue, terminating at the Long Island Depot.

The borings for the Broadway portion of this route had been previously made in connection with the preliminary work for the original subway contract, and the borings for the remaining portion, from the intersection of State street and Broadway, were let in three separate contracts, the first contract being for the river-wash borings, the second for the land-wash borings, and the third for the diamond-drill borings.

This paper will concern itself with the first and third contracts only, or to those pertaining solely to the river work. After preliminary soundings had been made along the proposed line, bids were invited for the wash borings, covering about 3,000 linear feet, measured from mean high water. Mr. F. W. Miller was the successful bidder, the bids varying from $1 per foot to $2.50.

The contractor's equipment for this work consisted of the

chartered steam-lighter "Grace Darling" (of about 65 tons), with a captain and crew of engineer, fireman, two deck-hands and a cook (the latter furnishing meals to the contractor's men as well as the crew); a superintendent, working foreman, and two men; sundry tools and piping, consisting of some 100 feet each, in short lengths, of ¾-inch wash-pipe and 2½-inch extra heavy casing-pipe.

The specifications cited, generally, the authority and conditions under which permits had been granted from the War Department and Superintendent of Anchorages for anchoring in the river and obstructing trafic; they also stated the approximate quantities, in linear feet, that the measurements were to be from mean high water, and gave the engineer in charge authority for the location of the borings, collection and preservation of samples, etc. While no difficulty was apprehended in locating the position of borings when occupied, a small problem suggested itself in finding some simple method by which these locations could be quickly determined on the ground, in order that reasonable intervals of spacing should be observed, in accord with the plans.

It was obviously impracticable to locate cross ranges up and down the river at intervals of about 100 feet, and it was also inexpedient to make an extensive triangulation to find such ranges as could be utilized, as owing to the swift tides and the heavy traffic it was not feasible to place instrument-men in position to make the location.

After considering one or two methods, the following one was finally adopted for both the wash borings and those with the diamond drill. Range-poles were erected on Pier 4, Manhattan, and Pier 17, Brooklyn, and permanent existing objects plainly visible from the river were utilized for the prolongation of the line on either end. On the Manhattan side the flag-pole on the South Ferry-house was selected, and well to the left of this (looking from the river) the flag-pole on the Barge-office was selected as another point in the general scheme of triangles. As the final triangulation of the tunnel line across the river had not then been made, it was necessary to locate the relative positions

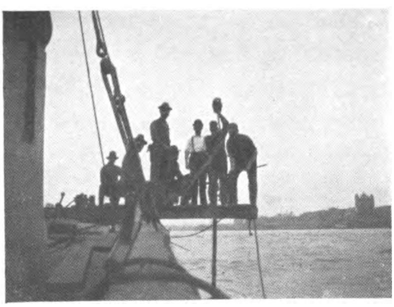
of these points, together with other emergency points, by making a small preliminary triangulation.

These points, together with 100-foot stations along the line of the tunnel, were then plotted, and the angle at these points, between the line and the Barge-office flag-pole, was calculated. It was thus seen that by approximating the intermediate angles between the 100-foot stations one's position on this line could be located with a sextant at the moment of crossing the line.

In order to comprehend the method of procedure in the work itself it will be necessary to understand something of the East River tide's movements, which are somewhat complicated.

For the purposes of this paper it is sufficient to state that in general the tide on the Manhattan side of the river runs up or down, as the case may be, some two hours after high or low tide water, and is slack for only 15 or 20 minutes, while on the Brooklyn side, the period of slack water covers some two hours following the period of low or high water. On the Manhattan side the ebb tide is very strong, reaching its maximum velocity at about four hours after the time of high water.

When there is a full moon there is a tidal range of about 6 or 7 feet (the mean range is 4½ feet) between high and low water, and with a high west wind to blow the water out of the bay, the range is greater, and the velocity of this flow is upwards of 6 miles an hour.



WASH BORING IN EAST RIVER. DRIVING DOWN 2½-INCH CASING-PIPE.

It is thus seen that while the borings from mid-stream towards the Brooklyn shore presented no especial difficulties, very little time was allowed for the completion of those on the Manhattan side, as the work had to be begun and completed during the slack-water period, or within an outside limit of 30 minutes.

After one or two experiences with anchors, three, weighing from 500 to 700 pounds each, were obtained, with from 400 to 600 feet of 5-inch line to each anchor. The original equipment of the steam-lighter provided a stationary platform projecting over the side of the boat from which the borings were to be made, but as this admitted of absolutely no shifting of the boat, a heavier movable platform was substituted, which allowed the boat to shift within a small space without damage to the borings.

A typical boring on the Manhattan side will now be described as descriptive of the general proceedings.

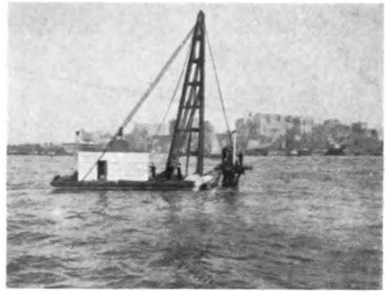
About one hour after the first high or low morning tide the lighter, with its equipment and men, proceeded to the scene of operation. The boat was made to cross the line as near the desired point as practicable; the angle was observed and a note made of it, and at the same time an up-and-down range was picked up and noted in conjunction with this angle. In this connection the lamp-posts on the bridge ranged in with Hecker's chimney, in Manhattan, proved to be specially helpful, and, after a few observations, locations of the borings could be made, with approximate correctness, with these ranges alone, and the sextant angle was used simply as a check.

After establishing the line and range, the boat was backed down stream, and the stern anchor was dropped some 400 feet from the line. The starboard and port bow anchors were then dropped up the stream, the three anchors forming the terminal points of the letter "Y." The boat was then pulled into position with steam-winches, and as soon as the tide began to slacken sufficiently the 2½-inch extra heavy casing-pipe (which had been made up in sufficient length to reach bottom, as indicated by the lead line) was hoisted up by the derrick and lowered to position, resting on the bottom. It was then driven down as far as possible with a hand maul, after which a ¾-inch jet-pipe was inserted, and water was pumped through it to wash out the material at the bottom.

The casing-pipe was thus alternately washed and driven down until the boring had reached the required depth, or was stopped

by rock or large boulders. In the middle of the river the greatest depth of boring required was 103 feet, or about 10 feet below the proposed base of rail of the tunnel. Samples of the wash borings were taken at the beginning and end of each boring, and at depths of change in the strata, the measurements of depths of these points being recorded.

Changes of strata could also be detected by noting the changes in resistance to the driving of the casing-pipe. The samples were preserved and afterwards placed in graduated tubes in the office of the chief engineer. After the completion of the boring the derrick was again utilized to pull up the casing-pipe, after which the anchors were hauled in to await another tide. The approach to the Manhattan shore is underlaid with rock, which has a covering of from 7 to 10 feet only of gravel and sand, and the 15 or 20 minutes of slack water scarcely gave time enough to accomplish the work of boring. In fact, several fruitless attempts were made to secure two or three of these borings before success attended the efforts of the contractor in that locality. This portion of the work was made hazardous, also, from the fact that it was directly in the line of the most congested traffic, and as the greatest depth of water (some 47 feet) occurs at this point, it was often difficult to get the pipe into position in time for the slack water, every minute of which had to be utilized to insure success. On one occasion the casing was broken by the tide after it had been driven into the bottom. Two collisions occurred during the progress of the work, one during slack water, near the Brooklyn shore, caused by the carelessness of a captain towing a loaded car-float, and another occurred just after putting out anchors in the ebb tide on the Manhattan side. The accident in this instance was due to the miscalculation of a tug's captain towing two loaded freight-car floats (one on each side), who tried to cross the bow of the anchored lighter, and who succeeded in staving in her bow-stem, parting her two bow-lines, and knocking her some 500 feet down stream. In view of the inequality in tonnage of the colliding craft, and the great velocity of the tide, the captain doubtless considered himself fortunate that his damages included only the repairs to the



EAST RIVER DIAMOND-DRILL BORINGS, SHOWING PILE-DRIVER AND STAGING IN PILE CLUSTER, WITH BORING-MACHINE IN OPERATION, OCT. 24, 1901.

lighter and the time lost to the contractor in dragging for the anchors and incidental work. The total number of wash borings made were 43, of which 7 were made in the slips between piers, and 2 through Pier 17, Brooklyn, the remaining 34 being made in the channel. The time required to make them ranged from 15 minutes to 2 hours, except that those through Pier 17 and adjacent thereto occupied an interval of about 5 hours each, as a porous gravel and sand was struck, which allowed the wash-water to run away instead of washing up material.

The distance between pier-head lines along the route of the tunnel is about 2,800 feet, so the spacing of the borings averaged a little over 80 feet. No attempt was made at first to get special borings, except that in general the Manhattan side was worked in the last of the ebb tide and the Brooklyn side in the last of the flood.

The piers and slips were reserved for foggy days or when it was impracticable otherwise to work in the channel. The deepest boring extended to a depth of 103 feet below mean high water, and the average depth of each was about 65 feet. The total time during which the contractor was engaged in the work was from August 12 to September 15, 1901.

The wash borings practically established the generally acknowledged fact that the rock on the Manhattan side sloped off gradually towards the middle of the river, and disappeared, so far as it concerned the territory of the trench, at a point some 1,000 feet from the pier-head line. The fact was also developed that a peak of rock jutted up slightly towards Brooklyn from the center of the river. The Manhattan rock was overlaid generally with sand or gravel, while all the rest of the strata, with

the exception of that at and for a few feet below the surface of the bottom, was the fine sand and clay or silt ordinarily found in subaqueous New York, and the further exception that the borings in the Brooklyn slip showed sand and gravel, each of the three borings being stopped by boulders within a few feet of the proposed final depth. . . .

Bids having been invited and received for the diamond-drill borings, actual work was begun shortly after the completion of the wash borings. The bids for the work ranged in prices from $9.25 to $16 a foot, the successful bidders being the United Engineering and Contracting Co., Mr. D. L. Hough, President, of 13–21 Park Row, New York.

The equipment for this work consisted of one American diamond drilling-machine, with 60 and 125 feet each of 4-inch and 6-inch extra heavy casing-pipe, and 125 feet of drill-rod; one special grooved core-lifter; one diamond bit set with six 2-carat diamonds, sundry tools, etc.; one pile-driver with superintendent and crew of 10 men, and one tug-boat. In addition to this there were for the three borings through the piers a stationary-engine boiler, and for the channel work various additional equipments, to be noted later.

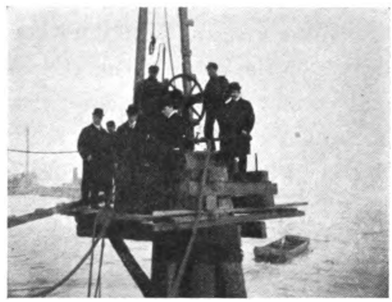
The drilling-machine was in charge of a drill-runner, — or, rather, one each for the night and day shifts, — and there was also a resident engineer for the contractor (besides the resident engineer in charge for the commissioners).

As the wash borings had indicated that the rock on the Manhattan side sloped gradually to an intersection with the proposed tunnel profile about 950 feet out from the pier line, and as another outcrop of rock was indicated slightly toward Brooklyn from the center, it was decided to make three channel borings through the Manhattan rock and two in the Brooklyn rock. Besides this, one boring through Pier 17 and one in the center of the river were decided upon, in order to investigate more definitely the conditions of these points.

The specifications in general stated the authority and regulations under which the borings were to be made, the precautionary signals to be used while anchored in the channel, gave

the approximate limits of borings (about 750 linear feet) to be made on lines 35 feet north or south of the center line, and to be measured from mean high water, all as directed by the engineer, to whom authority was given to locate and regulate the borings to be made, etc.

Boring No. 1 was made through Pier 17, Brooklyn, but as the casing-pipe was washed down without encountering rock, no special mention need be made of it. Signal-flags were then erected on the 35 north-and-south lines, on Pier 17, Brooklyn, and on the roof of one of the store-houses on Furman street, so that these lines could be picked up at any point on the river. For the up-and-down ranges, lamp-posts on the Brooklyn Bridge in line with Hecker's chimney, in Manhattan, had been noted in connection with the wash borings, as stated, and were picked up and utilized for the preliminary location.



EAST RIVER DIAMOND-DRILL BORINGS.
STAGING WITH BORING- AND DRILLING-MACHINE READY FOR OPERATION AT HOLE NO. 5.

The pile-driver used on this work was furnished by T. & A. Walsh, under contract with the United Engineering Company, and was in charge of K. Lande, superintendent. Four 500- to 700-pound anchors were tried at first, but as they could not hold the pile-driver in place, four 1-ton anchors were substituted, and with 200 feet each of 6-inch line and 300 feet of 4-inch line, no further difficulty was experienced in holding position in any tide.

On the morning of October 7, at about an hour before the last of the ebb tide, the pile-driver was towed out, and on reaching approximate position, a small anchor was dropped, and a buoy-boat attached thereto was left as a marker. Four anchors were then dropped so as to form the points of the letter "X," and the pile-driver pulled into the position indicated by the ranges and by sextant observations. A limit of 5 feet was established as the possible error for the location of these borings,

and two which were afterwards tested with a transit from a shore triangulation showed them to be off line by 6 inches in one case and 3 feet 6 inches in the other. For borings No. 2, 3, and 4, five 60-foot piles were used on which to erect a staging for the borings. These piles were driven in the form of a 5-point die, the center or king pile being the essential feature of the cluster. A chain was then attached to the king pile and carried around the outside of the other piles, being attached finally to a steam-winch, with which the four outside piles were drawn to the center one, and the chain made fast with staples and bolts. Extra turns were then taken with the chain, until the cluster was reasonably secure, after which it was bolted and braced as strongly as possible.

It was essential only that the piles should be driven and clustered before the flood tide began to run strong, as the bracing could be finished and the staging erected independently of the tide. It usually required 2 hours to drive the piles and get the preliminary work of clustering done, and some 10 or 12 hours thereafter to complete the bracing and staging, some 8 or 10 men being employed for the work. After driving the piles the work was carried on night and day without interruption, except that no work was done on Sundays, except watching and emergency work.

The 4-inch casing-pipe was next lowered to position through the platform, and the boring-machine, without the drilling attachment, consisting of an oscillating engine operating a winch, was then erected. The 1¼-inch drill-rod, in 5- and 10-foot lengths, and terminating in a perforated steel bit, called an "X" bit, was then lowered into the casing-pipe, to be used as a wash-pipe, the bit cutting through gravel and small stones. Attached to its upper end was a plug, with a nipple for line attachment; to this plug was attached a rope passing through an overhead pulley and thence to the winch. With one or two turns around the winch while in motion the drill-runner or his assistants could raise and drop the drill-rod, hammering it down into the sand, and by turning the casing-pipe with tongs it could be forced down into the opening thus made for it.

If boulders were struck a stick of dynamite was lowered into the pipe, which was pulled some 4 or 5 feet while the dynamite was exploded, after which the casing was dropped. If one or two blasts failed to shatter the boulder it was pronounced solid rock, and the hydraulic attachment for forcing down the drill was put into position and connected with the engine. The core-barrel, with its beveled ring for holding the core, was then attached to the end of the drill-rod, and to this was in turn attached the diamond bit with its setting of six 2-carat diamonds — three cutting inside and three cutting outside. The diameter of the core obtained was $\frac{7}{8}$-inch, while the hole cut in the rock had a diameter of about $1\frac{5}{8}$ inches.

Hole No. 2 was washed down to a depth of 100 feet without striking rock or boulders. No. 3 encountered a nest of boulders at the same elevation, as shown by the wash borings. After blasting, however, these boulders were penetrated, disclosing a blue clay and fine sand underlying them, which lasted for the full depth of boring, no rock being struck for 100 feet. No. 4, some 150 feet east of No. 2, encountered rock at a depth of some 58 feet, and was penetrated by the drill to a depth of 12 feet, disclosing the schistose rock common to New York, with streaks of quartzite rock.

For the borings on the Manhattan side, owing to the depth of water and greater strength of the tide, seven 70-foot piles were substituted for the five 60-foot ones, and 6-inch casing-pipe for the 4-inch previously used. Boring No. 5 was begun on the 9th of November and completed on the 19th without special interest. Rock was struck at about 60 feet and penetrated to a depth of 20 feet. In ordinary land borings the drill usually penetrates to a depth of 6 feet, *i.e.*, the full length of the core-barrel, before being pulled up. In the case of these borings, however, the vibration caused by the tide tended to dislodge the core, and the drill had to be pulled up for every 2 or 3 feet of penetration. Even then a large percentage of the core was lost, and in each instance this core had to be broken up with the X bit before the drill could again be put down.

After a sufficient degree of penetration into rock had been

EAST RIVER DIAMOND-DRILL BORINGS. A NEAR VIEW OF STAGING AND BORING- AND DRILLING-MACHINE. HOLE NO. 7.

obtained, the drill and bits were withdrawn and the casing raised, after which the staging was taken down and the piles pulled up. On the 19th of November the first attempt to get No. 6 (afterward No. 7) was made, and from then until the 5th of December the history of this portion of the boring is simply a chronicle of mishaps — five separate clusters of piles (three of which held the staging in place) were swept away by the carelessness of pilots of different craft in the strong ebb tide.

In addition to this there were more than twenty minor collisions, resulting in the loss of anchors and lines and in injury

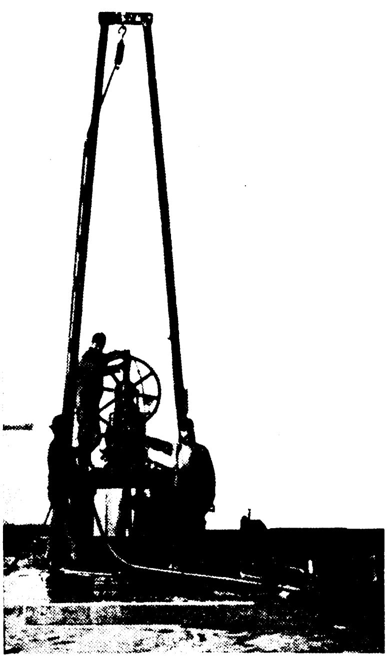
to the pile-drivers. So frequently did these accidents occur that the insurance was withdrawn, and in order to enable the work to be continued the contractor had to practically assume these insurances. Every expedient practicable was tried by the contractor, from anchoring an additional or "buffer" pile-driver up stream to calling on the police for protection. The buffer pile-driver, while warding off some collisions from the working pile-driver, succeeded in being so frequently damaged itself that it was finally towed into the dock.

The police protection consisted of a squad of six police, in reliefs of two each, placed at the disposal of the contractors, who engaged a tug to patrol the river up stream during the two ebb tides, with one relief of police constantly on board. As an extra precaution, the contractor decided to temporarily abandon No. 6, that is, to make the next inshore boring, hoping thereby that the craft would become more accustomed to the obstruction while slightly out of the lane of traffic, and thus be more on the lookout for it when it should return. This boring, finally called No. 6, was some 550 feet from the Manhattan pier line, and was some 200 feet west of No. 7 as eventually established and occupied. It was occupied and finished without special incident beyond the loss of one or two anchor-lines, which had to be dragged for, but just as the drill-rod and bit had been withdrawn, a scow in tow of a tug succeeded in bowling over the cluster and damaging the machine and casing-pipe. Another attempt was then made to drive No. 7, all possible expedition being used in getting the piles in place and the staging erected. It was impracticable, however, to work on the staging in the strong ebb tide, so that much valuable time was thus unavoidably lost.

The casing-pipe, however, was successfully washed down to rock at a depth of 63 feet, and on July 2, just at 6 o'clock, with the drill-rod, boring-machine, and diamond bit in place ready to begin boring, the steamship "Belvernon" (4,000 tons) came down on the strong ebb tide and struck the staging and entire equipment broadside, and made a complete wreck of everything except the pile-driver, which was knocked 800 feet down stream

till it rested on its own stern line. Coming as this did just as everything pointed to a successful and early completion of the channel work, it was particularly discouraging and exasperating.

The police tug gave the pilot warning, and on the witness-stand the latter acknowledged that he knew of the presence of the boring-machine, but that the accident was unavoidable. He was arrested and released on his own recognizance, owing to the fact that the vessel was carrying United States mail. The final settlement of the case has not yet been made. Next morning the broken pile-cluster, to which the drilling and boring-machines and 6-inch casing-pipe were still clinging, was towed ashore, and at slack water the two anchors were dragged for and recovered. With persistence and determination worthy of the highest praise the contracting company proceeded at once to procure new piles and erect another staging. A diver was sent down at slack water to see if any trace of the drill-rod, with its attached diamond bit, could be found, but the search was unavailing. This attempt to secure No. 7 was finally successful, and at a depth of 62 feet rock was struck on January 10, 1901, and after a penetration of 17 feet into rock the channel borings were declared satisfactorily finished on January 13, 1901.



EAST RIVER DIAMOND-DRILL BORINGS.
HOLE NO. 8 THROUGH PIER 4, MANHATTAN.

Before the drill-rod could be pulled up, however, a barge collided with the staging, jamming the diamond bit so that a diver had to be sent down to unscrew the drill-rod as far

as possible, leaving the rest of the rod and the bit in the hole. Two more borings were then made, one on Pier 4, striking rock at a depth of 30 feet and penetrating 41 feet into it, and one on Pier 3, finding the rock at 28 feet, the total depth of the boring being 60 feet, and one boring was also made on Water street, just outside of the ferry-houses. Rock was found here at a depth of 23 feet. The borings were made for and under the direction of the Rapid Transit Commissioners, of which Mr. Alexander E. Orr is president and Mr. W. B. Parsons chief engineer. Mr. Geo. S. Rice, deputy engineer, was in charge of the work, the writer acting as his representative on the ground.

In addition to the valuable strata obtained by the borings the fact was established by the contracting company that rigid pile-clusters can be established at will in the tidal rivers of New York, if the simple principle of the "king" or center pile, to which the others are drawn, is adhered to.

The writer desires to thank Mr. Rice for the courtesies extended him both in the work and in giving permission for the publication of this article.

RIVETS IN STRUCTURAL STEEL WORK.

C. J. TILDEN.

In dealing with the question of rivets in structural steel work the writer does not pretend, in what follows, to make anything like an exhaustive investigation of the subject. The intention is merely to note briefly some of the differences between theoretical and actual conditions, which have been noticed in an experience of some years in shop and drafting-room, and to suggest, if possible, a means of comparison, in this particular instance, between theory and practice.

A theoretically perfect rivet should fill the hole completely, be of homogeneous material throughout, and have two well-formed heads. The strength of a riveted joint depends, theoretically, on but two considerations: first, the shearing strength of the rivet material, usually soft steel; and, second, the number of rivets used. When comparatively thin plates are joined by rivets of large diameter, it may happen that the resistance of the metal to crushing is less than the shearing strength of one rivet; in which case the crushing or "bearing" value of the metal determines the value to be given to each rivet in calculating the strength of the joint. The question then arises, with what degree of safety may the designing engineer accept these theoretical assumptions, and how are they borne out by the conditions which occur in shop practice?

In the first place, the material of a rivet is not homogeneous. In a large majority of cases, it is probable that test pieces taken from different parts of a rivet after driving, assuming that such small pieces could be properly tested, would show widely different characteristics, and these totally different from similar tests of the same rivet before driving. A very good idea of the great difference in quality of rivet material after driving may be gained by watching for a few hours a shop gang engaged in cutting out rivets which have been condemned by the inspector. Sometimes the metal is hard, tough, and fibrous; then again

nearly as soft, to all appearances, as lead or pewter; and occasionally the rivet head will fly off at the first blow of the hammer, apparently almost as hard and brittle as glass.

A second noteworthy discrepancy in the design of riveted joints is the failure to take account of the action of the rivet heads in bringing the two or more surfaces into very close contact, so that a large amount of friction is developed. It is quite possible that this friction may amount to more than the shearing strength of the rivet. In any event, it is a very important factor in the strength of a riveted joint.

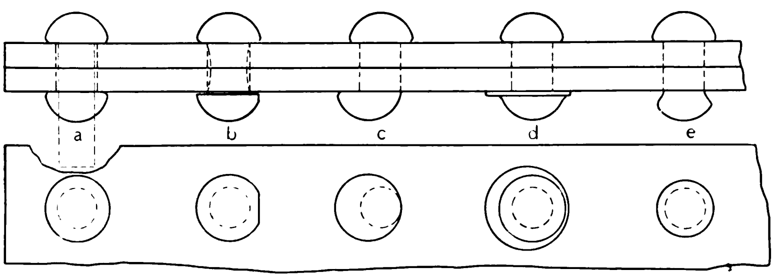


FIG. 1.

In the diagram, figure 1, are shown some of the more frequent imperfections in rivet work, resulting from carelessness of the workmen. At *a*, for comparison, is sketched a perfectly driven rivet. The original form is shown dotted, the "shank" being $\frac{1}{16}$ or $\frac{3}{32}$ of an inch less in diameter than the hole which it is to fill, and enough longer than the "grip," or length between heads, to allow the formation of the new head, and the squeezing out of the rivet material sufficiently to fill the hole completely. Both heads should be concentric with the shank, and the rivet should be perfectly tight, giving a clear, sharp ring when struck with a light hammer.

At *b* is shown a loose rivet which has been "caulked," with a cold-chisel, to make it appear tight under the inspector's hammer — a favorite trick of careless riveting-gangs, and often very difficult to detect; if suspected, a close examination should be made of the head of the rivet for signs of the caulking-tool, especially if the rivet has been generously bespattered with

fresh paint or tobacco-juice. Both these commodities, always plentiful in the shop, are favorite means of concealment for "scamped" work of this character. A result very similar to caulking, but much harder to discover, is sometimes secured by using the riveting-machine, or "bull," as it is familiarly known to the shop men, on the cold rivet. The movable cup of the "bull" is brought sharply against the rivet head, securing somewhat the effect of a blow, and this is repeated four or five times on each loose rivet. In general, this machine caulking is not very effective, but the writer has known instances where it has been successful. It is well-nigh impossible to tell from the appearance of the rivet head afterwards if this trick has been attempted. A very slight polish on the head of the rivet is about all the evidence that ever appears, and this is readily hidden by a dab of grease or dirt, or the ever-ready tobacco-juice. It is a form of "scamping" that is seldom resorted to, however, as it is more work than caulking with the cold-chisel, and far less likely to accomplish its purpose.

The sketch *b* also shows the probable result of heating the rivet unevenly. Where the heating is done in an ordinary portable forge, fired with coke, the forge-tender gets into the habit of heating only that part of the rivet which is to be upset to form the head, leaving the remainder comparatively cool. Referring to figure 2, for example, from the lower end of the rivet to, perhaps, the point *x*, the metal is at white heat; above that it cools rapidly until the head is practically "cold," often not even a dull red color. This uneven heating not only prevents the rivet from upsetting throughout its length, and so filling the hole, but is apt to injure the quality of the metal above the point *x*, owing to its being worked under the hammer at too low a temperature.

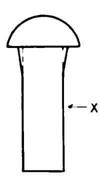


FIG. 2.

Careless manipulation of the riveting-machine may result in the condition shown at *c*, where the head is not concentric with the shank. The fault can be detected only by comparison with the other rivets in the joint, showing uneven spacing and irregular lines.

The condition shown at *d* results from too much metal in the shank of the rivet before driving, giving a "soldier-cap" head. The reverse of this is shown at *e*.

It must not be supposed that these defects are the only ones which occur in rivet work; they are only a few of the more frequent errors of this kind that may be observed in any shop. Combinations of two or more of the forms shown occur not infrequently, and an almost endless variety of changes may be rung on each one. Of the four types, *b* and *e* should be condemned unquestioningly whenever found, being not only bad workmanship but unreliable. *c* and *d* probably develop the full strength of the rivet, and may be allowed to pass if strength is the only consideration; but if the work is to be exposed they should be cut out and replaced, as they are sure to look ragged in finished work.

As to the actual difference in strength between a perfect rivet, as *a*, and any of the imperfect ones, it is impossible to judge with any degree of accuracy. In fact, if a test were made it is quite conceivable that a rivet such as *b*, or even *e*, might develop greater strength than *a*. About all that can be said is that this is not likely to happen, but rather the reverse, as a properly driven rivet is more likely to develop its full strength than one which is imperfect in any way. But this is not reducing the question to any scientific basis, and, indeed, it cannot

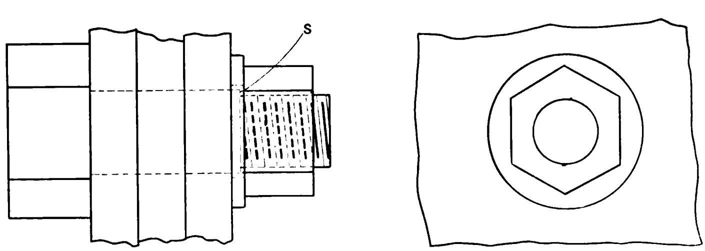


FIG. 3

be so reduced. Rigid specifications are required for riveted work, and the work in the shop is subjected to the most careful inspection, not because a carelessly driven rivet is less strong, by any definitely calculable percentage, than one which is properly

driven, but for the simple reason that careful and accurate work is more reliable.

The nearest approach to a theoretically perfect rivet is probably the turned bolt which is occasionally used for field connections (figure 3). In such cases it is the practice of some engineers to require the holes to be drilled instead of punched, or "subpunched and reamed" — that is, punched to a diameter about $\frac{1}{4}$ inch less than that of the bolt to be used and reamed to proper size. The bolt is turned to a driving fit, and the threaded part is of slightly reduced diameter, the shoulder, *s*, protecting the thread while the bolt is driven home. To keep the nut in place after it is screwed up tight, the projecting threaded end of the bolt is upset against the nut. In spite of the reliability of this connection, however, its high cost precludes its general use.

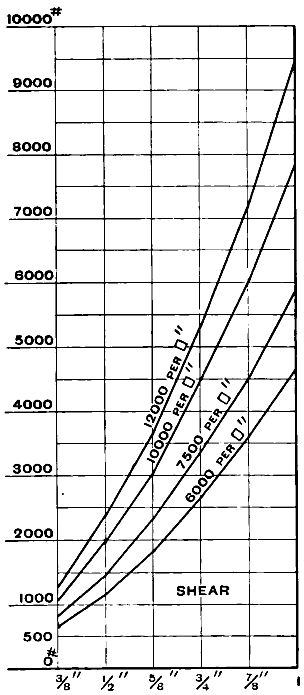


FIG. 4.

Figure 2 shows a form of rivet which has certain advantages and disadvantages over the ordinary shape. In this form the shank is slightly increased in diameter (exaggerated in the drawing) for a distance of $\frac{1}{2}$ to $\frac{3}{4}$ of an inch from the head. Directly under the head, at the base of the cone-like enlargement, the shank has the same diameter as the hole into which the rivet is to go — that is, from $\frac{1}{16}$ to $\frac{3}{32}$ of an inch larger than the main part of the shank. This is an advantage in the shop, where the rivet is sure to be uniformly heated throughout its length, as it insures the complete filling of the hole up to the rivet head. In the field, however, where the rivets are likely to be unevenly heated, such a design would be of doubtful advantage. A rivet of this shape might easily appear sound and tight under the inspector's hammer, and yet have been very imperfectly driven.

The shearing and bearing values of rivets, for various allowable unit stresses, are given in the form of tables in the steel hand-books. In figure 4, one of these tables of shearing strength is shown graphically by means of curves, the abscissas representing the rivet diameters in inches, and the ordinates the values of the shear in pounds. These curves are purely theoretical, based on assumed safe unit values for the shearing strength. The lower curve assumes the rivet material to be capable of resisting a shearing stress of 6,000 pounds per square inch; the others are for assumed stresses of 7,500, 10,000, and 12,000 pounds per square inch respectively, as noted. These are the safe unit stresses most commonly assumed in practice, and are all, of course, far below the actual shearing strength of the steel.

Now suppose a series of experiments made on riveted joints to determine their actual resistance to shearing forces. Such experiments could be made with rivets of different sizes, driven under different conditions, and from them the actual strength of a rivet under the given conditions deduced. The values thus obtained could be plotted on the same sheet with the theoretical curves, and through the plotted points curves of actual values drawn. A comparison of these two sets of curves could hardly fail to be interesting and instructive.

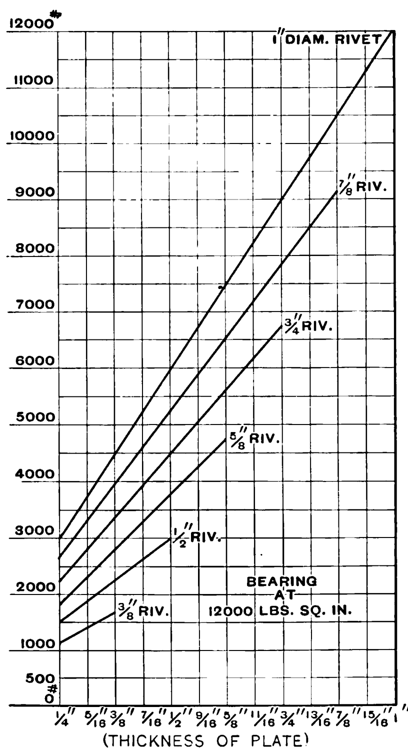


FIG. 5.

Figure 5 is a diagram somewhat similar to the foregoing, also compiled from the tables in the steel hand-books, showing graphically the bearing power of rivets of varying sizes on

different thicknesses of plate, assuming steel to bear safely a crushing stress of 12,000 pounds per square inch. An examination of the diagram will show the method of using it. For instance, assuming the metal to be good for 12,000 pounds' compression per square inch, the bearing power of a rivet 1 inch in diameter on a plate $\frac{3}{4}$ of an inch thick is 9,000 pounds; that of a $\frac{3}{4}$ inch rivet on a $\frac{5}{8}$ inch plate is about 5,600 pounds, etc. These are, of course, the theoretical values. By means of judicious experiments a set of actual values corresponding to the theoretical could be secured and plotted in the same way as suggested for the case of shear, and curves drawn in, showing actual conditions. The relation between actual and theoretical values of bearing power would then be made graphically comparable, as before.

An investigation, along these lines, of the relations existing between the theories assumed by designing engineers and the actual conditions of rivet work would make a valuable and instructive subject for a graduating thesis. With the unusually fine laboratory equipment of the Division of Engineering of Harvard University, the work of testing riveted joints of different kinds could be done very accurately, and there is little doubt but that any bridge-shop of good standing would be glad to coöperate in work of this character by furnishing, at cost, samples of rivet work to be tested. These samples should, of course, be made up under the personal supervision of the investigator, who would note fully all the conditions of their manufacture. A careful analysis and discussion of the results of these tests would not only furnish excellent training for the student of engineering, but would also make a valuable addition to engineering literature.

Structural engineering especially has advanced so rapidly that its followers have had little or no time to devote to its purely experimental phases. It would seem as if a great deal of such work could be put into the technical schools, with much profit alike to the student and to the profession he seeks to enter.

THE THERMOGRADE SYSTEM OF STEAM HEATING.

Walter E. Barnes.

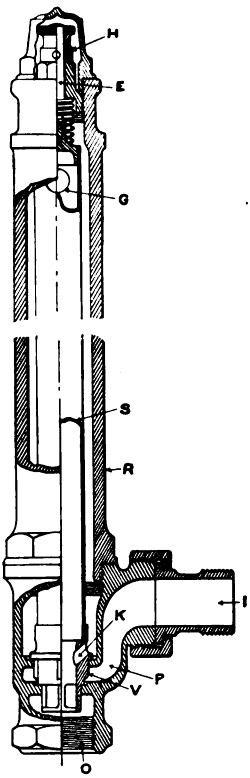
THE merits of steam heating, as a whole, over all other forms of heating, have been recognized for years, and its adaptability to distribution from a central source in almost any manner, or to any extent, has been seen to be of great advantage.

One great handicap, however, has been the fact that it has been practically impossible to regulate the amount of heat at the point of distribution, and thus, in addition to the discomfort experienced, the consumer was forced to use practically as much steam on a warm day as during the most severe cold snap. Another disadvantage lay in the fact that two and sometimes three valves had to be manipulated in connection with every radiator, and that even then water-hammer, air-binding, etc., oftentimes resulted. The Thermograde system of steam heating removes some of these drawbacks and incidentally confers several benefits, and, therefore, the following description may be of interest.

The Thermograde system of steam heating is one of the few strictly two-pipe low-pressure systems. The ordinary two-pipe system, so called, is in reality generally a three-pipe system — there being one supply- and one return-pipe to every radiating surface, and in addition a drip-pipe from the air-valve to carry off condensation and obnoxious odors.

The Thermograde uses but two pipes — one, the supply, being a steam-carrier, the other, the return, being both an air- and water-carrier. The fact that the returns are used as both air- and water-pipes is the first point of difference between this and the ordinary two-pipe system. The other distinction lies in the fact that the return side of the Thermograde system is free from pressure either of air or steam. In the common two-pipe system the return side is under practically the same pressure as the supply side, allowing for the slight losses caused from friction, etc., but in the Thermograde system the return side is open to the atmosphere at some point, either by venting the return risers at the top or venting the receiving tank, if one is used, or both.

Now, some means must be employed to prevent the pressure existing on the supply side of the Thermograde system from passing into the returns, and so the ordinary valve on the return end of each radiator is replaced by a combined automatic steam-trap and air-valve, or autovalve, which is shown in sectional view in the accompanying cut.



This valve is an automatic steam-trap and air-valve, constructed entirely of metal. When the radiating surface to which the valve is attached is fully heated, the valve closes automatically by the expansion of the zinc tube S, as explained below. Under all other conditions of the radiating surface — that is, when the surface is heated to half or three-quarters or anything less than full — the valve V remains open and off its seat, and allows the condensed water to drain freely through opening I to port P and the outlet O into the returns. When steam admitted to the radiating surface reaches the autovalve it passes through port P, and rises in the space between the zinc tube S and the iron column R, thence through the two openings G at top of the zinc tube and downward through the interior of the tube to the return side of the system. The passage of steam, as described, causes the expansion of the zinc tube, and valve V becomes seated. This prevents the further passage of steam from the radiating surface beyond port P of the autovalve, so that the zinc tubing in the valve gradually loses its heat from the first passage of steam, and contracts; with this contraction the valve is raised off its seat, and the water of condensation which has accumulated in the radiating surface drains away; the passage of additional steam by port P is also per-

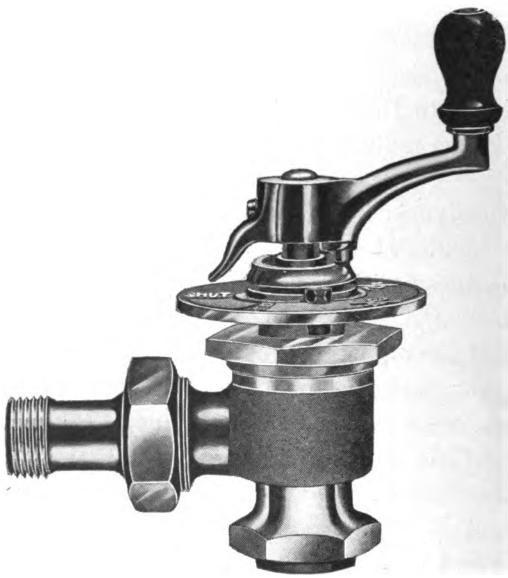
mitted, which results in another expansion and the seating of the valve again.

The effect of these actions is to keep the steam side of the autovalve free from water, while the expansive members gradually take a permanent temperature and position, which is that of allowing only sufficient steam to pass port P to maintain the temperature of the valve at the critical point. In other words, under the conditions described practically only the lower half of the autovalve is steam hot, and it thus passes the water of condensation into the returns and at the same time conserves the steam in the radiating surface and prevents its passing the valve.

The autovalve is instantly cleared of scale or sediment by lifting the handle H attached to rod E, thus raising the valve to an emergency position and allowing a full pressure of steam to sweep over the seat. This is accomplished without disarrangement of the adjustment of the valve — a novel feature not found in other similar automatic devices. It will also be noted that the autovalve can only be acted upon by steam, there being no danger of the hot water of condensation surrounding the zinc tube to cause expansion, as the moment water rises above the valve-seat it is drained into the returns by gravity through the two openings K.

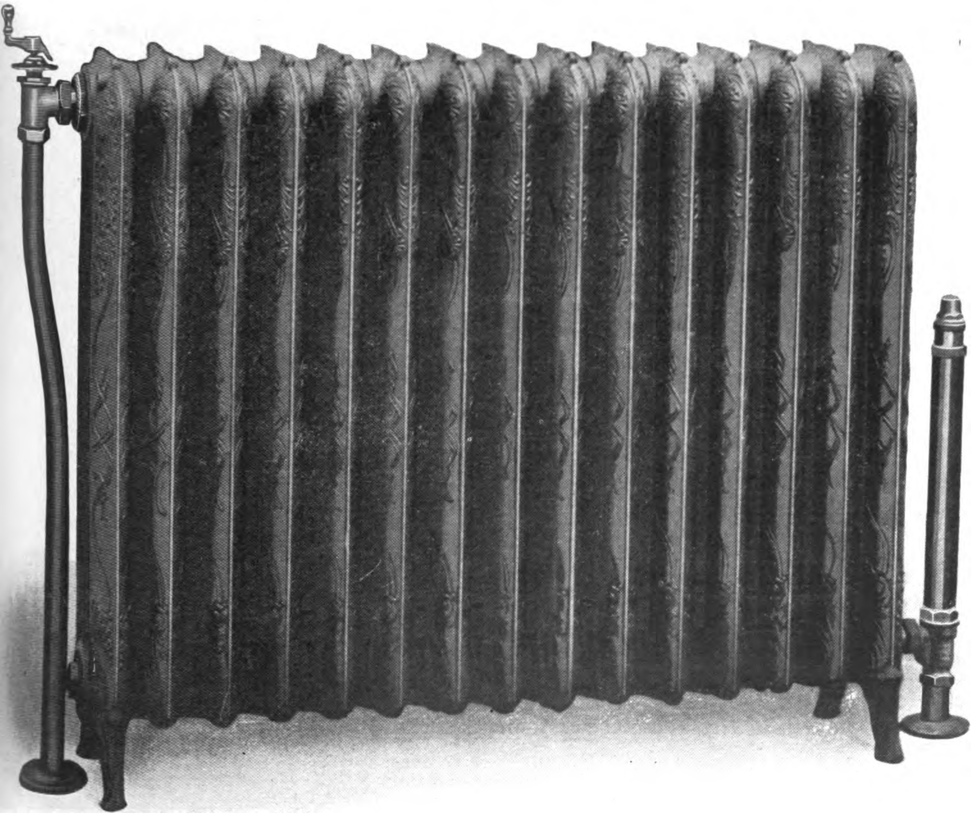
In the ordinary two-pipe heating system, owing to the fact that both sides of the system are under steam pressure, it is impossible to shut off any radiator by closing only the supply valve, as even under these conditions the steam will fill the radiator from the return side. In the Thermograde system, however, since steam pressure has been removed from the returns, it is not only possible to turn steam on or off from any radiator by manipulating only the supply valve, but it is also possible to admit just enough steam to a radiator so that any fractional part of the surface of that radiator will be sufficient to condense all the steam admitted. In other words, by varying the quantity of steam admitted to any radiator, it becomes possible to vary the extent of the effective heating surface of the radiator. For example, under a given pressure, a 50 square

feet radiator would normally condense a certain quantity of steam; if, then, only one-half that quantity be admitted to the radiator, only one-half the surface of the radiator will be necessary to condense all the steam admitted, and, therefore, in this particular case, only 25 square feet of the radiator will be effective heating surface.



As it is impossible with the ordinary radiator valves now used to secure a fine enough graduation of the effective heating surface, it became necessary to use the Thermograde supply, or control, valve. This valve is exceedingly simple in construction, having few parts and being little liable to disarrangement. It has a metal seat, and its dial is graduated to indicate the amount of heat obtained from the radiator. The valve-spindle is raised and lowered by the handle to which it is attached, and travels on the surface of a cam. The amount of opening is regulated by the pitch of the cam, which can be varied to suit the individual conditions under which different radiators may have to operate and the various pressures under which different plants may be run. Under ordinary conditions this pressure will be about 2 to 2½ pounds.

On this page is shown cut of a radiator equipped with the Thermograde valves. It will be noticed that the radiator is of the hot-water type, and that the control valve is connected at the top. This places in the most convenient position the only valve requiring operation, and as the steam is admitted at the top



of the radiator and the return taken out at the opposite end, the removal of air is effectually accomplished. All water of condensation drains away as it accumulates, and thus all water-hammer and kindred annoyances are avoided.

The fact that whatever steam pressure the plant is operating under exists solely on the supply side of the system, and that

there is no pressure on the return side, insures a positive circulation through every radiator — a matter of first consideration in any heating system. When steam is admitted to a radiator on the Thermograde system, it flows along the top passage and then descends through the flues toward the bottom passage, the air flowing out before it, through the autovalve into the return risers, and thus through the point of ventage into the atmosphere. The water of condensation drains away through the autovalve into the returns, and thus back to the boiler, until the steam, having filled the radiator, reaches the autovalve, and, heating up the zinc tube, causes it to expand and close. The valve remains closed but for a comparatively short time, however, as the closing of the valve frees the zinc tube from the heat of the steam and it gradually cools and contracts. If the radiator is run steadily at full heat for a time, the autovalve gradually takes a mean position, allowing the water to drain away, while at the same time preventing the steam from passing beyond the radiator.

If a radiator fully hot should be cooled down, the autovalve, being entirely relieved of the action of the steam, opens more freely and allows the air that is being drawn into the return risers from the atmosphere to pass into the radiator and fill the vacuum caused by the condensation of the steam that had filled the radiator.

Any system that is capable of effective regulation is of necessity much more economical than one that is not, but when the action of the autovalve is considered in connection with the regulation secured by the control valve, it becomes apparent that the matter of economy is hardly secondary to any other advantage to be derived by the use of the Thermograde system. The autovalve prevents the steam from passing beyond the end of each radiator, thus practically all the steam generated for heating purposes is conserved where it is most effective and where it is most desired — *i. e.*, in the radiators, and none of it passes into the returns to give forth a surplus not counted upon, which in some plants would amount to ten per cent. of the entire heating surface. Owing to the fact that the returns are simply air- and

water-carriers, it is not customary in this system to cover them, thus avoiding an item of considerable expense.

Briefly, the Thermograde system enables one to secure regulation of temperature by graduating the amount of heating surface in use; to obtain complete control of each radiator by the manipulation of only one valve, the supply, the return being automatic; to discard the use of all air-valves and drip-piping from them; to secure an economy of operation that is self-evident; a positive circulation; a saving of space, owing to the absence of one line of piping, and the avoidance of water-hammer and its disagreeable features. One point not to be overlooked is the fact that all of these advantages are secured by simple and natural means and without the use of any accessories, such as vacuum-pumps, etc.

In attempting to describe any subject at all out of the ordinary it is natural that comparisons between it and those with which we are most familiar should present themselves; but, when one attempts to follow out this course in connection with the Thermograde system, the conclusion is reached that such a method is practically useless, for the Thermograde occupies a field of its own. No other system attempts to do in its entirety what the Thermograde does. Some have started from one end of the radiator, so to speak, and some from the other, and have accomplished part of what the Thermograde does, but none have been able to grasp the problem wholly. When, therefore, one attempts to compare any heating system with the Thermograde, it will be found that such a system will have to be compared only with parts of the Thermograde.

The Automatic system, at first glance, seems to offer several advantages over the Thermograde, but, in reality, when one thoroughly looks into the matter, it is apparent, as a prominent engineer of Boston recently put it, that the chief function of the Automatic system is to prevent temporary over- or under-heating, rather than to do away with all personal attention. The Automatic does not attempt to effect a control of temperature by a graduation of heat emitted, which, of course, is the true way; it is practically as wasteful as the ordinary two-pipe system, and

the Thermograde system, as it requires only the manipulation of one valve to each radiator, probably does not demand any more personal attention than any automatic system with its variable thermostats. In simplicity of construction and operation the Automatic is not to be compared with the Thermograde.

The economy of Vacuum systems is much advertised, but this ordinarily does not carry great weight with one at all familiar with the subject. Vacuum systems are chiefly advocated in connection with exhaust-steam heating, but the economy that they seem to effect here is of little moment, for even in moderate weather considerable live steam is necessary to run the pumps used to produce the vacuum, and, furthermore, practically as much steam is used for this purpose in warm weather as in cold. The economy of using exhaust steam is indisputable, but that it is necessary to employ a vacuum in connection with it to produce the best results is a fallacy. Two or three decades ago, when these systems were first introduced, they had a field of their own, and to a limited extent they are to-day of benefit in some cases. When first brought forth, they made possible the economical use of exhaust steam in connection with heating systems whose piping was very small, viewed from the standpoint of to-day, and where twenty, thirty, and even forty pounds pressure had previously been employed to produce the best results. In these instances the back pressure produced at the engine by trying to force the exhaust through such a system of piping made its practical use an impossibility. When a vacuum was applied to the return side, however, it necessarily showed a large measure of economy, and in these cases was of benefit. To-day, however, piping is well proportioned, plants are designed to operate on a low pressure, and vacuum systems are practically limited in their application to those few cases where, from force of circumstances, what might be termed artificial means have to be employed to produce a circulation.

ON ELECTRIC CONDUCTING LINES OF UNIFORM CONDUCTOR AND INSULATION RESISTANCE, IN THE STEADY STATE.

A. E. KENNELLY,
Professor of Electrical Engineering.

SOME of the quantitative relations of uniformly leaky lines are not known so generally as the importance of the subject demands.

The subject naturally falls within the domain of telegraphy, but extends also into telephony and into electric power transmission. It applies to all long uniform conductors.

If a perfectly insulated line is freed at the distant end, and connected to a steady e. m. f. *e* at the sending end, or home end, then the pressure remains uniform from one end to the other, as shown in figure 1, while the current-strength is, by hypothesis, everywhere zero.

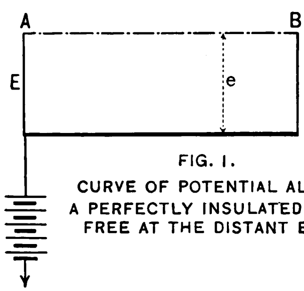


FIG. 1.
CURVE OF POTENTIAL ALONG A PERFECTLY INSULATED LINE, FREE AT THE DISTANT END

Moreover, if the line be grounded at the distant end, the pressure falls steadily along the line, as in figure 2, while the current-strength remains uniform, as in figure 3.

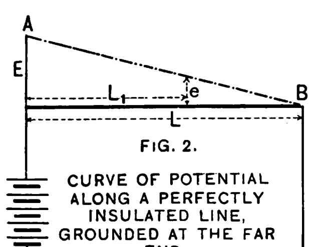


FIG. 2.
CURVE OF POTENTIAL ALONG A PERFECTLY INSULATED LINE, GROUNDED AT THE FAR END.

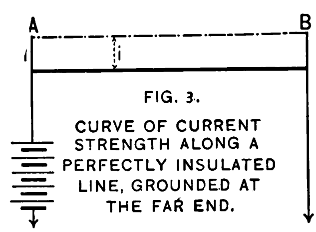


FIG. 3.
CURVE OF CURRENT STRENGTH ALONG A PERFECTLY INSULATED LINE, GROUNDED AT THE FAR END.

The equation relating to figure 1 is $e = E$ volts . . (1)
The equation relating to figure 2 is $e = E - IL_1r$ volts (2)
The equation relating to figure 3 is $i = I$ amperes . (3)

Where L_1 is the length of the line in miles or kilometers from the sending end, and r is the uniform conductor-resistance per unit length.

The curves of potential and current in figures 1, 2, and 3 are, of course, rectilinear.

If resistances be inserted into the line of figure 2, as by the injection of instruments or apparatus into the line, the line A B will either be a new straight line, or a series of parallel straight lines, according as the abscissas are selected to units of resistance or to units of distance.

Again, if a leak or leaks are applied to the line at definite points, the effect will be to make a pressure curve of a series of straight lines and a current curve of similar straight lines no longer parallel to each other.

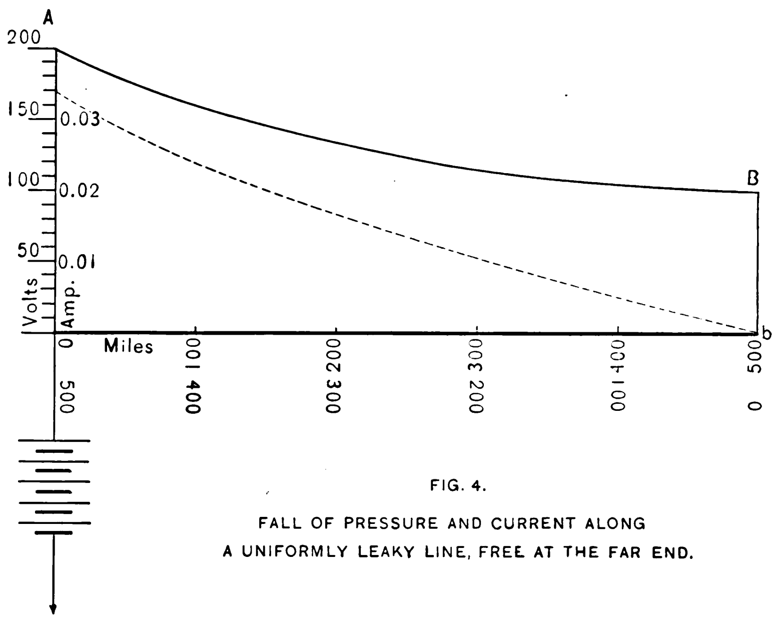


FIG. 4.

FALL OF PRESSURE AND CURRENT ALONG A UNIFORMLY LEAKY LINE, FREE AT THE FAR END.

Consequently, the diagrams of pressure and of current on a line of perfect insulation are essentially rectilinear diagrams. From an arithmetical point of view, Ohm's Law is the key to all of them.

If, however, the line has a uniform insulation-resistance of R ohm-miles, or ohm-kilometers, corresponding to a uniform leakage-conductance, or leakance, of $\frac{1}{R} = g$, mhos per mile, or mhos

per kilometer, then the simple rectilinear curve of potential in figure 1 becomes a catenary, or curve of a free uniform chain, as in figure 4, where the full line A B is a catenary curve, or hyperbolic cosine curve, indicating the fall of potential or voltage from 200 volts at A to 100 volts at B, in an assumed distance of 500 miles; while the dotted line represents the leakage current-strength to a different scale, falling from 33.9 milliamperes at A to zero at B. This dotted line is a hyperbolic sine curve.

The remarkable property connecting these two curves A B and A b is that at any abscissa, or distance from A, the slope of one is proportional to the ordinate of the other. Thus at A the slope or gradient of the potential line A B is a fall of 0.46 volts per mile, and the current is 0.0338 ampere, a numerical ratio of 14.0. At another point, such as that midway along the line, the slope of potential is a fall of 0.186 volts per mile, and the current is 0.013 ampere, likewise in the ratio 14. A similar condition applies to the slope of the current line and the corresponding ordinate of the potential line.

The pressure and current along a line of uniform resistance and leakance are subject to the following conditions:

$$\mathbf{e} = \mathrm{E} \cosh \mathrm{L}_1 \alpha - \mathrm{I} \mathrm{r}_0 \sinh \mathrm{L}_1 \alpha \qquad \text{volts} \quad . \quad . \quad (4)$$

$$\text{and} \quad \mathbf{i} = \mathrm{I} \cosh \mathrm{L}_1 \alpha - \frac{\mathrm{E}}{\mathrm{r}_0} \sinh \mathrm{L}_1 \alpha \qquad \text{amperes} \quad . \quad (5)$$

where **e** and **i** are the pressure and current at a distance of L_1 miles (or kilometers) from the sending end, E and I the pressure and current at the sending end ($L_1 = 0$); while the constants α and r_0 are defined by the relations

$$\alpha = \sqrt{\frac{\mathrm{r}}{\mathrm{R}}} = \sqrt{\mathrm{rg}} \qquad \text{per mile} \quad (6)$$

where r is the conductor-resistance in ohms per mile or kilometer, R is the insulation-resistance in ohm-miles or ohm-kilometers, and $g = \frac{1}{R}$ is the insulation-conductance in mhos per mile or kilometer. This constant α is numerically a small fraction of the dimensions $(\text{length})^{-1}$, and is commonly called the "*attenuation-constant*" of the line.

$$r_0 = \sqrt{rR} = \sqrt{\frac{r}{g}} \qquad \text{ohms . . (7)}$$

r_0 is thus a resistance equal to the geometrical mean of the conductor and dielectric resistances referred to unit-length of line. The resistance r_0 is sometimes called the "*surge-resistance*," because it is, in generalized form, the resistance which the line offers to free waves and surges. It is also the resistance offered by the line when the length of line is so large that terminally reflected disturbances are negligible.

If we consider a telegraph line with a conductor-resistance of 10 ohms per mile ($r = 10$), and an insulation resistance of 2 megohm-miles ($R = 2{,}000{,}000$), then the attenuation-constant for that line will be

$$\alpha = \sqrt{\frac{10}{2{,}000{,}000}} = \frac{1}{\sqrt{200{,}000}} = \frac{1}{447.2} = 0.002236 \text{ per mile,}$$

and $r_0 = \sqrt{10 \times 2{,}000{,}000} = \sqrt{20 \times 10^6} = 4.472 \times 10^8 = 4472$ ohms

With fairly well insulated telephone and telegraph circuits the attenuation-constant (referred to the mile as unit of length) is of the order $\frac{1}{1000}$, or 0.001. With very well insulated lines, such as lines in dry weather or subterranean and submarine cables, α may be $\frac{1}{10{,}000}$, or even $\frac{1}{100{,}000}$; while for commercial telegraph and telephone lines in leaky condition it may be $\frac{1}{100}$. Consequently α may be said to range from $\frac{1}{100{,}000}$ to $\frac{1}{100}$. The value of α for one and the same line will increase as the unit of length adopted, being about 60 per cent. greater when the unit of length is the mile than when the unit is the kilometer. On the other hand, the attenuation-length $L\alpha$ is independent of the unit of length selected, and its dimensions are zero, or those of a mere numeric.

The physical meaning of α is the Naperian logarithm of the attenuation of outgoing waves per unit-length of line. Any wave undergoes in transmission over one mile (or kilometer) an attenuation in the ratio of $\frac{1}{\epsilon^{\alpha}} = \epsilon^{-\alpha}$. Thus, with $\alpha = 0.001$ (on the mile-unit of length) the attenuation of waves running over one mile will be $\epsilon^{-0.001} = 10^{-0.0004343} = \frac{1}{1.001}$; so that a wave of ori-

ginal amplitude would be diminished to $\frac{1}{1.001}$ or $\frac{1}{\epsilon^a}$ or ϵ^{-a} after running one mile, and would be diminished to ϵ^{-ax} after running x miles.

Equation (4) involves a knowledge of the initial current-strength I. But in order to express the potential at any point along the line when freed at the far end, we may eliminate I and write

$$e = E\,(\cosh L_1 a - \sinh L_1 a \tanh La) \qquad \text{volts} \quad . \quad (8)$$

which gives the potential at distance L_1, from A, figure 4, in terms of E and of hyperbolic functions of L_1 and L.

If we solve (8) for the pressure at the distant free end of the line,

$$L_1 = L \quad \text{and } e = E \operatorname{sech} La \qquad \text{volts} \quad . \quad . \quad (9)$$

If we call La the *attenuation-length* of the line, then the equation states that, considering the impressed voltage at the home end of the line as unity, the voltage at the distant free end will be the hyperbolic secant of the attenuation-length.

Consider, for example, the case depicted in figure 4, where L = 500 miles, r = 13.94 ohms per mile, R = 2 megohm-miles, and E = 200 volts. Then $a = 0.002634$. The attenuation-length is $La = 1.317$ and sech 1.317 = 0.5. The pressure at the distant end is, therefore, $200 \times 0.5 = 100$ volts.

If two lines freed at their far ends have the same attenuation-length, or La, their percentage drop of pressure at the distant ends will be the same, no matter what their individual lengths may be.

Similarly, the length of line which will give 50 per cent. loss of pressure at the free distant end is the quotient of 1.317 by the attenuation-constant. Thus, with $a = 0.0001$ for a highly insulated line, the length is 13,170 miles, while for a very leaky line, with $a = 0.01$, the length is 131.7 miles.

When the attenuation-length La is very short, its secant is nearly unity, and an accurate value is not easily found from tables. An expansion, however, leads to the approximate equation applicable to such cases

$$e = E\left(1 - \frac{L^2a^2}{2}\right) \qquad \text{volts} \quad . \quad . \quad (10)$$

This equation is readily proved by placing a total leak of Lg mhos in the middle of the line, and finding the drop of the leakage-current through $\frac{Lr}{2}$, half the line-resistance, or the resistance from the sending end to the leak.

The following table illustrates the use of formulas (9) and (10), and gives the amount of the applied pressure reaching the distant free end of lines having different lengths and attenuation-constants. Thus, for $a = 0.001$, and $L = 200$, the value is 0.9803, or 98.03 per cent.

	$a =$	0.0001	0.0005	0.001	0.0025	0.005	0.0075	0.01
Length Miles.	50	0.9999	0.9997	0.9988	0.992	0.970	0.930	0.8868
	100	0.9999	0.9987	0.9950	0.970	0.887	0.772	0.6481
	200	0.9998	0.9950	0.9803	0.887	0.648	0.425	0.2658
	300	0.9995	0.9888	0.9566	0.772	0.425	0.209	0.094
	400	0.9992	0.9803	0.9250	0.648	0.266	0.094	0.037
	500	0.9988	0.9695	0.8868	0.530	0.163	0.048	0.014
	600	0.9982	0.9566	0.8435	0.425	0.094	0.022	0.005
	700	0.9975	0.9416	0.7967	0.337	0.060	0.011	0.002
	800	0.9968	0.9250	0.7477	0.266	0.037	0.005	0.001
	900	0.9960	0.9066	0.6978	0.209	0.022	0.002	0.0003
	1000	0.9950	0.8868	0.6480	0.163	0.014	0.001	0.0001

To find the pressure at a point distant L_1 miles (or kilometers) from the home end, when the distant end is free, formula (8) may be used.

A simpler formula may, however, be obtained by taking the distance L_2 of the point from the far free end. The required relation is

$$e = e_B \cosh L_2 a \qquad \text{volts} \quad . \quad (11)$$

where e_B is the pressure at the far free end, B. This is expressed by formula (9) in terms of the pressure at the home end. Substituting from (9) we obtain

$$e = E\,\frac{\cosh L_2 a}{\cosh La} \qquad \text{volts} \quad . \quad (12)$$

Formula (11) shows that, when the line is free at the distant end, the pressure increases towards the home end as the hyperbolic cosine of the attenuation-distance from the far end. Thus, taking the case presented in figure 4, in which $r = 13.94$, $R = 2{,}000{,}000$, $L = 500$, and $E = 200$ volts. The pressures at each 100-mile distance along the line may be required. The first step is to find $\alpha = \sqrt{\frac{13.94}{2{,}000{,}000}} = 0.002634$. Then to find the pressure at B $= E \operatorname{sech} (500 \times 0.002634) = 200 \operatorname{sech} 1.317 = 100$ volts. Starting along the line from B, the attenuation-lengths at successive 100-mile distances are $100 \times 0.002634 = 0.2634$, 0.5268, 0.7902, 1.0536, 1.317. The pressures at these points are, therefore,

$$
\begin{aligned}
e_{100} &= 100 \times \cosh 0.2634 = 103.5 \text{ volts.}\\
e_{200} &= 100 \times \cosh 0.5268 = 114.1 \text{ volts.}\\
e_{300} &= 100 \times \cosh 0.7902 = 132.9 \text{ volts.}\\
e_{400} &= 100 \times \cosh 1.0536 = 154.9 \text{ volts.}\\
E = e_{500} &= 100 \times \cosh 1.317 \ \ = 200. \text{ volts.}
\end{aligned}
$$

These are the pressures found in figure 4, on the line AB, at the said distances from B.

Equation (12) may also be written in the form

$$e = E \frac{\cosh (L - L_1)\,\alpha}{\cosh L\,\alpha} \qquad \text{volts} \qquad (13)$$

in which all distances are measured from A, the sending end. Formulas (8) and (13) are equivalent, and one may readily be transformed into the other.

Equations (4) and (5) express the pressure and current at any point along a line in terms of the initial pressure and current at the sending end. The following equations, (14) and (15), express the results in terms of the pressure and current at the receiving end:

$$\mathbf{e} = e \cosh L_2\alpha + i\,r_0 \sinh L_2\alpha \qquad \text{volts} \quad . \quad (14)$$

$$\mathbf{i} = i \cosh L_2\alpha + \frac{e}{r_0} \sinh L^2\alpha \qquad \text{amperes} \qquad (15)$$

where $\mathbf{e}$ and $\mathbf{i}$ are respectively the pressure and current at distance L_2 miles or kilometers from the distant end; while α and

r_0 are the attenuation-constant and surge-resistance, as already defined. The pressure and current at the distant or receiving end are denoted by e and i.

In the case of perfect insulation, or $a = 0$, equations (4), (5), (14), and (15) become respectively

$$\mathbf{e} = E - I\ r\ L_1 \qquad \text{volts} \quad . \quad . \quad . \quad (16)$$

$$\mathbf{i} = I \qquad \text{amperes} \quad . \quad . \quad (17)$$

$$\mathbf{e} = e + i\ r\ L_2 \qquad \text{volts} \quad . \quad . \quad . \quad (18)$$

$$\mathbf{i} = i \qquad \text{amperes} \quad . \quad . \quad (19)$$

which are the ordinary expressions for constant current and resistance-drop of pressure along a direct-current line.

If we free the far end of the line, we have the condition of $= 0$ in equations (14) and (15), so that

$$\mathbf{e} = e \cosh L_2 a \qquad \text{volts} \quad . \quad . \quad . \quad . \quad (20)$$

$$\mathbf{i} = \frac{e}{r_0} \sinh L_2 a \qquad \text{amperes} \quad . \quad . \quad . \quad (21)$$

$$\text{and } \frac{\mathbf{e}}{\mathbf{i}} = r_f = r_0 \coth L_2 a \qquad \text{ohms} \quad . \quad . \quad . \quad . \quad (22)$$

This means that the resistance offered at any point by the line beyond, when the far end is free, is the product of the surge-resistance of the line, and the hyp: cotangent of the attenuation-length.

Consider, for example, a line of $r = 10_\omega$ per mile, and $R =$ 2.5 megohm-miles. Then $a = \frac{1}{500}$ and $r_0 = 5000$ ohms, by formulas (6) and (7). If the distant end of such a line be freed, the resistance offered by 300 miles of such line will be

$$5000 \times \coth \tfrac{300}{500} = 5000 \coth 0.6 = 5000 \times 1.862 = 9310 \text{ ohms.}$$

When the line is very long, so that $L_2 a$ becomes greater than 3, then the hyp: cotangent becomes very nearly unity; so that

$$r_f = r_0 \qquad (\text{when } L_2 = \infty) \quad . \quad (23)$$

or the resistance offered by the line is equal to the surge-resistance r_0.

We have hitherto considered telegraph lines which employ a ground return-circuit. In telephony metallic circuits are used, *i.e.*, two parallel conductors. Both wires may be regarded as

normally possessing a like conductor resistance and like leakance. They may each be reduced to the preceding case of a single wire and ground return, by dividing the impressed e. m. f. by 2, as also the terminal resistance at the receiving end, and multiplying the insulation by 2. Thus a loop-telephone circuit 500 miles long, containing 1000 miles of wire, having a conductor-resistance of 10 ohms per mile of loop, and an insulation-resistance between the sides of the loop of 2 megohm-miles, with a battery of 80 volts at the sending end, and a resistance of 200 ohms at the receiving end, as indicated in figure 5, would be the equivalent of two independent wires, each with ground return, as in figure 6, and each with 40 volts at A, 100 ohms at B, 5 ohms per mile, and 1 megohm-mile.

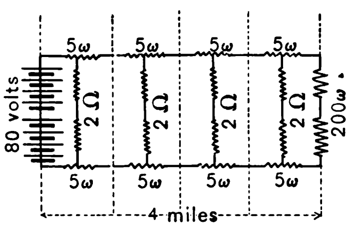


FIG. 5.

DIAGRAM OF FOUR MILES OF LINE WITH 10ω PER LOOP MILE AND 2 MEGOHM-MILES.

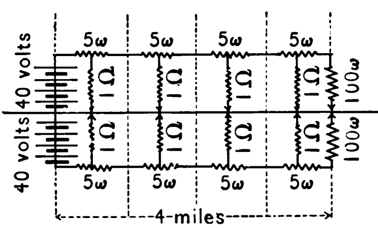


FIG. 6.

DIAGRAM OF FOUR MILES COMPOSED OF TWO SEPARATE CIRCUITS WITH GROUND RETURN, EACH HAVING 5ω PER MILE AND 1 MEGOHM-MILE.

In general, if r_1 be the resistance per mile of loop (ohms);
g_1 be the leakance per mile of loop (mhos),
while r and g are the corresponding quantities per mile of single wire, referred to ground return,

$$r = \frac{r_1}{2} \text{ ohms, and } g = 2\,g_1 \qquad \text{mhos} \quad . \quad (24)$$

$$\alpha_1 = \sqrt{r_1\,g_1} = \sqrt{rg} = \alpha \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (25)$$

$$r_{1o} = \sqrt{\frac{r_1}{g_1}} = \sqrt{\frac{2\,r}{\frac{g}{2}}} = 2\,r_o \qquad \text{ohms} \quad . \quad (26)$$

The attenuation-constant is the same for unit length of loop, and for unit length of either wire referred to ground return. The surge-resistance of the loop is double the surge-resistance of

either wire, but the loop is assumed to contain double the e. m. f. impressed on either wire, so that the current strengths are the same at any point of the line on either basis. It is, perhaps, somewhat simpler to deal with every case by reference to the ground-return circuit, either actual or inferential. Consequently, it is preferable to confine attention to the properties of ground-return circuits, for the sake of simplicity, since any loop-circuit may be immediately reduced to this case.

A uniform direct-current line is defined electrically by the two constants r and g, the conductor-resistance, and dielectric conductance. But a line may be equally defined by the two constants α and r_0, the attenuation-constant, and the surge-resistance. In fact,

$$\alpha\, r_0 = \sqrt{rg} \times \sqrt{\frac{r}{g}} = r \qquad \text{ohms} \quad . \quad . \quad (27)$$

$$\text{and } \frac{\alpha}{r_0} = \sqrt{rg} \div \sqrt{\frac{r}{g}} = g \qquad \text{mhos} \quad . \quad . \quad (28)$$

The two defining constants α and r_0 are fundamental constants in relation to the progression of electric waves along wires; while the other two constants r and g are fundamental in the measurable properties of such wires in the steady state, after an indefinitely great number of electric waves or circuitations of waves around the circuit have produced summation currents. Thus α and r_0 are primordially fundamental.

The surge-resistance r_0, referred to direct-current lines in the steady state, may vary from about 500 ohms, with very leaky lines, to about 500,000 in very highly insulated lines. This surge-resistance is generally very different in magnitude from that encountered by advancing waves during the initial period preceding the steady state.

Resistance of the Line at the Sending End.

When the distant end of the line is freed, we have seen from equation (22) that the resistance of the line of length L_2, as measured at its sending end, is

$$r_f = r_0 \coth L_2\alpha \qquad \text{ohms} \quad . \quad (29)$$

When the line is put directly to ground at the distant end, we may write $e = o$ in equations (14) and (15), which then reduce to

$$\mathbf{e} = i\, r_o \sinh L_2 a \qquad \text{volts} \quad . \quad (30)$$

$$\text{and } \mathbf{i} = i \cosh L_2 a \qquad \text{amperes} \quad (31)$$

$$\text{or; } r_g = \frac{\mathbf{e}}{\mathbf{i}} = r_o \tanh L_2 a \qquad \text{ohms} \quad . \quad (32)$$

Consequently the product of the resistances of the line at the sending end, when grounded and freed at the distant end respectively, is the square of the surge-resistance; or

$$r_f \times r_g = r_o \coth L_2 a \times r_o \tanh L_2 a = r_o^2 \qquad \text{ohms} \quad (33)$$

$$\text{and } r_o = \sqrt{r_f\, r_g} \qquad \text{ohms} \quad (34)$$

so that the surge-resistance is always a third proportional, or geometric mean, of the free and ground resistances at the sending end.

When the length L_2 is very short, or when the attenuation-length $L_2 a$ is very small, we obtain the approximate expressions

$$\tanh L_2 a = L_2 a \quad . \; . \; . \; . \; . \; . \; . \; . \quad (35)$$

$$\coth L_2 a = \frac{1}{L_2 a} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (36)$$

$$\text{so that } r_f = \frac{r_o}{L_2 a} = \frac{R}{L_2} \qquad \text{ohms} \quad (37)$$

$$\text{and } r_g = r_o L_2 a = L_2 r \qquad \text{ohms} \quad . \quad (38)$$

These express the ordinary conditions of perfectly insulated lines.

When the line is put to ground at the distant end through a resistance r_d ohms, we may substitute $e = i r_d$ in equations (14) and (15), and we obtain

$$r_D = r_o \left\{ \frac{\tanh L_2 a + \dfrac{r_d}{r_o}}{1 + \dfrac{r_d}{r_o} \tanh L_2 a} \right\} \qquad \text{ohms} \quad (39)$$

This is the general equation of sending-end resistance, under which equations (29), (32), (37), and (38) represent specific cases. Thus, considering the line indicated in figure 4, for

which $L_2 = 500$, $a = 0.002634$, and $r_o = 5120$ ohms, what will be the resistance of the line at the sending end when the distant end is (*a*) grounded, (*b*) freed, (*c*) grounded, through 1000 ohms? Here $L_2 a = 500 \times 0.002634 = 1.317$.

By formula (32):

$$r_g = 5120 \tanh 1.317 = 5120 \times 0.866 = 4440 \text{ ohms.}$$

By formula (29):

$$r_f = 5120 \coth 1.317 = 5120 \times 1.155 = 5913 \text{ ohms.}$$

By formula (39):

$$r_D = 5120 \left\{ \frac{\tanh 1.317 + \frac{1000}{5120}}{1 + \frac{1000}{5120} \tanh 1.317} \right\} = \frac{5120 \times 1.0612}{1.169} = 4648.$$

The general equation of sending-end resistance (39) can be converted into the form

$$r_D = r_o \left\{ \tanh (L_2 + L_r) a \right\} \quad \text{ohms} \quad . \quad (40)$$

where $\tanh L_1 a = \frac{r_d}{r_o}$; or $L_r = \frac{1}{a} \tanh^{-1} \frac{r_d}{r_o}$

This expresses the extra length of line L_r which, put directly to ground at the distant end, is equivalent to the resistance r_d. Thus, in the immediately preceding case

$$r_d = 1000,\ r_o = 5120;\ \frac{r_d}{r_o} = \frac{1000}{5120} = 0.1953;\ \tanh^{-1} \frac{r_d}{r_o} = 0.198$$

$$L_r = \frac{0.198}{0.002634} = 75.2 \text{ miles.}$$

Consequently the effect of 1000 ohms between line and ground at the receiving end is the same on the sending-end resistance as 75.2 miles of extra line directly to ground. The actual resistance of 75.2 miles is $75.2 \times 13.485 = 1014$ ohms.

Equivalent Resistance of the Line at the Receiving End.

If the line be grounded at the receiving end while an e. m. f. of e volts is impressed at the sending end, we may enter formula (14) with the implied condition $e = 0$, and we obtain at once

$$I_B = \frac{e}{r_o \sinh L_2 a} = \frac{e}{r_o} \operatorname{cosech} L_2 a \quad \text{amperes} \quad (41)$$

The current flowing through the receiving end to ground is,

therefore, such as would flow through a simple circuit of resistance $r_0 \sinh L_2 a$ ohms under the e. m. f. e volts. Consequently the virtual resistance of the line, as judged from the receiving end, is $r_0 \sinh L_2 a$ ohms.

Thus, in the case represented by figure 4, where $L_2 = 500$ miles, and $a = 0.002634$, and $e = 200$ volts, $r_0 = 5120$ ohms, we have for the received current strength

$$I_B = \frac{200}{5120 \times \sinh 1.317} = \frac{200}{5120 \times 1.732} = 0.0225 \text{ ampere.}$$

If the line is put to ground at the receiving end through a resistance r_d ohms, the pressure at the receiving end of the line will be $e \doteq i r_d$ volts, and entering formula (14) with this condition we obtain

$$I_B = \frac{e}{r_0 \sinh L_2 a + r_d \cosh L_2 a} \quad \text{amperes} \quad (42)$$

Consequently, the resistance of the receiving device in such a line is magnified in the ratio of $\cosh L_2 a$ as far as concerns its influence upon the received current strength. In the case last considered, if the line be grounded through 1000 ohms, the received current will be

$$I_B = \frac{200}{5120 \times 1.732 + 1000 \cosh 1.317} = \frac{200}{8868 + 2000} = 0.0184$$

Influence of Line Length upon the Current Delivery and Current Efficiency.

Confining attention to formula (41), and ignoring the resistance of any receiving apparatus at the distant end of a line, it is evident that the strength of current which can be delivered at the distant end with a given impressed e. m. f. at the sending end of a uniformly leaky line varies inversely as the hyperbolic sine of the attenuation-length, or that fraction of the actual length which is represented numerically by the attenuation-constant. It also varies directly as the hyperbolic cosecant of the attenuation-length.

The current strength at the sending end of a line grounded at the receiver is, by (32),

$$I_A = \frac{e}{r_0 \tanh L_2 a} = \frac{e}{r_0} \coth L_2 a \qquad \text{amperes} \qquad (43)$$

Of this current, we know from (41) the amount delivered at the distant end. Consequently the ratio of the delivered to the offered current is

$$\frac{I_B}{I_A} = \frac{\dfrac{e}{r_0} \operatorname{cosech} L_2 a}{\dfrac{e}{r_0} \coth L_2 a} = \operatorname{sech} L_2 a \quad . \quad . \quad . \qquad (44)$$

The current-efficiency of a line grounded at the distant end through a receiver of negligible resistance, or of a metallic-return loop circuit working through a distant receiver of negligible resistance, is the hyperbolic secant of the attenuation-length. Thus, in the case illustrated by figure 4, we have seen that the outgoing current is $\frac{200}{4440} = 0.0450$ amperes, when the distant end is grounded, and the received current strength is 0.0225 ampere. The current-efficiency of the line is thus $\frac{0.0225}{0.0450} = 0.50$. The attenuation-length of the line is $500 \times 0.002634 = 1.317$, of which the hyperbolic secant is 0.50, according to (44).

When the attenuation-length is very small, an approximate formula substituted for (44) is

$$\frac{I_B}{I_A} = 1 - \frac{(L_2 a)^2}{2} \qquad (45)$$

Points of Discontinuity.

If a resistance be inserted in a homogeneous leaky line, it will virtually divide the line into two sections over each of which the preceding formulas will apply. Consequently, to compute the complete steady state, it is necessary to commence at the terminal of the line, where the pressure and current are known, thence to determine the pressure and current at the inserted resistance, and then repeat the process beyond the resistance on the second section.

Thus, consider a uniform aerial telegraph line, A C, 600 miles long, grounded at C through a relay of 200 ohms, and delivering 10 milliamperes in that relay, the line having an attenuation-constant of $\alpha = \frac{1}{500}$ and a surge-resistance of 5000 ohms. ($r = 10$ ohms and $R = 2{,}500{,}000$ ohms.) What will be the pressure and current at A, the sending end?

Here applying formulas (14) and (15) with $e = 200 \times 0.01 = 2$ volts, and $i = 0.01$ ampere, we obtain with $L_2\alpha = 600 \times \frac{1}{500} = 1.2$.

$$\mathbf{e} = 2 \cosh 1.2 + 0.01 \times 5000 \times \sinh 1.2 \qquad \text{volts.}$$

$$\mathbf{i} = 0.01 \cosh 1.2 + \frac{2}{5000} \sinh 1.2 \qquad \text{ampere.}$$

so that $\mathbf{e} = 2 \times 1.8107 + 50 \times 1.5095 = 79.095$ volts at A.

and $\mathbf{i} = 0.01 \times 1.8107 + 0.0004 \times 1.5095 = 0.0187$ ampere.

If, however, a relay of 200 ohms resistance be inserted half way along the line, at B, the line becomes broken into two 300-mile sections of attenuation-length $300 \times \frac{1}{500} = 0.6$. We now have to find the pressure and current at B just beyond the inserted relay. The formulas (14) and (15) now give

$$\mathbf{e} = 2 \cosh 0.6 + 0.01 \times 5000 \times \sinh 0.6 = 34.2 \text{ volts.}$$

$$\mathbf{i} = 0.01 \cosh 0.6 + 0.0004 \sinh 0.6 = 0.0121 \text{ ampere.}$$

On the near side of the B relay the current will be the same as on the far side, *i. e.*, 0.0121 ampere, while the pressure will be the "drop" of that current through 200 ohms plus the pressure at the far side — viz., $34.2 + 0.0121 \times 200 = 36.62$ volts. Reëntering equation (14) and (15) with these values of e and i, we can compute the steady state of the first 300-mile section thus:

$$\mathbf{e} = 36.62 \cosh 0.6 + 0.0121 \times 5000 \times \sinh 0.6 = 81.95 \text{ volts.}$$

$$\mathbf{i} = 0.0121 \cosh 0.6 + 0.00732 \sinh 0.6 = 0.0198 \text{ ampere at A.}$$

The insertion of the relay at B has had the effect of raising the required pressure at the sending end from 79.095 to 81.95 volts, and the current at A from 0.0187 to 0.0198 ampere. These values would be required at A in order to deliver 10 milliam-

peres through the relay at C, if the relay at B were removed but the attenuation-length were increased from 1.2 to 1.23, which corresponds to a virtual increase of attenuation-constant from 0.002 to 0.00205. That is to say, the loading of the line with a resistance at B has the same effect at the termini A and C as though the line had remained unloaded but had a certain increase in attenuation-constant.

The preceding proposition is of general application. If we consider a homogeneous leaky line loaded by the insertion of uniform resistances R_p at uniform distances L miles, as indicated in figure 7, then we may compare a section of this line, A B,

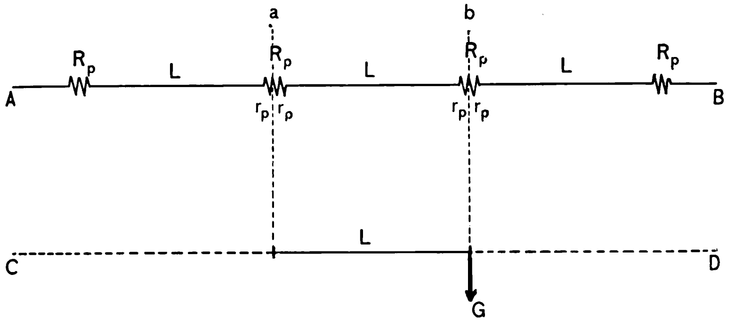


FIG. 7 DIAGRAM OF A LINE LOADED WITH A RESISTANCE R_p OHMS AT UNIFORM DISTANCE OF L MILES.

with a corresponding length of an equivalent line, C D, having a greater attenuation-constant, but without any discontinuities or inserted resistances.

Confining attention to the section a b of the line A B, we may regard the section as extending from the middle of the resistance R_p at a to the middle of the resistance R_p at b. Or if $r_p = R_p \div 2$, the section will contain $r_p + L + r_p$, and each successive section will be similarly constructed. If we assume the section a b grounded at b, and a current of i amperes flowing there to ground, the pressure at the end of the line-section near b will be $i\, r_p$ volts. The current at a will then be, by formula (15),

$$I_a = i \cosh L\alpha + \frac{i\, r_p}{r_0} \sinh L\alpha \qquad \text{amperes} \quad (46)$$

Where a is the attenuation-constant $\sqrt{rg}$, obtained in the ordinary way for the line section of length L miles. Now, referring to the same length L of equivalent line C D, figure 7, grounded at B, but without resistances, we have from formula (15) with $e = 0$

$$I_a = i \cosh La^1 \qquad \text{amperes} \quad . \quad (47)$$

where a^1 is the new or equivalent attenuation-constant. The two values of I_a obtained from equations (46) and (47) must agree. Therefore, equating (46) and (47) and canceling i as a common factor, we obtain

$$\cosh La^1 = \cosh La + \frac{r_p}{r_o} \sinh La \quad . \quad . \quad . \quad . \quad . \quad (48)^*$$

This enables the equivalent attenuation-constant of the loaded line to be computed. For example, if a line of $a = \frac{1}{500}$ and $r_o = 5000$ ohms, has a resistance of $R_p = 100$ ohms, inserted every 10 miles ($L = 10$), what will be the equivalent attenuation-constant of the line considered as unloaded and continuous?

Here $La = \frac{1}{50} = 0.02$, and $r_p = 50$ ohms,

$$\begin{aligned} \text{so that } \cosh La^1 &= \cosh 0.02 + \tfrac{50}{5000} \sinh 0.02 \\ &= 1.0002 + 0.0002 = 1.0004 \\ La^1 &= 0.028 \\ a^1 &= 0.0028 \end{aligned}$$

The attenuation-constant of the line has been increased by the loading from 0.002 to 0.0028, or 40 per cent., while the actual conductor-resistance of the line has been doubled. The line now behaves as though it had a uniform conductor-resistance per mile of

$$r^1 = R(a^1)^2 \qquad \text{ohms} \quad . \quad . \quad (49)$$

where R is the original insulation-resistance. In this case $r^1 = 0.0028^2 \times 2,500,000 = 19.6$ ohms. The insulation-resis-

* The formula (48) may also be derived from equation (14) by considering the section a b as freed at b.

tance and leakage-conductance are the same in the virtually continuous line as in the loaded line; so that

$$\frac{r_0^1}{r_0} = \frac{a^1}{a} \qquad \text{. . . . . (50)}$$

The surge-resistance of the equivalent continuous line is increased above the original surge-resistance in the same ratio as the attenuation-constant increases.

It will be found that all the preceding formulas for continuous-current circuits apply, without symbolical change but with extended meaning, to alternating-current circuits of uniform resistance, leakance, inductance, and capacity.

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Editorial.

THE Fifth Annual Dinner of the Harvard Engineering Society was held on May 13 at the Westminster Hotel. Professor Hollis presided and called on A. G. McAvity, '03, to act as toastmaster. The following responded to toasts:— Professor Hollis, Professor Adams, Professor Johnson, Professor Kennelley, Professor Marks, Mr. Saldăna, '97, G. S. Meem, '04, J. P. H. Perry, '03, T. Lindsley, '04, H. D. Grinnell, '03. The Dinner was the best attended and the most successful in the history of the Society.

APPENDIX

HYPERBOLIC FUNCTIONS

Hyperbolic trigonometry has important applications in graphics, statics, mechanics and cartography. Moreover, it is the language in which the phenomena of the transmission of electric currents finds natural expression. The laws governing the passage of electric waves and currents over uniform conductors are expressed with charming simplicity in hyperbolic functions. In any other symbolic language their expression is labored, lengthy and roundabout.

The following synopsis of hyperbolic trigonometry will probably be sufficient for the purposes of the student of electrical engineering. It deals with the trigonometry of real hyperbolic angles. This concerns the flow of continuous electric currents over conductors of uniform resistance and leakance. The flow of alternating currents over conductors of uniform resistance, leakance, capacitance and inductance forms a sequel and a natural extension of the subject, involving the trigonometry of complex hyperbolic angles (reals plus imaginaries). This extension of the subject must be relegated to future consideration.

A remarkable analogy connects the circular and hyperbolic trigonometrical functions. This is presented in Figs. 1 and 2. Fig. 1 shows a circular arc of g E A E′ f′ of radius O A, which we may consider as of unit length. Fig. 2 shows a rectangular hyperbolic arc; i.e., part of a hyperbola, whose asymptotes O S, O S′ are rectangular, or mutually perpendicular. The semi-transverse axis O A of this hyperbola, corresponding to the radius O A of Fig. 1, is taken of length unity for the sake of simplicity.

In Fig. 1, a radius vector, or movable radius, such as O E, of constant length, is pivoted at the centre O and may be made to run over the circle, counterclockwise, with its tip E. As the tip of this radius vector moves over the circular arc A, b, c, d, E,

f, g, it describes a circular angle. This angle is, of course, capable of being measured and expressed in several ways; e.g., in degrees, or in grades: but the particular way here considered is fundamental in theory, and is called radian measure. A circular angle is unity in this measure when it is equal to a radian, and a radian is the 2πth part of a complete revolution or $\frac{360}{2\pi}$ $= 57°, 17', 45''$, approximately, in degree measure. In Fig. 1, the angle A O E is drawn to measure one radian. The angles A O b, A O c, A O d, A O E, A O f similarly indicate angles of magnitudes 0.25, 0.5, 0.75, 1.0 and 1.25 respectively, in radian measure. The right angle A O g has $\frac{\pi}{2}$ or 1.5708 radians.

But there is another way of viewing the matter which is more pertinent to the analogy to be developed. Considering the unit angle A O E, and its image A O E′, the double angle E O E′ in Fig. 1 includes a shaded circular segment. The area of this shaded double-sector is just unity. If the radius O A be made one inch, and the angle A O E is one radian, then the area of the double sector E O E′ is just one square inch. We may say, then, that the magnitude of a circular angle is numerically equal to the area of the corresponding double circular segment (with radius unity). Thus the angle A O c is 0.5, because the area of the double circular segment c o c′ is 0.5 square inch.

Considering now Fig. 2, a radius vector, such as O E, of variable length, is pivoted at O, and may be made to run over the hyperbola with its tip E, counterclockwise. As the tip of this radius vector moves over the hyperbolic arc A, b, c, d, E, f, it describes a hyperbolic angle. This hyperbolic angle is measured by the area of the double hyperbolic sector, just as the circular angle is measured by the area of the double circular sector. Thus, the hyperbolic angle A O E is unity, because the shaded area in Fig. 2 of the double hyperbolic sector E′ O E is one square inch, if O A is one inch in length. Similarly, the hyperbolic angles A O b, A O c, A O d, A O E and A O f have respective magnitudes of 0.25, 0.5, 0.75, 1.0 and 1.25; because the corresponding double hyperbolic segments b O b′, c O c′, d

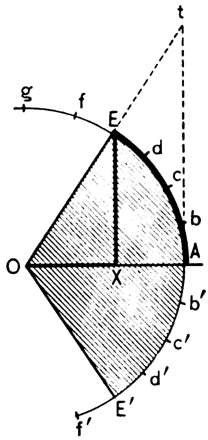

FIG. 1.
CIRCULAR SECTOR
AND CIRCULAR FUNCTIONS

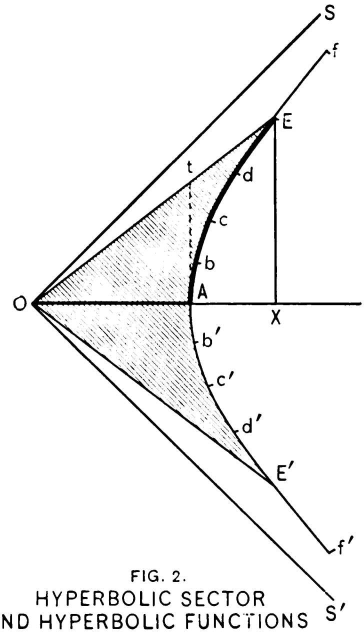

FIG. 2.
HYPERBOLIC SECTOR
AND HYPERBOLIC FUNCTIONS

O d′, E O E′, f O f′, have respective areas of 0.25, 0.5, 0.75, 1.0 and 1.25.

The hyperbolic angle A O E of Fig. 2, which has been drawn to represent unit hyperbolic angle, must, therefore, be carefully distinguished from the circular angle formed by the lines O E and O A. In the case represented this circular angle would be about 37°, 17′, 30″ or 0.651 radians; while the hyperbolic angle A O E as above defined is 1.

In Fig. 1, the uniformly moving tip E of the radius vector describes equal circular angles in equal times. In Fig. 2, the uniformly moving tip E of the radius vector describes successively smaller hyperbolic angles in equal times. In other words, the distances along the arc corresponding to equal hyperbolic angles of 0.25 —— A b, b c, c d, d E and E f, continually increase. For large hyperbolic angles the radius vector O E has to be extended prodigiously. At infinity, the radius vector coincides with the asymptote O S. The circular radius vector remains constant at unity, as the circular angle increases. The hyperbolic radius vector goes on forever beeween unity and infinity, as the hyperbolic angle increases.

In Fig. 1 the orthogonal projection O X of the radius vector O E upon the initial unit line O A is the circular cosine of the angle A O E. In the case presented A O E = 1 and O X = 0.5403; so that the circular cosine of 1 is 0.5403; or cos 1 = 0.5403 as far as four decimal places.

Similarly, in Fig. 2, the orthogonal projection O X of the radius vector O E, upon the initial unit line O A, produced, is the hyperbolic cosine of the hyperbolic angle A O E. In the case presented, A O E = 1, and O X = 1.543 (as far as 3 decimal places); so that the hyperbolic cosine of 1 is 1.543; or cosh 1 = 1.543.

In Fig. 1, the length of the perpendicular X E is the sine of the circular angle A O E (disregarding the question of the direction of X E in space). In the case presented A O E = 1 and X E = 0.841 (as far as 3 decimals); so that sin 1 = 0.841.

Similarly in Fig. 2, the length of the perpendicular X E is the sine of the hyperbolic angle A O E (disregarding the direction

of the line X E). In the case presented A O E = 1, and X E = 1.175; so that sinh 1 = 1.175.

Again in Fig. 1, the length of the perpendicular A t, is the tangent of the circular angle A O E = angle A O t (disregarding questions of direction). In this case angle A O E = 1 and A t = 1.557; so that tan 1 = 1.557.

Similarly in Fig. 2, the length of the perpendicular A t, is the tangent of the hyperbolic angle A O E (ignoring direction). In this case angle A O E = 1 and A t = 0.7616 so that tanh 1 = 0.7616.

In hyperbolic trigonometry, the secant, cosecant and cotangent are the reciprocals of the cosine, sine and tangent, respectively; just as in circular trigonometry.

In Fig. 1, as the extremity E of the radius vector pursues its circular path and describes an increasing angle, the cosine diminishes from +1 to —1 and then back again to +1. The sine does the same with a lag of 90° or 1.57 radians. As the circular angle augments, these two functions oscillate between the limits ± 1 with perfect regularity. The tangent periodically runs between + infinity and — infinity. In Fig. 2, the corresponding conditions are quite different. As the hyperbolic angle increases, the sine and the cosine go on increasing at an ever increasing rate, but they approach each other in magnitude, or tend to become equal. The tangent increases from 0 to 1. At infinity of hyperbolic angle, the point t reaches the asymptote O S, and the tangent A t is then = O A = 1. Long before infinity it very nearly reaches unity.

The diagram of Fig. 3 shows the graphs of the sine, cosine and tangent of hyperbolic angles in heavy lines, also the secant, cosecant and cotangent in dotted lines. It will be seen that when the hyperbolic angle reaches 3, the sine and the cosine have practically joined together and have run off the diagram at 10. At 7.5 the limit of the diagram for hyperbolic angles, the sine and cosine are over 900. The tangent and cotangent very nearly merge together at 3 and beyond, each tending towards unity. The sine, cosine, cotangent and cosecant all take very large values at some angle; but the tangent and secant confine

themselves between the limits 0 and 1 for all positive hyperbolic angles.

The following formulæ are frequently useful in dealing with hyperbolic functions. They are arranged for reference.

$$\sinh \alpha = \frac{\epsilon^{\alpha} - \epsilon^{-\alpha}}{2} \qquad \cosh \alpha = \frac{\epsilon^{\alpha} + \epsilon^{-\alpha}}{2} \qquad \tanh \alpha = \frac{\epsilon^{\alpha} - \epsilon^{-\alpha}}{\epsilon^{\alpha} - \epsilon^{-\alpha}}$$

$$\sinh \alpha = \alpha + \frac{\alpha^3}{3\,!} + \frac{\alpha^5}{5\,!} + \qquad \cosh \alpha = 1 + \frac{\alpha^2}{2\,!} + \frac{\alpha^4}{4\,!} +$$

$$\operatorname{cosech} \alpha = \frac{1}{\sinh \alpha} \qquad \operatorname{sech} \alpha = \frac{1}{\cosh \alpha} \qquad \coth \alpha = \frac{1}{\tanh \alpha}$$

$$\cosh^2\alpha - \sinh^2\alpha = 1\ ;\ 1 - \tanh^2\alpha = \operatorname{sech}^2\alpha\ ;\ \tanh \alpha = \frac{\sinh \alpha}{\cosh \alpha}$$

$$\sinh 2\alpha = 2 \sinh \alpha \cosh \alpha\ ;\ \cosh 2\alpha = \cosh^2\alpha + \sinh^2\alpha$$

$$\tanh 2\alpha = \frac{2 \tanh \alpha}{1 + \tanh^2\alpha}\ ;\ \coth 2\alpha = \frac{\coth^2\alpha + 1}{2 \coth \alpha}$$

$$\sinh^2\alpha = \frac{\cosh 2\alpha - 1}{2}\ ;\ \cosh^2\alpha = \frac{\cosh 2\alpha + 1}{2}$$

$$\tanh \frac{\alpha}{2} = \frac{\sinh \alpha}{1 + \cosh \alpha} = \frac{\cosh \alpha - 1}{\sinh \alpha} = \sqrt{\frac{\cosh \alpha - 1}{\cosh \alpha + 1}}$$

$$\sinh (\alpha \pm \beta) = \sinh \alpha \cosh \beta \pm \cosh \alpha \sinh \beta$$

$$\cosh (\alpha \pm \beta) = \cosh \alpha \cosh \beta \pm \sinh \alpha \sinh \beta$$

$$\sinh (\alpha + \beta) + \sinh (\alpha - \beta) = 2 \sinh \alpha \cosh \beta$$

$$\sinh (\alpha + \beta) - \sinh (\alpha - \beta) = 2 \cosh \alpha \sinh \beta$$

$$\tanh (\alpha \pm \beta) = \frac{\tanh \alpha \pm \tanh \beta}{1 \pm \tanh \alpha \tanh \beta}\ ;\ \coth (\alpha \pm \beta) = \frac{\coth \alpha \coth \beta \pm 1}{\coth \beta \pm \coth \alpha}$$

$$\sinh^{-1} \alpha = \log (\alpha + \sqrt{\alpha^2 + 1})\ ;\ \cosh^{-1} \alpha = \log (\alpha + \sqrt{\alpha^2 - 1})$$

$$\frac{d}{d\alpha} \sinh \alpha = \cosh \alpha\ ;\ \frac{d}{d\alpha} \cosh \alpha = \sinh \alpha\ ;\ \frac{d}{d\alpha} \tanh \alpha = \operatorname{sech}^2 \alpha$$

$$\frac{d}{d\alpha} \coth \alpha = - \operatorname{cosech}^2 \alpha$$

$$\frac{d}{d\alpha} \sin^{-1} \alpha = \frac{1}{\sqrt{\alpha^2 + 1}}\ ;\ \frac{d}{d\alpha} \cos^{-1} \alpha = \frac{1}{\sqrt{\alpha^2 - 1}}$$

$$\int \sin \alpha\, d\alpha = \cosh \alpha\ ;\ \int \cosh \alpha\, d\alpha = \sinh \alpha$$

$$\int \tanh \alpha\, d\alpha = \log \cosh \alpha\ ;\ \int \coth \alpha\, d\alpha = \log \sinh \alpha$$

$$\sinh j\alpha = j \sin \alpha\ ;\ \cosh j\alpha = \cos \alpha\ ;\ \tanh j\alpha = j \tan \alpha \text{ where } j = \sqrt{-1}.$$

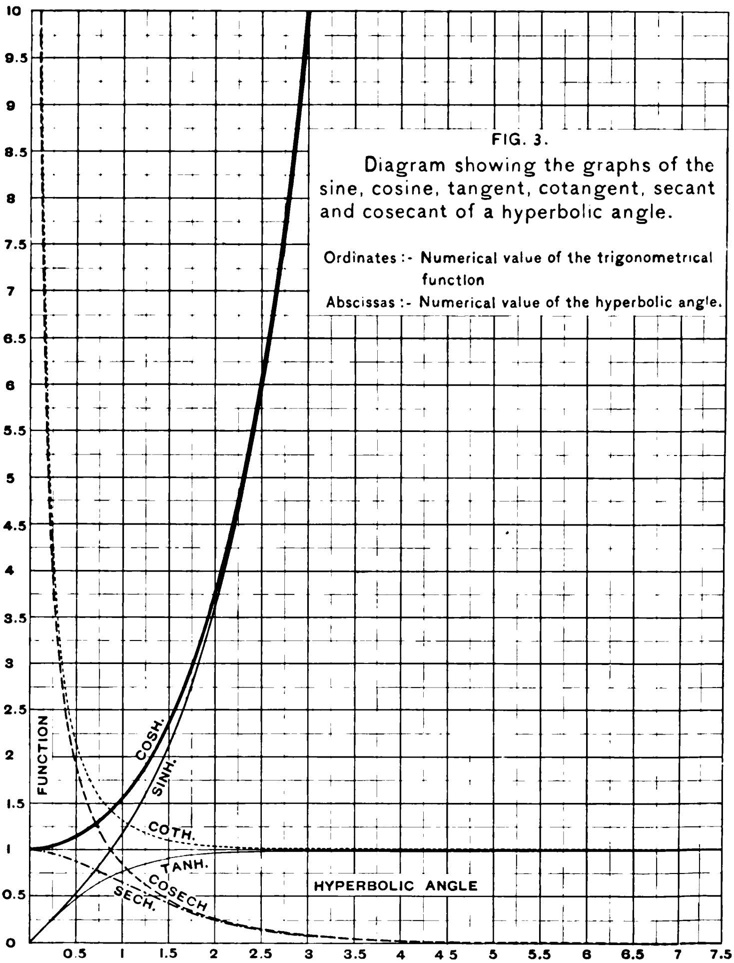

FIG. 3.

Diagram showing the graphs of the sine, cosine, tangent, cotangent, secant and cosecant of a hyperbolic angle.

Ordinates :- Numerical value of the trigonometrical function

Abscissas :- Numerical value of the hyperbolic angle.

TABLE OF SINES, COSINES, TANGENTS, COTANGENTS, SECANTS AND COSECANTS OF HYPERBOLIC ANGLES.

The Sines, Cosines, and Tangents have been taken from Ligowski's Tables published in Berlin in 1890. The Cotangents, Secants, and Cosecants have been deduced from the preceding quantities.

Φ	Sinh. Φ	Cosh. Φ	Tanh. Φ	Coth. Φ	Sech. Φ	Cosech. Φ	Φ
0.00	0.	1.000	0.	∞	1.00	∞	**0.00**
0.01	0.010000	1.000050	0.01000	100.	0.9999	100.	0.01
0.02	0.020001	1.000200	0.02000	50.	0.9998	50.	0.02
0.03	0.030005	1.000450	0.02999	33.34	0.9995	33.333	0.03
0.04	0.040011	1.000800	0.03998	25.013	0.9992	24.99	0.04
0.05	0.050021	1.001250	0.04996	20.016	0.9987	19.992	0.05
0.06	0.060036	1.001801	0.05993	16.686	0.9982	16.657	0.06
0.07	0.070057	1.002451	0.06989	14.308	0.9975	14.274	0.07
0.08	0.080085	1.003202	0.07983	12.527	0.9968	12.487	0.08
0.09	0.090122	1.004053	0.08976	11.141	0.9959	11.097	0.09
0.10	0.100167	1.005004	0.09967	10.033	0.9950	9.983	**0.10**
0.11	0.110222	1.006056	0.10956	9.128	0.9940	9.073	0.11
0.12	0.120288	1.007209	0.11943	8.373	0.9928	8.314	0.12
0.13	0.130366	1.008462	0.12927	7.735	0.9916	7.669	0.13
0.14	0.140458	1.009816	0.13909	7.189	0.9902	7.120	0.14
0.15	0.150563	1.011271	0.14888	6.716	0.9888	6.642	0.15
0.16	0.160684	1.012827	0.15865	6.303	0.9873	6.223	0.16
0.17	0.170820	1.014485	0.16838	5.939	0.9857	5.854	0.17
0.18	0.180974	1.016244	0.17808	5.615	0.9840	5.525	0.18
0.19	0.191145	1.018104	0.18775	5.325	0.9822	5.232	0.19
0.20	0.201336	1.020067	0.19737	5.067	0.9803	4.967	**0.20**
0.21	0.211547	1.022131	0.20696	4.832	0.9784	4.726	0.21
0.22	0.221779	1.024298	0.21652	4.618	0.9763	4.509	0.22
0.23	0.232033	1.026567	0.22603	4.425	0.9742	4.310	0.23
0.24	0.242311	1.028939	0.23549	4.246	0.9719	4.127	0.24
0.25	0.252612	1.031413	0.24492	4.083	0.9695	3.959	0.25
0.26	0.262939	1.033991	0.25430	3.932	0.9671	3.803	0.26
0.27	0.273292	1.036672	0.26363	3.793	0.9646	3.659	0.27
0.28	0.283673	1.039457	0.27290	3.664	0.9620	3.525	0.28
0.29	0.294032	1.042346	0.28214	3.544	0.9591	3.400	0.29
0.30	0.304520	1.045339	0.29131	3.433	0.9566	3.284	**0.30**
0.31	0.314989	1.048436	0.30043	3.328	0.9537	3.175	0.31
0.32	0.325489	1.051638	0.30951	3.231	0.9511	3.072	0.32
0.33	0.336022	1.054946	0.31852	3.140	0.9479	2.976	0.33

Φ	Sinh. Φ	Cosh. Φ	Tanh. Φ	Coth. Φ	Sech. Φ	Cosech.Φ	Φ
0.34	0.346589	1,058359	0.32748	3.053	0.9447	2.885	0.34
0.35	0.357190	1.061878	0.33637	2.973	0.9416	2.800	0.35
0.36	0.367827	1.065503	0.34522	2.897	0.9385	2.719	0.36
0.37	0.378500	1.069234	0.35399	2.825	0.9353	2.642	0.37
0.38	0.389212	1.073073	0.36271	2.757	0.9319	2.569	0.38
0.39	0.399962	1.077019	0.37136	2.693	0.9285	2.500	0.39
0.40	0.410752	1.081072	0.37995	2.632	0.9250	2.434	**0.40**
0.41	0.421584	1.085234	0.38847	2.574	0.9215	2.372	0.41
0.42	0.432457	1.089504	0.39693	2.519	0.9178	2.312	0.42
0.43	0.443374	1.093883	0.40532	2.467	0.9141	2.256	0.43
0.44	0.454335	1.098372	0.41365	2.417	0.9103	2.201	0.44
0.45	0.465342	1.102970	0.42190	2.370	0.9066	2.149	0.45
0.46	0.476395	1.107679	0.43009	2.325	0.9025	2.099	0.46
0.47	0.487496	1.112498	0.43820	2.282	0.8988	2.051	0.47
6.48	0.498646	1.117429	0.44624	2.241	0.8949	2.006	0.48
0.49	0.509845	1.122471	0.45421	2.202	0.8909	1.961	0.49
0.50	0.521095	1.127626	0.46211	2.164	0.8868	1.919	**0.50**
0.51	0.532398	1.132893	0.46995	2.128	0.8827	1.878	0.51
0.52	0.543754	1.138274	0 47769	2.093	0.8785	1.839	0.52
0.53	0.555164	1.143769	0.48538	2.060	0.8743	1.801	0.53
0.54	0.566629	1.149378	0.49299	2.028	0.8700	1.765	0.54
0.55	0.578152	1.155101	0.50052	1.998	0.8658	1.730	0.55
0.56	0.589732	1.160941	0.50797	1.969	0.8614	1.696	0.56
0.57	0.601371	1.166896	0.51536	1.940	0.8570	1.663	0.57
0.58	0.613070	1.172968	0.52266	1.913	0.8525	1.631	0.58
0.59	0.624831	1.179158	0.52990	1.887	0.8480	1.601	0.59
0.60	0.636654	1.185465	0.53704	1.862	0.8435	1.571	**0.60**
0.61	0.648540	1.191891	0.54413	1.838	0.8390	1.542	0.61
0.62	0.660492	1.198436	0.55112	1.814	0.8344	1.514	0.62
0.63	0.672509	1.205101	9.55805	1.792	0.8298	1.487	0.63
0.64	0.684594	1.211887	0.56490	1.770	0.8251	1.461	0.64
0.65	0.696748	1.218793	0.57166	1.749	0.8205	1.435	0.65
6.66	0.708970	1.225822	0.57836	1.729	0.8158	1.410	0.66
0.67	0.721264	2.232973	0.58498	1.709	0.8110	1.387	0.67
0.68	0.733630	1.240247	0.59152	1.690	0.8065	1.363	0.68
0.69	0.746070	1.247646	0.59798	1.672	0.8015	1.340	0.69
0.70	0.758584	1.255169	0.60437	1.655	0.7967	1.318	**0.70**
0.71	0.771174	1.262818	0.61067	1.637	0.7919	1.297	0.71
0.72	0.783840	1.270593	0.61691	1.621	0.7870	1.276	0.72
0.73	0.796586	1.278495	0.62306	1.605	0.7821	1.255	0.73

Φ	Sinh. Φ	Cosh. Φ	Tanh. Φ	Coth. Φ	Sech. Φ	Cosech. Φ	Φ
0.74	0.809411	1.286525	0.62914	1.590	0.7773	1.235	0.74
0.75	0.822317	1.294683	0.63516	1.574	0.7724	1.216	0.75
0.76	0.835305	1.302971	0·64108	1.5599	0.7675	1.1972	0.76
0.77	0.848377	1.311390	0.64693	1.5457	0.7625	1.1787	0.77
0.78	0.861533	1.319939	0.65271	1.5320	0.7576	1.1607	0.78
0.79	0.874776	1.328621	0.65842	1.5188	0.7527	1.1431	0.79
0.80	0.888106	1.337435	0.66403	1.5059	0.7477	1.1259	**0.80**
0.81	0.901525	1.346383	0.66959	1.4934	0.7427	1.1092	0.81
0.82	0.915034	1.355466	0.67507	1.4813	0.7377	1.0928	0.82
0.83	9.928635	1.364684	0.68047	1.4696	0.7327	1.0768	0.83
0.84	0.942328	0.374039	0.68580	1.4582	0.7278	1.0612	0.84
0.85	0.956116	1.383531	0.69107	1.4470	0.7228	1.0459	0.85
0.86	0.969999	1.393161	0.69626	1.4362	0.7178	1.0309	0.86
0.87	0.983980	1.402931	0.70137	1.4258	0.7128	1.0163	0.87
0.88	0.998058	1.412841	0.70642	1.4156	0.7078	1.0020	0.88
0.89	1.012237	1.422893	0.71139	1.4057	0.7028	0.9881	0.89
0.90	1.026517	1.433086	0.71629	1.3961	0.6978	0.9737	**0.90**
0.91	1.040899	4.443423	0.72114	1.3867	0.6928	0.9607	0.91
0.92	1.055386	1.453905	0.72591	1.3776	0.6878	0.9475	0.92
0.93	1.069978	1.464531	0.73060	1.3687	0.6828	0.9346	0.93
0.94	1.084677	1.475305	0.73522	1.3600	0.6778	0.9219	0.94
0.95	1.099484	1.486225	0.73979	1.3517	0.6728	0.9095	0.95
0.96	1.114402	1.497295	0.74427	1.3436	0.6678	0.8973	0.96
0.97	1.129431	1.508514	0.74870	1.3356	0.6629	0.8854	0.97
0.98	1.144573	1.519884	0.75306	1.3279	0.6579	0.8737	0.98
0.99	1.159829	1.531406	0.75736	1.3204	0.6529	0.8621	0.99
1.00	1.175201	1.543081	0.76159	1.3130	0.6480	0.8509	**1.00**
1.01	1.190691	1.554910	0.76576	1.3059	0.6431	0.8395	1.01
1.02	1.206300	1,566895	0.76987	1.2989	0.6382	0.8290	1.02
1.03	1.222029	1.579036	0.77391	1.2921	0.6333	0.8183	1.03
1.04	1.237881	1.591336	0.77789	1.2855	0.6284	0.8078	1.04
1.05	1.253857	1.603794	0.78181	1.2791	0.6235	0.7975	1.05
1.06	1.269958	1.616413	0.78566	1.2728	0.6186	0.7874	1.06
1.07	1.286185	1.629194	0.78946	1.2666	0.6138	0.7777	1.07
1.08	1.302542	1.642138	0.79320	1.2607	0.6090	0.7677	1.08
1.09	1.319029	1.655245	0.79688	1.2549	0.6042	0.7581	1.09
1.10	1.335647	1.668519	0.80050	1.2492	0,5993	0.7487	**1.10**
1.11	1.352400	1.681959	0.80406	1.2437	0.5945	0.7393	1.11
1.12	1.369287	1.695567	0.80757	1.2382	0.5898	0.7302	1.12
1.13	1.386312	1.709345	0.81102	1.2330	0.5850	0.7215	1.13

Φ	Sinh. Φ	Cosh. Φ	Tanh. Φ	Coth. Φ	Sech. Φ	Cosech. Φ	Φ
1.14	1.403475	1.723294	0.81441	1.2279	0.5803	0.7125	1.14
1.15	1.420778	1.737415	0.81775	1.2229	0.5755	0.7038	1.15
1.16	1.438224	1.751710	0.82104	1.2180	0.5708	0.6953	1.16
1.17	1.455813	1.766180	0.82427	1.2132	0.5662	0.6869	1.17
1.18	1.473548	1.780826	0.82745	1.2085	0.5616	0.6786	1.18
1.19	1.491430	1.795651	0.83058	1.2040	0.5569	0.6705	1.19
1.20	1.509461	1.810656	0.83365	1.1995	0.5523	0.6625	**1.20**
1.21	1.527644	1.825841	0.83668	1.1952	0.5477	0.6546	1.21
1.22	1.545979	1.841209	0.83965	1.1910	0.5431	0.6468	1.22
1.23	1.564468	1.856761	0.84258	1.1868	0.5385	0.6392	1.23
1.24	1.583115	1.872499	0.84546	1.1828	0.5340	0.6317	1.24
1.25	1.601919	1.888424	9.84828	1.1789	0.5296	0.6242	1.25
1.26	1.620884	1.904538	0.85106	1.1750	0.5251	0.6170	1.26
1.27	1.640010	1.920842	0.85380	1.1712	0.5206	0.6098	1.27
1.28	1.659301	1.937339	0.85648	1.1675	0.5162	0.6026	1.28
1.29	1.678758	1.954029	0.85913	1.1640	0.5118	0.5957	1.29
1.30	1.698382	1.970914	0.86172	1.1604	0.5074	0.5888	**1.30**
1.31	1.718177	1.987997	0.86428	1.1570	0.5030	0.5820	1.31
1.32	1.738143	2.005278	0.86678	1.1537	0.4987	0.5753	1.32
1.33	1.758283	2.022760	0.86925	1.1504	0.4944	0.5687	1.33
1.34	1.778599	2.040445	0.87167	1.1472	0.4901	0.5623	1.34
1.35	1.799093	2.058333	0.87405	1.1441	0.4858	0.5559	1.35
1.36	1.819766	2.076427	0.87639	1.1410	0.4816	0.5495	1.36
1.37	1.840622	2.094729	0.87869	1.1380	0.4773	0.5433	1.37
1.38	4.861662	2.113240	0.88095	1.1351	0.4732	0.5372	1.38
1.39	1.882887	2.131963	0.88317	1.1323	0.4690	0.5311	1.39
1.40	1.904302	2.150898	0.88535	1.1295	0.4649	0.5252	**1.40**
1.41	1.925906	2.170049	0.88749	1.1268	0.4608	0.5192	1.41
1.42	1.947703	2.189417	0.88960	1.1241	0.4568	0.5134	1.42
1.43	1.969695	2.209004	0.89167	1.1215	0.4527	0.5077	1.43
1.44	1.991884	3.228812	0.89370	1.1189	0.4486	0.5020	1.44
1.45	2.014272	2.248842	0.89569	1.1165	0.4446	0.4964	1.45
1.46	2.036862	2.269098	0.89765	1.1140	0.4407	0.4909	1.46
1.47	2.059655	2.289580	0.89958	1.1116	0.4367	0.4855	1.47
1.48	2.082654	2.310292	0.90147	1.1093	0.4329	0.4802	1.48
1.49	2.105861	2.331234	0.90332	1.1070	0.4290	0.4749	1.49
1.50	2.129279	2.352410	0.90515	1.1048	0.4251	0.4697	**1.50**
1.51	2.152910	1.373820	0.90694	1.1026	0.4212	0·4645	1.51
1.52	2.176757	2.395469	0.90870	1.1005	0.4174	0.4594	1.52
1.53	2.200821	2.417356	0.91042	1.0984	0.4137	0.4543	1.53

Φ	Sinh. Φ	Cosh. Φ	Tanh. Φ	Coth. Φ	Sech. Φ	Cosech. Φ	Φ
1.54	2.225105	2.430486	0.91212	1.0963	0.4099	0.4494	1.54
1.55	2.249611	2.461859	0.91379	1.0943	0.4062	0.4444	1.55
1.56	2.274343	2.484479	0.91542	1.0924	0.4025	0.4398	1.56
1.57	2.299302	2.507347	0.91703	1.0905	0.3988	0.4350	1.57
1.58	2.324490	2.530465	0.91860	1.0886	0.3952	0.4302	1.58
1.59	2.349912	2.553837	0.92015	1.0868	0.3916	0.4255	1.59
1.60	2.375568	2.577464	0.92167	1.0850	0.3879	0.4209	**1.60**
1.61	2.401462	2.601349	0.92316	1.0832	0.3844	0.4164	1.61
1.62	2.427596	2.625495	0.92462	1.0815	0.3809	0.4119	1.62
1.63	2.453973	2.649902	0.92606	1.0798	0.3774	0.4075	1.63
1.64	2.480595	2.674575	0.92747	1.0782	0.3739	0.4031	1.64
1.65	2.507465	2.699515	0.92886	1.0765	0.3704	0.3988	1.65
1.66	2.534586	2.724725	0.93022	1.0750	0.3670	0.3945	1.66
1.67	2.561960	2.750207	0.93155	1.0735	0.3636	0.3903	1.67
1.68	2.589591	2.775965	0.93286	1.0719	0.3602	0.3862	1.68
1.69	2.617481	2.802000	0.93415	1.0704	0.3569	0.3820	1.69
1.70	2.645632	2.828315	0.93541	1.0690	0.3536	0.3780	**1.70**
1.71	2.674048	2.854914	0.93665	1.0676	0.3503	0.3740	1.71
1.72	2.702731	2.881797	0.93786	1.0662	0.3470	0.3700	1.72
1.73	2.731685	2.908969	0.93906	1.0649	0.3438	0.3661	1.73
1.74	2.760912	2.936432	0.94023	1.0636	0.3405	0.3622	1.74
1.75	2.790414	2.964188	0.94138	1.0623	0.3373	0.3584	1.75
1.76	2.820196	2.992241	0.94250	1.0610	0.3342	0.3546	1.76
1.77	2.850260	3.020593	0.94361	1.0597	0.3310	0.3508	1.77
1.78	2.880609	3.049247	0.94470	1.0585	0.3279	0.3471	1.78
1.79	2.911246	3.078206	0.94576	1.0573	0.3248	0.3435	1.79
1.80	2.942174	3.107473	0.94681	1.0561	0.3218	0.3399	**1.80**
1.81	2.973397	3.137051	0.94783	1.0550	0.3187	0.3363	1.81
1.82	3.004916	3.166942	0.94884	1.0539	0.3158	0.3328	1.82
1.83	3.036737	3.197150	0.94983	1.0528	0.3128	0.3293	1.83
1.84	3.068860	3.227678	0.95080	1.0517	0.3098	0.3258	1.84
1.85	3.101291	3.258528	0.95175	1.0507	0.3069	0.3224	1.85
1.86	3.134032	3.289705	0.95268	1.0497	0.3040	0.3191	1.86
1.87	3.167086	3.321210	0.95359	1.0487	0.3011	0.3157	1.87
1.88	3.200457	3.353047	0.95449	1.0477	0.2982	0.3125	1.88
1.89	3.234148	3.385220	0.95537	1.0467	0.2954	0.3092	1.89
1.90	3.268163	3.417732	0.95624	1.0457	0.2926	0.3059	**1.90**
1.91	3.302504	3.450585	0.95709	1.0448	0.2897	0.3028	1.91
1.92	3.337176	3.483783	0.95792	1.0439	0.2870	0.2997	1.92
1.93	3.372181	3.517329	0.95873	1.0430	0.2843	0.2965	1.93

Φ	Sinh. Φ	Cosh. Φ	Tanh. Φ	Coth. Φ	Sech. Φ	Cosech. Φ	Φ
1.94	3.407524	3.551227	0.95953	1.0422	0.2816	0.2935	1.94
1.95	3.443207	3.585481	0.96032	1.0413	0.2789	0.2904	1.95
1.96	3.479234	3.620093	0.96109	1.0405	0.2762	0.2874	1.96
1.97	3.515610	3.655067	0.96185	1.0397	0.2736	0.2844	1.97
1.98	3.552337	3.690406	0.96259	1.0389	0.2710	0.2815	1.98
1.99	3.589419	3.726115	0.96331	1.0380	0.2684	0.2786	1.99
2.00	3.626860	3.762196	0.96403	1.0373	0.2658	0.2757	**2.00**
2.01	3.66466	3.79865	0.96473	1.0365	0.2632	0.2729	2.01
2.02	3.70283	3.83549	0.96541	1.0358	0.2607	0.2701	2.02
2.03	3.74138	3.87271	0.96608	1.0351	0.2582	0.2673	2.03
2.04	3.78029	3.91032	0.96675	1.0344	0.2557	0.2645	2.04
2.05	3.81958	3.94832	0.96740	1.0337	0.2533	0.2618	2.05
2.06	3.85926	3.98671	0.96803	1.0330	0.2508	0.2596	2.06
2.07	3.89932	4.02550	0.96865	1.0323	0.2484	0.2565	2.07
2.08	3.93977	4.06470	0.96926	1.0317	0.2460	0.2538	2.08
2.09	3.98061	4.10430	0.96986	1.0310	0.2436	0.2512	2.09
2.10	4.02186	4.14431	0.97045	1.0304	0.2413	0.2486	**2.10**
2.11	4.06350	4.18474	0.97101	1.0298	0.2389	0.2461	2.11
2.12	4.10555	4.22558	0.97159	1.0293	0.2366	0.2436	2.12
2.13	4.14801	4.26685	0.97215	1.0286	0.2344	0.2411	2.13
2.14	4.19089	4.30855	0.97274	1.0280	0.2321	0.2386	2.14
2.15	4.23419	4.35067	0.97323	1.0275	0.2298	0.2362	2.15
2.16	4.27791	4.39323	0.97375	1.0269	0.2276	0.2338	2.16
2.17	4.32205	4.43623	0.97426	1.0264	0.2254	0.2314	2.17
2.18	4.36663	4.47967	0.97477	1.0259	0.2232	0.2290	2.18
2.19	4.41165	4.52356	0.97524	1.0254	0.2211	0.2267	2.19
2.20	4.45711	4.56791	0.97574	1.0249	0.2189	0.2244	**2.20**
2.21	4.50301	4.61271	0.97622	1.0243	0.2168	0.2221	2.21
2.22	4.54936	4.65797	0.97668	1.0239	0.2147	0.2198	2.22
2.23	4.59617	4.70370	0.97714	1.0234	0.2126	0.2176	2.23
2.24	4.64344	4.74989	0.97758	1.0229	0.2105	0.2154	2.24
2.25	4.69117	4.79657	0.97803	1.0224	0.2085	0.2132	2.25
2.26	4.73937	4.84372	0.97847	1.0220	0.2064	0.2110	2.26
2.27	4.78804	4.89136	0.97888	1.0216	0.2044	0.2089	2.27
2.28	4.83720	4.93948	0.97929	1.0211	0.2024	0.2067	2.28
2.29	4.88683	4.98810	0.97970	1.0207	0.2005	0.2047	2.29
2.30	4.93696	5.03722	0.98010	1.0203	0.1985	0.2026	**2.30**
2.31	4.98758	5.08684	0.98049	1.0199	0.1966	0.2005	2.31
2.32	5.03870	5.13697	0.98087	1.0195	0.1947	0.1985	2.32
2.33	5.09032	5.18762	0.98124	1.0191	0.1928	0.1965	2.33

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2.34	5.14245	5.23879	0.98161	1.0187	0.1909	0.1945	2.34
2.35	5.19510	5.29047	0.98198	1.0183	0.1890	0.1925	2.35
2.36	5.24827	5.34269	0.98233	1.0180	0.1872	0.1905	2.36
2.37	5.30196	5.39544	0.98268	1.0177	0.1854	0.1886	2.37
2.38	5.35618	5.44873	0.98302	1.0173	0.1835	0.1867	2.38
2.39	5.41093	5.50256	0.98335	1.0169	0.1817	0.1848	2.39
2.40	5.46623	5.55695	0.98368	1.0166	0.1800	0.1829	**2.40**
2.41	5.52207	5.61189	0.98399	1.0163	0.1782	0.1811	2.41
2.42	5.57847	5.66739	0.98431	1.0159	0.1765	0.1793	2.42
2.43	5.63542	5.72346	0.98462	1.0156	0.1747	0.1775	2.43
2.44	5.69294	5.78010	0.98492	1.0153	0.1730	0.1757	2.44
2.45	5.75103	5.83732	0.98522	1.0150	0.1713	0.1739	2.45
2.46	5.80969	5.89512	0.98551	1.0147	0.1696	0.1721	2.46
2.47	5.86893	5.95352	0.98579	1.0144	0.1680	0.1704	2.47
2.48	5.92876	6.01250	0.98607	1.0141	0.1663	0.1687	2.48
2.49	5.98918	6.07209	0.98635	1.0138	0.1647	0.1670	2.49
2.50	6.05020	6.13229	0.98661	1.0135	0.1631	0.1653	**2.50**
2.6	6.69473	6.76901	0.98403	1.0110	0.1477	0.1494	**2.6**
2.7	7.40626	7.47347	0.99101	1.0091	0.1338	0.1350	**2.7**
2.8	8.19192	8.25273	0.99263	1.0074	0.1212	0.1221	**2.8**
2.9	9.05956	9.11458	0.99396	1.0060	0.1097	0.1104	**2.9**
3.0	10.01787	10.06766	0.99505	1.0050	0.0937	0.09982	**3.0**
3.1	11.07645	11.12150	0.99595	1.0041	0.0899	0.0903	**3.1**
3.2	12.24588	12.28665	0.99668	1.0033	0.0814	0.0816	**3.2**
3.3	13.53788	13.57476	0.99728	1.0027	0.0736	0.0739	**3.3**
3.4	14.96536	14.99874	0.99778	1.0022	0.0667	0.0668	**3.4**
3.5	16.54263	16.57282	0.99818	1.0018	0.0604	0.0604	**3.5**
3.6	18.28546	18.31278	0.99851	1.0015	0.0646	0.0547	**3.6**
3.7	20.21129	20.23601	0.99878	1.0012	0.0494	0.0495	**3.7**
3.8	22.33941	22.36178	0.99900	1.0010	0.0447	0.0448	**3.8**
3.9	24.69110	24.71135	0.99918	1.0008	0.0405	0.0405	**3.9**
4.0	27.28992	27.30823	0.99933	1.0007	0.0366	0.0366	**4.0**
4.1	30.16186	30.17843	0.99945	1.0006	0.0331	0.0332	**4.1**
4.2	33.33567	33.35066	0.99955	1.0005	0.0300	0.0300	**4.2**
4.3	36.84311	36.85668	0.99963	1.0004	0.0271	0.0271	**4.3**
4.4	40.71930	40.73157	0.99970	1.0003	0.0245	0.0245	**4.4**
4.5	45.00301	45.01412	0.99975	1.0003	0.0222	0.0222	**4.5**
4.6	49.73713	49.74718	0.99980	1.0002	0.0201	0.0201	**4.6**
4.7	54.96904	54.97813	0.99983	1.0002	0.0182	0.0182	**4.7**

Φ	Sinh. Φ	Cosh. Φ	Tanh. Φ	Coth. Φ	Sech. Φ	Cosech. Φ	Φ
4.8	60.75109	60.75932	0.99986	1.0001	0.0165	0.0165	**4.8**
4.9	67.14117	67.14861	0.99989	1.0001	0.0149	0.0149	**4.9**
5.0	74.20321	74.20995	0.99991	1.0001	0.0135	0.0135	**5.0**
5.1	82.0079	82.0140	0.99993	1.00007	0.01219	0.01219	**5.1**
5.2	90.6334	90.6389	0.99993	1.00007	0.01103	0.01103	**5.2**
5.3	100.1659	100.1709	0.99994	1.00006	0.00998	0.00998	**5.3**
5.4	110.7009	110.7055	0.99995	1.00005	0.00903	0.00903	**5.4**
5.5	122.3439	122.3480	0.99996	1.00004	0.00818	0.00818	**5.5**
5.6	135.2114	135.2150	0.99997	1.00003	0.00740	0.00740	**5.6**
5.7	149.4320	149.4354	0.99998	1.00002	0.00669	0.00669	**5.7**
5.8	165.1483	165.1513	0.99998	1.00002	0.00606	0.00606	**5.8**
5.9	182.5174	182.5201	0.99998	1.00002	0.00548	0.00548	**5.9**
6.0	201.7132	201.7156	0.99999	1.00001	0.00496	0.00496	**6.0**
6.1	222.9278	222.9300	1.	1.	0.00449	0.00449	**6.1**
6.2	246.3735	246.3755	1.	1.	0.00406	0.00406	**6.2**
6.3	272.2850	272.2869	1.	1.	0.00367	0.00367	**6.3**
6.4	300.9217	300.9233	1.	1.	0.00332	0.00332	**6.4**
6.5	332.5701	332.5716	1.	1.	0.00301	0.00301	**6.5**
6.6	367.5469	367.5483	1.	1.	0.00272	0.00272	**6.6**
6.7	406.2023	406.2035	1.	1.	0.00246	0.00246	**6.7**
6.8	448.9231	448.9242	1.	1.	0.00223	0.00223	**6.8**
6.9	496.1369	496.1379	1.	1.	0.00202	0.00202	**6.9**
7.0	548.3161	548.3170	1.	1.	0.00182	0.00182	**7.0**
7.1	605.9831	605.9839	1.	1.	0.00165	0.00165	**7.1**
7.2	669.7150	669.7158	1.	1.	0.00149	0.00149	**7.2**
7.3	740.1496	740.1503	1.	1.	0.00135	0.00135	**7.3**
7.4	817.9919	817.9925	1.	1.	0.00122	0.00122	**7.4**
7.5	904.0209	904.0215	1.	1.	0.00111	0.00111	**7.5**

NOVEMBER, 1903

HARVARD ENGINEERING JOURNAL

DEVOTED TO THE INTERESTS OF ENGINEERING AND ARCHITECTURE AT HARVARD UNIVERSITY

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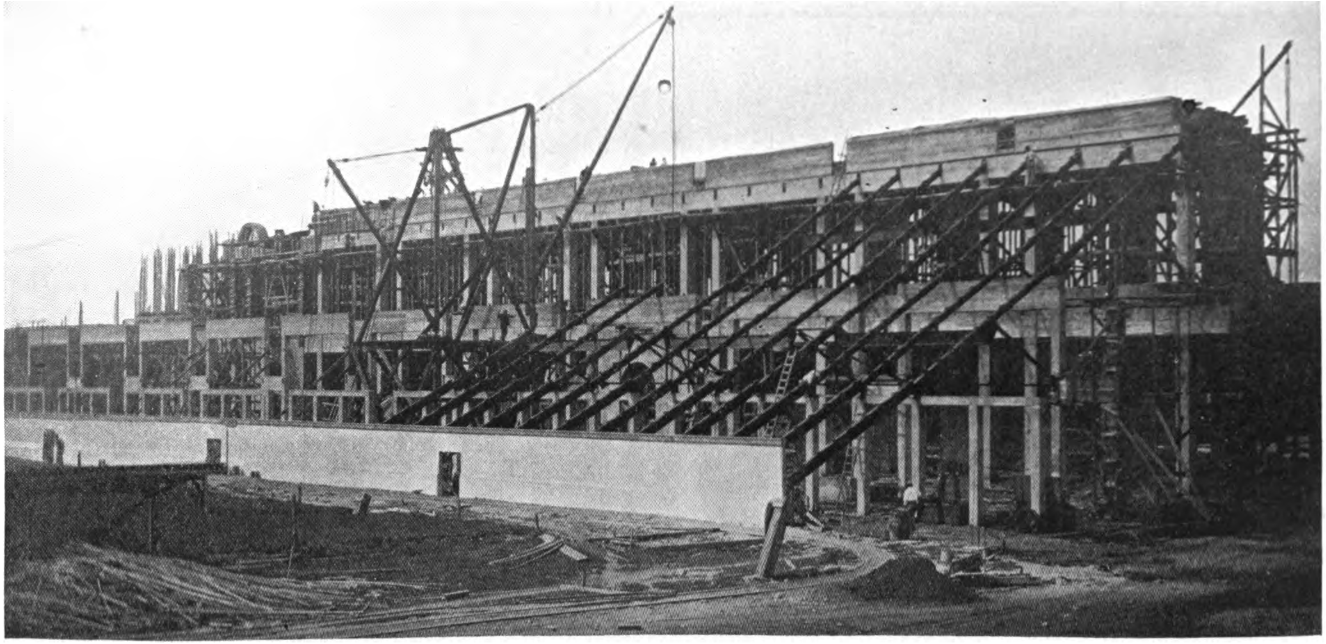


THE STADIUM

View of west arm, showing steel construction and concrete supporting columns

HARVARD ENGINEERING JOURNAL

Devoted to the interests of Engineering and Architecture at Harvard University

VOL. II NOVEMBER, 1903 NO. 3

THE HARVARD STADIUM.

CHARLES MAYO HARRINGTON, '04.

To the students returning to Cambridge after their vacations there is an interest in seeing what changes have been made in college buildings or grounds during the summer. There is always some improvement, something new or something altered; but this year found a change greater than usual, representing not only a summer's hard work but several years of study, planning, and devising. The throngs who saw the Yale baseball game on Soldiers' Field the day before Class Day did not realize that in less than four months the place could be so transformed that, in place of the bleachers stretching out on each side of the diamond, would rise a structure so large and imposing, so strong and solid, so permanent as the new steel-concrete stand called the Stadium. The inception of this idea, the growth through many changing forms, the hours of consultation and study, and the financiering form a story by themselves. This article will simply tell of the summer's work, the difficulties overcome, and the results achieved.

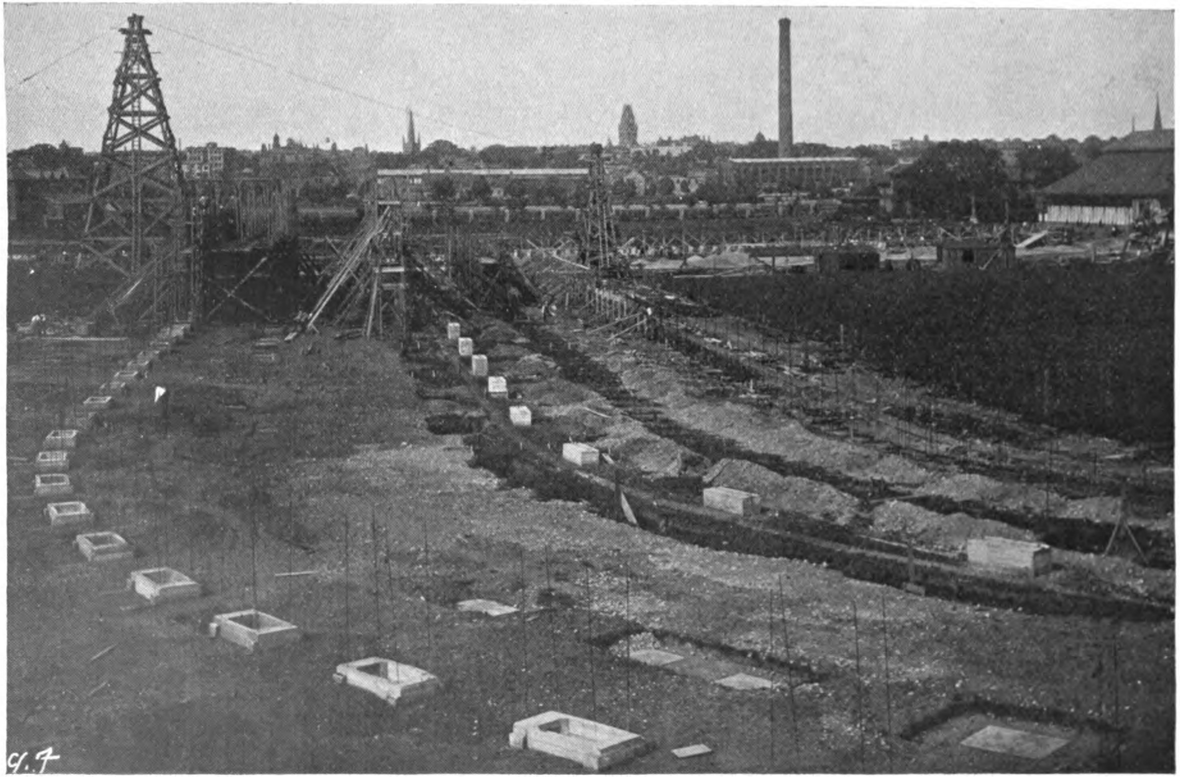
The Stadium is essentially a Harvard product. The class of '79 gave a generous sum of money, augmented by another from the Athletic Association. The architectural designs were made by Charles F. McKim, h '90, already associated with so many of Harvard's buildings and gates, assisted by George B.

De Gersdorff, '88. The preliminary working drawings wer made in the office of Joseph R. Worcester, '82, as consulting engineer. Prof. Hollis has had entire charge of the work from the beginning. Prof. Johnson was chief engineer, and the corps of assistants with but few exceptions has been made up of graduates and students of the University.

The Stadium presented many new engineering problems, for nothing like it in size and construction had been built in this country. It is a pioneer in steel-concrete work. Probably many large bridge piers or canal locks, such as those at Sault-St.-Marie, contain more tons of concrete than were used here, but they are all in great solid masses and not in building form. In shape the Stadium is a huge letter U, each arm being 95 feet wide from the outer wall to the parapet next to the inclosed field. From the open end, which is toward the north, the straight parallel sides extend 363 feet of the point of curve. The closed end is a semicircle with an outside radius of 210 feet, making the entire structure 573 feet long and 420 feet wide.

Before Commencement the work had begun. The first step was surveying the field and staking out the foundations. It is a common belief that Soldiers' Field is a great marsh, and visitors often asked how much piling was used. The western part of the field is marshy, but where the Stadium stands is firm, hard clay and gravel, affording excellent foundation and making it only necessary to dig below the reach of frost. Not a single pile was used. Stout wooden boxes were placed in these holes, carefully centered and plumbed, and then filled with broken stone concrete. In these foundations steel rods were placed horizontally. All the steel rods used in reinforcing the concrete throughout the structure are square in section and were twisted cold, thereby increasing their elastic limit. They range in size from ½-inch to 1-inch square. Vertical rods are imbedded in the foundations and extend up throughout the length of all columns. They are placed one near each corner of the column, and are held in place by other rods bent into rectangular shape and placed horizontally at intervals.

On these foundations a large force of carpenters built strong



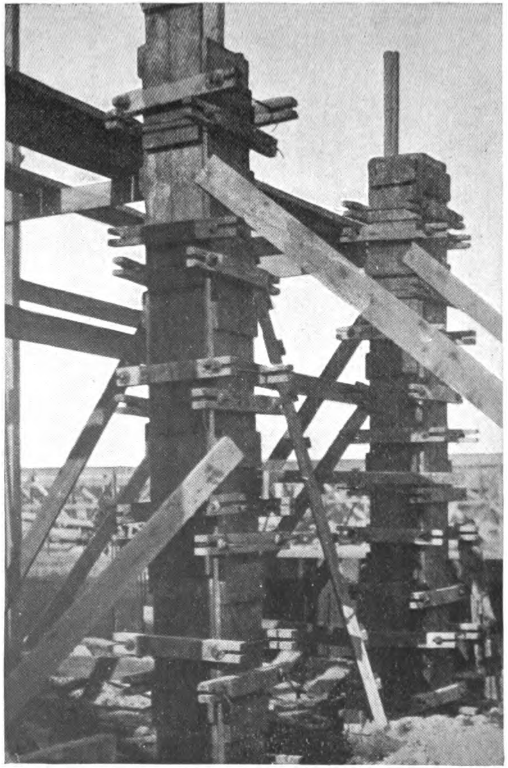
GENERAL VIEW OF FOUNDATIONS

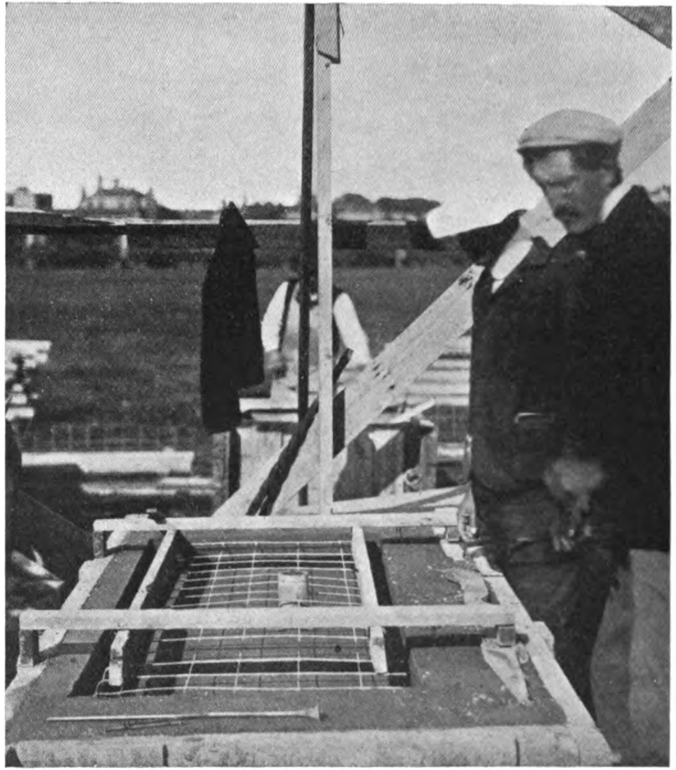
This shows the vertical steel rods around which the columns are built

wooden box-forms, the inside dimensions of which were the exact size of the columns. These forms were bolted together so that they could be taken apart easily and erected again further along. After being carefully centered and plumbed they were securely braced, and as soon as a few were in place the concrete filling began. When filled the forms were allowed to remain a week to give the concrete a chance to set thoroughly. The same method of construction was used with the walls and parapets. Similar forms were built for the girders that connect the columns. For the latter a small quantity of concrete was first placed in the bottom of the form and steel rods were then laid in horizontally, proportioned in size and quantity to take up all the tension in the girder. In addition the girders had vertical hoops or stirrups specially designed and placed where they offer the necessary resistance to any possible tendency to crack. All the wooden forms were well painted with crude oil to prevent sticking.

The general design of the concrete superstructure is first a parapet 9 feet high adjoining the field. On the field side this is finished with a moulded base and cap. On the other side it has low piers at intervals. Twenty-four feet behind this parapet rises the second row of columns, each 14 inches square. These are connected by girders into groups, the center of each group coming under an aisle of the seats above. The columns of the third and forth rows become progressively larger and higher, and are tied together as before with cross girders, the largest of which are 2 feet wide and 5 feet deep at a height of 50 feet from the ground.

The outer wall is 3 feet thick, with hollow spaces at intervals where great strength is not required. It is pierced with a series of arched openings. Between the fourth row and the outer wall are two floors, 25 feet and 50 feet above the ground, forming promenades running completely around the building. These floors also are of steel-concrete. A solid parapet separates the upper promenade from the seats. At intervals it is thickened into pedestals from which will rise a row of handsome Roman Doric columns, supporting a concrete flat roof over the prome-





DETAILS OF CONSTRUCTION

The removable forms in which the columns are cast

The sand mould for casting the seats, in which the steel netting is placed

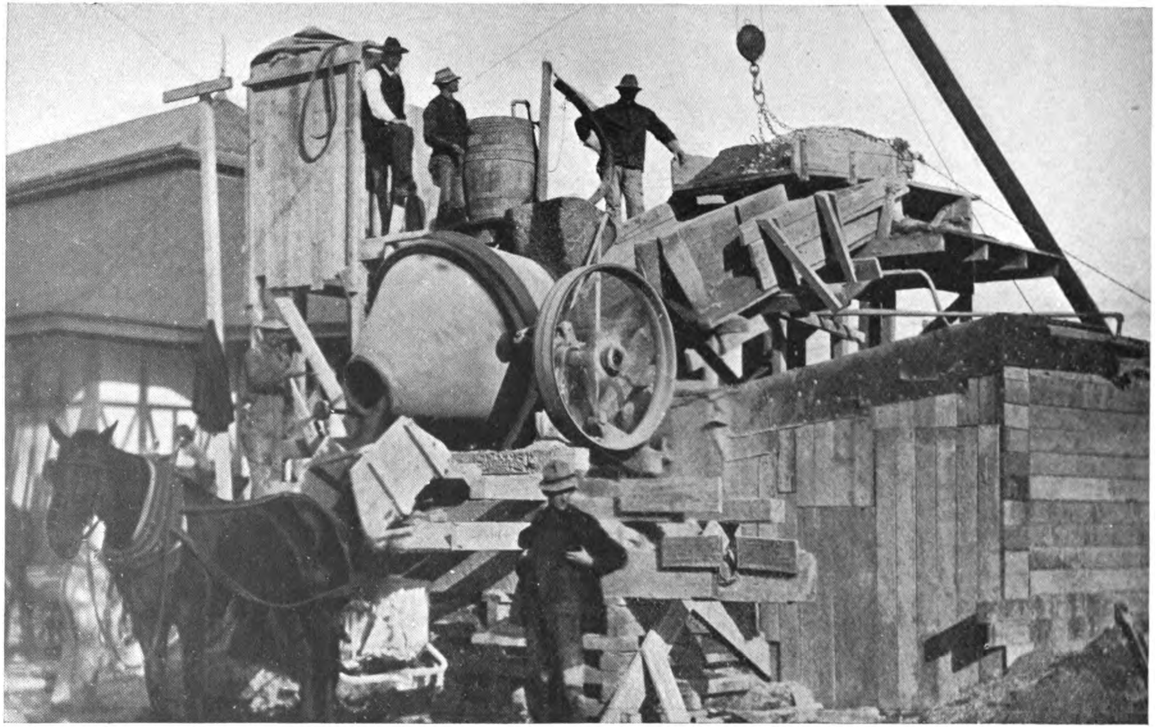
nade. Both the side toward the field and the outer side are to be crowned by a massive and dignified cornice of concrete. There will be no roof over the seats.

From the tops of the low piers of the parapet next the gridiron steel I beams, resting on the tops of the successive columns and their girders, extend to the upper girders of the fourth row. At intervals these beams are extended into pairs of triangular trusses, the other ends of which are let into the sides of certain of the columns. These beams do not have one continuous pitch but rise in three slopes, each a little steeper than the one below, the three approximating a curve that will give an equal view of the field from all points. It is a matter of interest that after these slopes had been worked out and decided upon, Mr. McKim in looking over some drawings of ancient stadia found a Roman stadium in Northern Africa having exactly the same three slopes.

On the upper sides of the steal beams are bolted metal knees with their upper surfaces level. On these are placed the seat slabs. The seats are a particularly interesting feature, for they are all of concrete and are cast in sand moulds exactly as iron is cast in a foundry. In fact, an open-air foundry 900 feet long was built on the field and equipped with three traveling cranes. Wooden patterns were made, moulding sand and heavy flasks were used, and the concrete was of such consistency as would allow it to be poured. Each slab has a broad tread and a narrow riser. Close to the under surface of the tread and bent up into the riser is a layer of electrically welded wire mesh, with a twisted steel rod in the bottom of the riser at the bend. When in place on the structure the two back corners of the tread rest on the steel knees of the I beams, and the entire length of front edge rests on the top of the riser of the seat below.

As soon as cast, all the seats were smoothed and finished on the edges and under side while they were still in the mould, where they were allowed to harden for at least three days before the flasks were broken open. Then they were moved into the field, covered with burlap and left to season and harden slowly.

The entrances to the Stadium are all from the outside. Com-



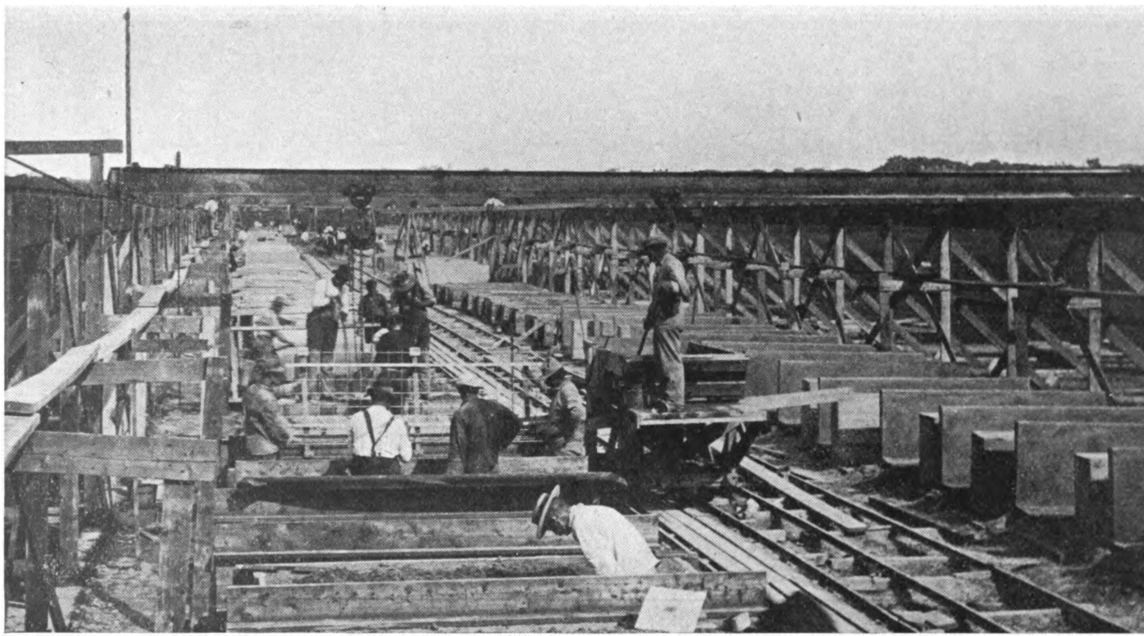
THE CONCRETE MIXER

ing up three steps, which run entirely around the structure, the visitor to the games will enter a corridor having a gravel floor. This space can be used as a running-track in wet weather, for its length from one end of the Stadium around the curve to the other is a full quarter of a mile. From this lower corridor concrete walks lead inward to some three dozen staircases, which open directly into the lower part of each aisle. Eight larger staircases at equal intervals lead to the second-floor corridor, from which passageways and short flights of stairs lead out at many points to the seats. All of the staircase structure is concrete and was very largely cast in place. At the upper end of each aisle an opening through the parapet gives entrance to the upper promenade.

One must stand on this promenade to get the best idea of the size of the Stadium. High as it is, one can see easily every part of the gridiron and every play that may be made. From the outer side the views over the Charles River west to Mt. Auburn and east to Boston are very fine. The State House seems but a short distance away and Memorial Hall appears hardly farther than the opposite river bank.

The end of each arm of the Stadium will be closed by a plain wall, following in shape the slope of the seats. Square towers surmount the outer corners, projecting from both the end and the outer wall and giving a finish to them both. In height the towers will be same as the roof of the promenade, 72 feet. At the south end the center of the curve has a break in the sweep of the lower parapet. It is a deep recess with a handsome doorway, affording an entrance to that end of the field. Space has been reserved for a quarter-mile running-track and a full football field within the inclosure of the Stadium. At the very beginning the gridiron was fenced off. Not a team has been on it, and now its turf is as firm and green as any player could wish.

The materials of construction used in the Stadium, with the exception of the steel and cement, are of local interest. People crossing the bridge had seen a mountain of sand piled high on the river bank east of the road. This was all taken from the bed of the river near the West Boston bridge and brought in

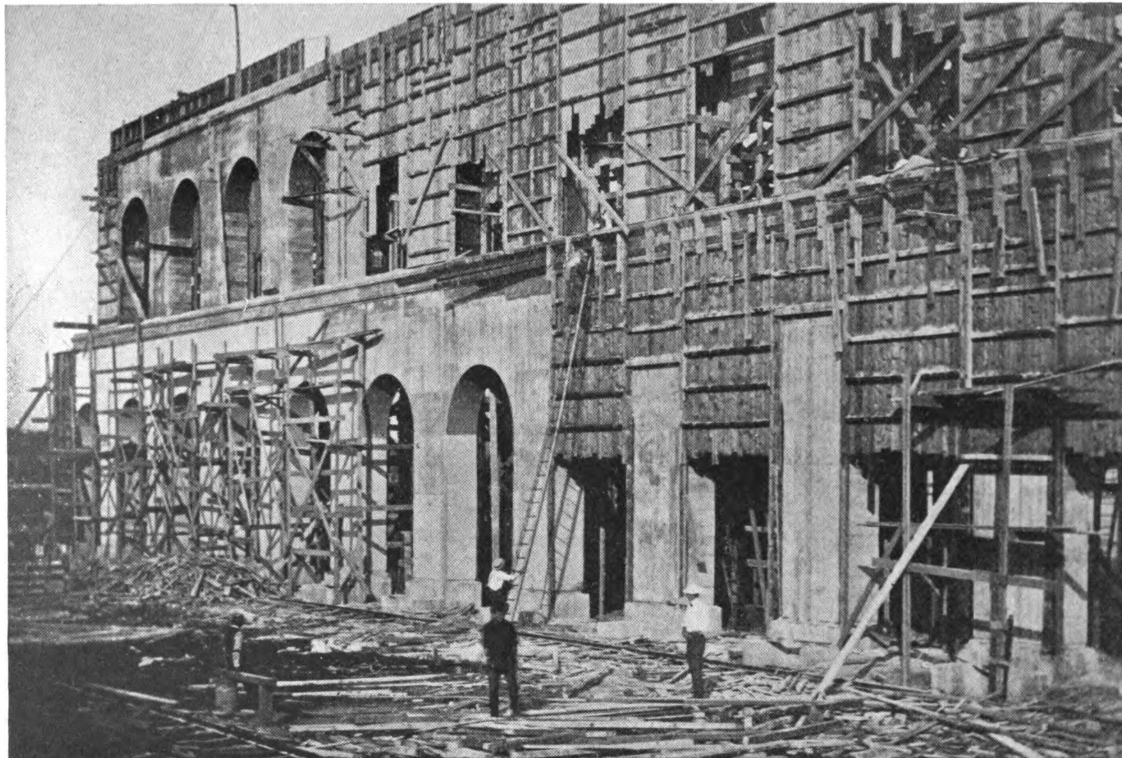


THE "FOUNDRY" WHERE THE SEATS ARE CAST

scows to this point, where a large dredger armed with an "orange-peel" bucket lifted it out, a cartload at a single filling of the bucket, and deposited it on the shore. In all 5,000 cubic yards were piled there and then teamed to the field. The broken stone used is Roxbury pudding-stone from local quarries. At times the work required 100 tons of stone a day. Nearly all the cement used was the Lehigh brand, with a smaller quantity of Atlas brand. The steel beams and trusses are the product of the Boston Bridge works. The concrete construction was done by the Aberthaw Construction Co. of Boston.

To handle this immense undertaking many mechanical appliances have been installed. A sawmill driven by a gasolene engine has been kept busy all summer cutting stock for the wooden forms. When this engine was first set up the exhaust-pipe had no muffler. The reports of the exhaust were loud and sharp. One afternoon when the engineer had left for a few minutes these reports suddenly became immensely louder. They sounded as though the engine were being blown up in a series of explosions. The engineer hurried back with others of the workmen only to find an unsophisticated member of the Engineering Society sitting astride of the exhaust-pipe holding an immense megaphone over the end!

Two large mixers prepared the concrete. One supplied that for the building while the other was used for the seats. At the beginning of the job two pairs of heavy timber towers were built at the northern end, spanning each leg of the structure. Between the towers of each pair was swung a heavy cable, guyed to "dead-men" across the field, and on this was rigged a trolley that lifted huge buckets of concrete to platforms, from which it was wheeled in barrows to its final location. A hoisting-engine was placed in each of the lower towers. As the work progressed these towers were moved on tracks until they reached the center of the curve. Two traveling cranes, each with two booms, were used for setting the steel beams. East of the Stadium a stone-crusher was placed to supplement the supply from the quarries, and the same engine drove an air-compressor that supplied the power for pneumatic tools for drilling and



GENERAL VIEW OF OUTER WALL

Showing the wooden forms partly removed

chipping. A similar tool was used to dress the surface of the parapets, outer walls, and other exposed places, giving them what is known as a "rough-picked" surface. It is not very different from rough granite in appearance, although in all the structure there has been no attempt to imitate stone in any way. Massiveness, dignity, and simplicity of form in harmony with the nature of the material used are the characteristics of the design. It is frankly built of concrete. The seating capacity of the structure is a little over 21,000, allowing a very liberal space for each person. It can be made to hold more without much crowding.

This is the story of the work of a summer. From the field has risen this massive pile, unique, alone, a pioneer in concrete construction. To those who have watched it grow, who know the strength that lies hidden in its fibres, who have spent the days in its study or its execution, it is more than a grand-stand. It is an experience. They have been a part of it and it will ever be a part of them.

TRAIN RESISTANCE.

C. O. MAILLOUX.

Lecture before the Harvard Engineering Society, May 12, 1903.

GENTLEMEN: — I must first express my appreciation of the honor which has been done me in being invited to lecture before you. I then wish to apologize for the somewhat hasty manner in which I begin my lecture. It is largely due to the great hospitality of Harvard and her worthy sons. I have been entertained in such splendid manner during the last few hours, and I have enjoyed myself so much, that I quite forgot this lecture. I could easily have dispensed with entering the lecture-room at all. This is my first appearance in the atmosphere of Harvard, and by way of testimonial I may say I have found it so very agreeable that I would be quite willing to stay here all the time.

The subject which we are to discuss this evening is, obviously, a subdivision of the general subject of "train movement." It represents one of the factors of a very important portion of the total power consumed in the propulsion of a railway car or train. Our discussion might properly begin with a preliminary analysis of train energy and power, and a summary investigation of the dynamical or kinematical principles involved in the motion of a train. We could, in fact, profitably utilize considerable time — more than the time of one lecture — in presenting and discussing these principles. In this case, however, owing to lack of time, we shall have to content ourselves with a rapid glance at the fundamental laws, and to summarize briefly those which are indispensable for our discussion, before passing to the particular subject which is before us.

As we will have occasion to make frequent allusions, in this lecture and in the next lecture, to the ideas implied by the words "energy," "power," "force" and "motion," we should have a definite clear understanding as to the sense in which these words are used and are to be taken. It is worth while noting that scientific men do not all agree in regard to the exact meaning of these terms.

I am pleased to express the opinion that the best definitions of these and of cognate terms have come from Massachusetts; they are to be found in a little book entitled "Matter, Energy, Force and Work," by the late Professor Silas W. Holman. I regard this book as the authoritative gospel of energy, force and motion. I have perused its contents several times, each time with renewed interest and pleasure, derived from the logical, simple, yet thorough and exhaustive manner in which the author handles his subject. I have a kind of veneration for that book and its author. I have never taken up the volume without regretting that I never had the privilege of personal acquaintance with Professor Holman, or the opportunity to express as I would wish, while he still lived, and to himself, my high regard for his work, and, above all, my profound sympathy for him in the painful sickness which made him physically helpless for so long time previous to his death. I feel privileged, in coming to lecture in Massachusetts, to pay honor and do reverence to one of her sons who has distinguished himself so much, especially in the physics of matter, energy, force, motion and work. The following are Prof. Holman's definitions, which I accept, and which I present to your attention as the best definitions known to me:

Energy. The power to change the state of motion of a body (p. 20).

Force. That action of energy by which it produces a tendency to change in state of motion of bodies (p. 41).

Work. That action of energy by which it produces motion in a free body or produces or maintains the motion of a body against resisting forces (p. 117).

Motion. Change of relative position (p. 12). This definition involves the definition of "position," which may be defined as the "relation of a certain body to certain points of reference." As the points of reference are, of necessity, arbitrary, it follows that position is only *relative* (p. 12).

Velocity. The time-rate of motion. Velocity may be constant, increasing or diminishing. We also may have *average* or *mean* velocity, and *instantaneous* velocity (p. 14).

State of Motion. The instantaneous velocity and direction of

motion of a particle referred to any chosen point and line (p. 15).

Acceleration. Increase in velocity (p. 14). It may be continuous or sudden. *Total acceleration* is the total increment of velocity in any stated time (p. 15). *Rate of acceleration* is the time-rate of increase of velocity $\frac{(dv)}{(dt)}$ (p. 15).

Inertia. The inability of a body to change its own state of motion (p. 15).

I have indicated in each case the page of Prof. Holman's book where the corresponding definition is to be found. There are many other equally good definitions in this valuable work. I urge all of you who have not yet read the book to do so at the first opportunity. You will find that every word used in these definitions, and in formulating all important principles or laws, has evidently been weighed and considered very carefully by Prof. Holman, and that the phraseology was the result of a thorough process of selection and elimination. The book is interesting, even aside from its high scientific merit, as a rare example of careful, logical, scientific use of language in expressing scientific thought, for the purpose of saying precisely what is meant and of meaning precisely what is said, and no more. The definitions which I have cited, with two exceptions, accord more or less perfectly with the generally accepted definitions. The two definitions which are at variance with the generally accepted definitions are those of "*acceleration*" and of "*rate of acceleration.*" Prof. Holman defines acceleration to be an *increment of velocity*, while the definition usually accepted takes acceleration to mean the time-rate of variation of velocity, or the *first* derivative of velocity with respect to time. This quantity has been called by Prof. Holman the *time-rate of acceleration*, notwithstanding the fact that the term "rate of acceleration" is usually employed to designate the *second* derivative of velocity. I have adopted Prof. Holman's nomenclature and used it in my paper on the "Plotting of Speed Time Curves." * Aside from my deep respect for Prof. Holman, I think the definition is inherently preferable

* Transactions of American Institute of Electrical Engineers, 1902, Vol. XIX.

to the accepted one. Distance, mathematically considered, is an increment of motion, which is independent of time. Velocity is a time-function of this change of motion, Likewise, if we adopt Prof. Holman's definition, acceleration is an increment of velocity, which is independent of time, and rate of acceleration is an increment of velocity divided by the time in which this increment takes place.

When we come to deal with *energy*, we recognize two different kinds, which are distinguished by the terms "potential" and "kinetic." We measure energy, as is well known, by taking the product of two separate factors, one of which may be called the "distance" factor, and the other the "force" factor. The distance is, of course, equal to the distance between the "initial" and the "final" states of position of the body measured along a certain line called the "path." The force represents the agency which operates to transfer the body from the *initial* to the *final* "state of position." When the force acts uniformly during the whole operation of the transfer, the energy is equal to the product of the force by the distance. We therefore have the equation:

$$E = F \times S.$$

When the force is not uniform at every point along the path of motion, however, it is necessary, in estimating the amount of energy, to subdivide the distance into an indefinitely great number of very small parts; we then take, for each such small part, the product of the average force, by the corresponding distance, and, finally, make a summation of the whole series of such products. In this case the formula becomes:

$$E = \int_{s'}^{s''} f' \, ds.$$

When we come to measure *power*, we likewise use the product of two factors, namely a "force-factor" and a "distance-factor." The force-factor is the same as in the case of energy; the distance-factor, however, is somewhat different. While it includes distance, it also includes the *time* in which this distance is covered; that is to say, it is a time-rate of distance variation;

in other words, it is a distance divided by a time-value, which is a velocity. Hence, we say (in the case of power) that the "distance-factor" is a "velocity"; because velocity is, in effect, the *rate* at which *distance* is covered, and power may, therefore, be measured by taking the product of the force and the velocity which are its factors, or

$$P = f\frac{ds}{dt} = fv.$$

We can now state this more concisely as follows: The unit of measurement in the case of energy represents the products of a force by a *distance;* in the case of power, it represents the product of a force by a *distance-rate.*

The relations of these quantities, — force, energy, power, velocity, etc., — as we all know, depend upon and are governed by the now universally recognized principle or doctrine, sometimes called the law, of the conservation of energy, which asserts, first, that energy can neither be created nor destroyed, its sum total being constant in the universe, and second, that a given amount of energy of one form may be converted into a corresponding amount of energy of some other form. As a corollary of this principle, we have the well-known theorem that "action is equal to reaction."

The application of these principles is that, in all manifestations of energy, the "effect" must be equal to the "cause," although opposed to it in direction, and that the effect is necessarily the precise measure of the cause, when both cause and effect are fully defined and adequately evaluated in equivalent and comparable units. It follows that the energy which, in the words of Prof. Holman, confers on substance the "power to change the position of a body," admits of being evaluated by reference to the effort required either to reproduce that change, or else to prevent it. This effort is called the *equivalent effort.*

Guided by these principles, if we should find, when dealing with energy in any concrete case, or, in a given cycle of operation, a discrepancy between the amount of energy known to be initially present, as the *causative force*, and that which is still found present at the end of the cycle, in the *equivalent effort*,

we will conclude at once that the portion of energy which has disappeared is, in reality, only apparently lost, having been converted into other forms whereby it becomes transferred to other points, and thus becomes dissipated without participating in the useful result. We thus come to distinguish practically between two forms of energy, which might be termed the "useful" energy and the "lost" energy. We sometimes designate these forms by the terms "recoverable" energy and "non-recoverable" energy. These terms are useful in cases where the energy brought into play becomes, wholly or partly, converted into kinetic energy of momentum, or potential energy of position, both of which may be considered forms of *stored* energy, and, consequently, are recoverable energy, at least theoretically.

It is oftentimes convenient to segregate the different forms or manifestations of energy and to estimate or evaluate each form separately by reference to the effect (equivalent effort) corresponding to each. In such a case we use, for each item or portion of the total energy, an equation like the general equation for energy, and we have only to take the sum of these items, thus individually expressed, in order to obtain the aggregate or total energy in the given case. The equation would then take the following form:

$$E = \int f\, ds \pm \int f'\, ds \pm \int f''\, ds \pm \text{etc.},$$

there being as many terms on the right side of the equation as there are different kinds of "force" involved in the particular case.

These preliminary considerations will enable us to obtain a clear idea of the phenomena involved in the motion of a train, a car, or a vehicle of any kind. This motion involves two kinds of forces, which we will conveniently distinguish as the *moving* forces and the *resisting* forces respectively.

We have mentioned action and reaction. The *moving* forces represent the "action," i. e., the energy acting to propel the car, whether this energy come from an outside source or whether it be already present in the car itself, owing to its position, as in the case of a car on a down grade, or else owing to its state of

motion, such as in the case of a car running by momentum. These forces would take the + sign in the preceding equation.

The *resisting forces* represent the "reaction," i. e., the energy which is abstracted from the moving forces. A portion of the energy thus abstracted is what might be termed lost or non-recoverable energy, it being immediately converted into heat or into other forms of energy which generate heat. The other portion is energy that is really *stored*, either in giving the car momentum, or in raising it to a higher position, as in propelling a car on an up grade. These forces would take the — sign in the preceding equation.

The motion of the car in every instance is a resultant effect, which is determined by, and can be used to measure, the difference between the sum of the moving forces and the sum of the resisting forces. Now, we could not, obviously, have definite knowledge of the difference between two such quantities unless we have definite knowledge of both quantities themselves. In this case, it so happens that our knowledge of the "moving" forces is at present more extended than our knowledge of the "resisting" forces. Our knowledge of the moving forces is, in fact, fairly complete and quite satisfactory, but we are far from having reached this condition in the case of the resisting forces. We know, as already stated, that these resisting forces are of two kinds, including those which, like inertia and gravity, can produce a resisting effect by storing the energy of which they cause the absorption, and another kind, which, like friction, can produce a resisting effect only by immediately dissipating that energy. We have adequate knowledge of the physical and kinematical properties of the resisting forces of the first kind. There is a definite and well-known relationship between the height to which a body is lifted and the energy required to lift it. There is also a fixed relationship between the motion of a body, the mass of that body, and the energy required to impart that motion to the said mass. For this reason, these forms of resisting force, being thus susceptible of exact mathematical computation, have been termed, not inaptly, "mathematical" resistances, in contradistinction to the second kind of resistance, which might be termed "physical" resistance.

We do not, unfortunately, possess the same complete satisfactory knowledge concerning the physical resistances. These resistances are precisely the resistances defined and included in the term "train resistance."

Train Resistance. The subject of train resistance is far from new. It is as old as railroading itself. The bibliography of the subject, in fact, shows that it has, more or less continuously and systematically, received some attention from railroad engineers and from scientific men from the very beginning of railroading.

The reason why train resistance is now mentioned and discussed so much more frequently than formerly is, simply, that the further evolution of our present methods of transportation, in the direction of higher speeds, requires that we should go far more deeply into the subject and obtain a better knowledge of it than has hitherto been necessary.

Let us first agree upon the meaning of the term "train resistance."

We are familiar with the word "resistance," as indicating, in general, that which produces or determines an opposing effect; and we employ it in mechanics in a specific sense, to denote whatever opposing effect acts to prevent or diminish motion. We might, therefore, consider the term "train resistance" as a special technical use of the word resistance in the same sense as a resistance opposing motion, but including, categorically, all resistances, in other words, all resisting forces, opposing the motion of a train or car. The term, as used, however, does not have a meaning quite so broad. It is restricted to the "physical" resistances involving friction in some form, directly or indirectly, and constituting the portion of energy which is characterized as the lost or non-recoverable energy, and it does not include the "mathematical resistances" already referred to, which constitute the recoverable energy.

Train energy may, therefore, be defined as the resisting force representing the force factor which has to be introduced into the general equations for energy and power, in order to enable us to get numerical values for the "lost" energy and power, or the energy or power directly converted into heat by frictional effects of

all kinds. How do we estimate this force? It is usually estimated in "pounds per ton," a specific term which railroad men have been wont to use to indicate the number of pounds of "pulling" or "tractive" effort required for each ton of weight of the car or train, to set it or keep it in motion. The reason for the adoption of this unit may be related to the fact that the first measurements of train resistance were made with a "traction-dynamometer," a device which indicates the pull (in pounds) exerted in the direction of motion. The accepted unit of train-weight in all railroad transportation estimates, computations, etc., being the *ton* (of 2,000 lbs. if it be a "short" ton, and of 2,240 lbs. if it be a "long" ton, and 2,204 lbs. if it be a "metric" ton), it was natural to divide the total tractive effort by the total number of tons and get a figure expressing the tractive effort, in pounds per ton — a figure which is obviously useful in determining the power required per ton, the cost of power per ton, and eventually the profit or loss per ton.

I have drawn on the blackboard a diagram showing the principles involved in the determination of what is called the coefficient of friction. The horizontal line represents a prepared frictional surface on which is placed a body whose frictional resistance is to be measured. A string attached to this body extends horizontally a certain distance and then passes down over a pulley, being put under tension by a weight attached at the other end. The amount of tension can be regulated by varying the weight. It may be increased until the pull, or the tractive effort exerted upon the horizontal portion of the string, is sufficient to overcome the frictional resistance of the body and to make it slide over the prepared surface. Knowing the weight which caused the motion and also the weight of the body moved, we can establish a relation between the two weights, and thus express the frictional resistance in terms of the force required to overcome it, i.e., the "equivalent effort," represented by the tension produced in the string. We do this when we take the ratio of the pulling force (equal in this case to the weight applied) to the weight of the body experimented upon. This ratio, called the "coefficient of friction," is a percentage figure.

Hence, when multiplied by 100 it expresses the number of pounds of pull that is required, per hundred pounds of the particular body experimented upon, to overcome its frictional resistance. There being twenty hundred-weights in a "short" ton (2,000 lbs.) it follows that if we multiply the coefficient of friction by 2,000 the resultant figure will be "pounds per ton" — the unit generally used in measuring and comparing train-resistance values. The term "pounds of resistance per ton," therefore, might be considered as being a "coefficient of friction" differently expressed. The term expresses the propelling force which has to be applied to the car or train, for each ton of its weight, to move it. It has been objected to because certain portions of the total train resistance (notably the air resistance) are entirely independent of the train weight; but since it is very convenient and it is the general practice to make calculations, and especially to make comparisons, of energy and power by reference to the amount of energy or power *per ton*, it would be necessary or desirable, in most cases, to reduce the total train resistance to some form that indicates its relation or proportion with respect to the total weight (tons) of the car or train. The "pounds per ton" unit has the advantage of exhibiting this relation. It is always easy to obtain the "total pounds of train resistance," which are, obviously, equal to the product of the "pounds per ton" by the "number of tons."

We have already noted that the term "train resistance" usually includes only the "physical" or "frictional" resistances, and does not include the so-called "mathematical" resistances. We will not, therefore, dwell at length upon the "mathematical" resistances. We will only note, summarily, their principal characteristics. They are of two kinds, including: (*a*) the resistance which represents the extra force exerted to *lift* a car or train when *ascending* a grade; and (*b*) the resistance which represents the extra force exerted to *increase the speed* of a car or train when "acceleration" takes place. In each case the force exerted is the "force factor" of a certain "power" which causes energy to be *stored* in the car or train — as "potential" energy, or "energy of position" in the first case, and as "kinetic"

energy, or "energy of motion," in the second case. It is obvious that we have here, as in all cases involving storage of energy, a reversible phenomenon or process. When the car *descends* a grade, or when its *speed decreases* (i. e., when "retardation" takes place), a contrary effect will be produced; power will then be developed at the expense of the stored energy, and, instead of a *resisting* force, there will be a *moving* force in each case. These "moving forces," due to stored energy, are familiar to us. In a car which is "coasting" on a down grade, the propelling power is that derived from stored *potential* energy. In a car which is "running by momentum," on a level track, the propelling power is that derived from the stored *kinetic* energy.

The term "mathematical resistance" is, obviously, applicable only when there are "resisting forces," causing storage of energy. The "grade" resistance is somewhat analogous to the "spurious resistance of polarization" expressing the "equivalent effect" of the counter E. M. F. in an electrolytic cell (for example, in a storage battery which is being charged). The "acceleration" resistance is analogous to the spurious resistance of self-induction, that is to say, to the reactance component of impedance, which, it is well known, is expressed in terms of electric resistance in ohms. It is desirable to have a general term that will apply equally to both the "resisting" and "moving" effects of grades and of acceleration and retardation. I have used the term "extrinsic force" for this purpose. The expression "extrinsic *resisting* force" means the same thing as "mathematical resistance"; while the expression "extrinsic *moving* force" means the opposite effect, corresponding to cases where the *force* is obtained from the stored energy.

It is proper to point out that the extrinsic forces due to grades and to acceleration or retardation may, in some cases, neutralize or complement each other. Thus, when a car is accelerating on a down grade, the extrinsic force due to the grade acts as a propelling force which reduces the load placed on the motors. In this case the stored potential energy in the car is converted into, and remains stored as, kinetic energy. The reverse can

also take place. The "slowing down" (retardation) of a car when it begins to ascend a grade is a familiar instance. A portion of the kinetic energy stored in the car disappears, as such, as the speed diminishes, but it reappears as potential energy, being expended in lifting up the car on the grade, thereby reducing the amount of power which has to be developed by the motors.

It may be of interest, before leaving the subject of "mathematical resistance," to note that the definite relationship already mentioned as existing between the weight of a body and the height to which it is lifted, though generally recognized as a fundamental physical fact, has been questioned in one instance. A French engineer, M. Barbier, has stated that the whole theoretical effect of a grade does not appear in the energy stored in ascending, or given out in descending, the grade. This peculiar result was obtained in connection with some elaborate dynamometric tests made for the purpose of determining the train resistance under different operating conditions. According to M. Barbier, the pull exerted on the dynamometer in ascending a grade is only nine-tenths (0.9) of what ought to be required, and in descending a grade, the pull due to the train weight is only nine-tenths of the theoretical or "mathematical" amount. I whish to caution you against "Barbier's coefficient." I still think the principle of the conservation of energy is valid. In my opinion the results found by M. Barbier were not general results, but specific results due to special conditions. In descending a grade the cars of a train come closer together, and there is a sidewise swerving or buckling tendency whereby the flange friction is increased, especially in the case of cars like the European standard cars, which have single rigid trucks instead of double pivoted trucks. Hence the train resistance was in reality *greater* than the average value found on a level track. In ascending a grade, the car couplings being under tension and there being less lost motion, the cars were kept in better alignment, there was less flange resistance, and the train resistance was therefore *less* than the average value found on a level track. The momentum effect would also tend to increase the dis-

crepancy, since, as we have already seen, the kinetic energy stored in a train is capable of developing power which assists in lifting the train, the amount of power developed being influenced, in each particular, by the rate of fall of speed $\left\{\frac{-dv}{dt}\right\}$ in ascending grade. It is my opinion that in this country, where all cars, except the shorter street-cars, are provided with double pivoted trucks, the discrepancy between the theoretical and the practical "grade-effect" will be a negligibly small quantity.

Grade resistance is far more important in steam railroad than in electric railroad work. There is in steam railroad engineering an important technical term which is scarcely used or even known in electric railroad engineering — the *maximum grade.* Nearly every road has its "maximum" grade, which, we should note as an interesting fact, is seldom above, and in many cases considerably below, two per cent. In electric railroad work there is really no limiting grade. I have heard of grades as high as 13.5 per cent — which, it is needless to say, are quite beyond the possibilities of the steam locomotive.

We shall have occasion to return to mathematical resistance, and to the extrinsic forces, in the next lecture. We now pass on to the resistances of frictional character, which properly constitute train resistance. I use the general term "extrinsic" to designate these frictional resistances, because their equivalent effort, as we shall find in the next lecture, is the force factor of the non-recoverable power which is actually and unavoidably dissipated by the motion of the car or train.

(*To be continued.*)

VILLAS ABOUT ROME.

EDWARD T. P. GRAHAM, .
Austin Travelling Fellow, 1900–01.

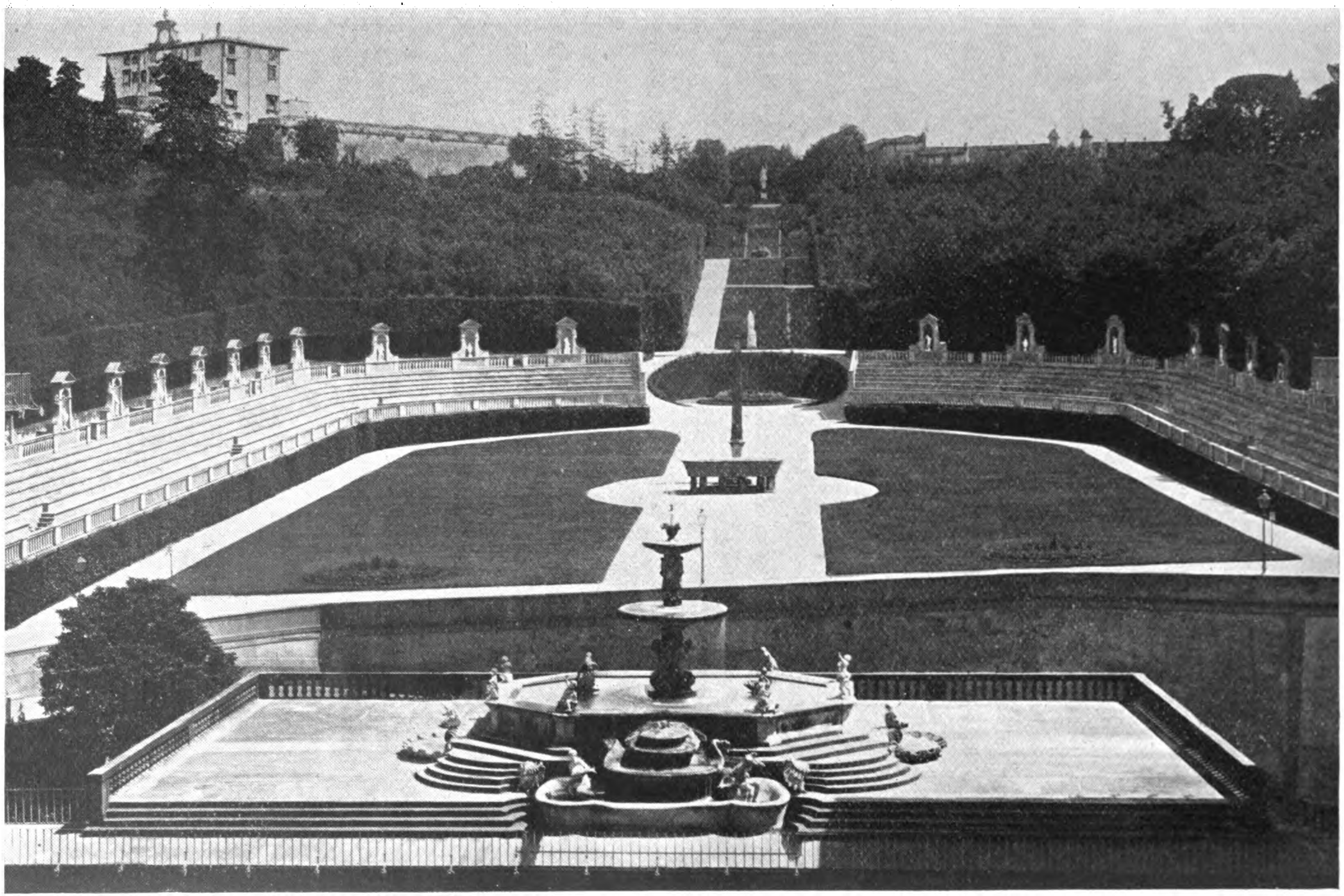
THE garden and the villa have at all times been important in the art of Italy. The mild climate, the perpetual sunshine of a southern summer day, the desire for cooling shade, and the lack of rigorous winter, make the garden and outdoor life most desirable.

That the Romans loved nature and outdoor life is known from their authors, from the wall decorations of Roman and Pompeian houses, and from the villas about Naples at Baies and Capri, and from those at Tivoli, Frascati and Albano. Hadrian's villa near Tivoli is one of the greatest ruins of antiquity. There that much traveled emperor surrounded himself with the most beautiful works that he had seen in his travels — peristyles, Greek and Roman theaters, baths, basilicas, temples and palaces. The hilly country rising beyond the Roman Campagna was a favorite retreat for the Romans from city life. Frascati and the shores of Lakes Albano and Nemi were the sites of villas then as they are now. The remains of Domitian's villa, overlooking Lake Albano, give some idea of the grandeur of these estates. In the garden of the Villa Barberini are terraced hillsides supported by subterranean galleries, and the outlines of extensive buildings, all that remains of Domitian's villa. This villa occupied the crest of ground overlooking on one side the lake and on the other the Roman Campagna.

Not so much is known of Roman villas in other parts of Italy, but the lovely valley of the Arno and the shores of Lakes Como and Maggiore were the sites of Roman villas then, just as today they are dotted with the abodes of a later race.

Italian gardens may be geographically divided into three groups — the southern, including those of Naples and Sicily, the central, those of Rome, Tuscany and Venetia, and thirdly those of the northern lakes.

The southern gardens occur on the steep rocky slopes over-



BOBOLI GARDENS

looking the sea, and are but a succession of narrow terraces, wherever it is possible to get a foothold on the abruptly rising shores about Amalfi and along the east coast of Sicily. The formal garden of the royal palace at Caserta, near Naples, is unique in southern Italy. Its water-works are very extensive.

Of the Tuscan group, one of the most notable is the Boboli garden, especially beautiful near the Pitti Palace. About Florence, on the hills of Fiesole, San Miniato and the neighboring heights, deserving special mention are the villas of Petraia and Castello; also about Siena are a few villas. But none of these equal the best Roman examples.

Along the Brenta, on the low lands bordering the lagoons, were the villas of the Venetians. To-day one may see in a journey from Venice to Padova many villas, but none of such extent or beauty as those near Rome.

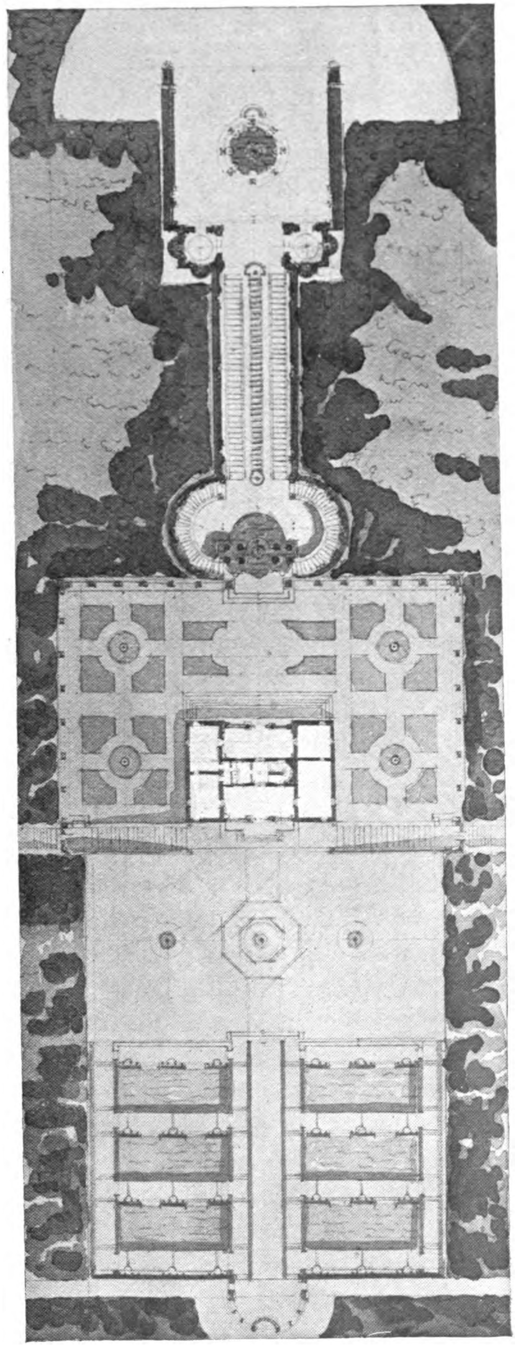
On the northern lakes are the villas of Carlotta and Melzi on Lake Como, and others about the Borromean Islands in Lake Maggiore.

The Roman gardens interest us most in point of design, for there the villa and garden architecture was best understood and attempted on the largest scale.

The Renaissance awoke in the people a love and desire for the civilization of their predecessors. It found Rome rich, the center of the Christian world, and the home of the most cultured art-loving men of Europe.

The churchmen of the time had much taste and built generously. Renaissance Rome rose on and in reality was but a thin crust, covering the older city and civilization. Cardinals and princes reared their villas, churches and palaces on earth deep in which was buried much of Rome's material greatness. Men became interested in the past and humanistic study carried them back to ancient times. This resulted in the Renaissance and gave the gardens, palaces and villas that delight us to-day.

Many of Rome's fine gardens have disappeared before modern change and progress. The Via Giulia was once the fashionable boulevard of the city. Its palaces had gardens sloping to the banks of the Tiber. Now the river is bordered by quays and



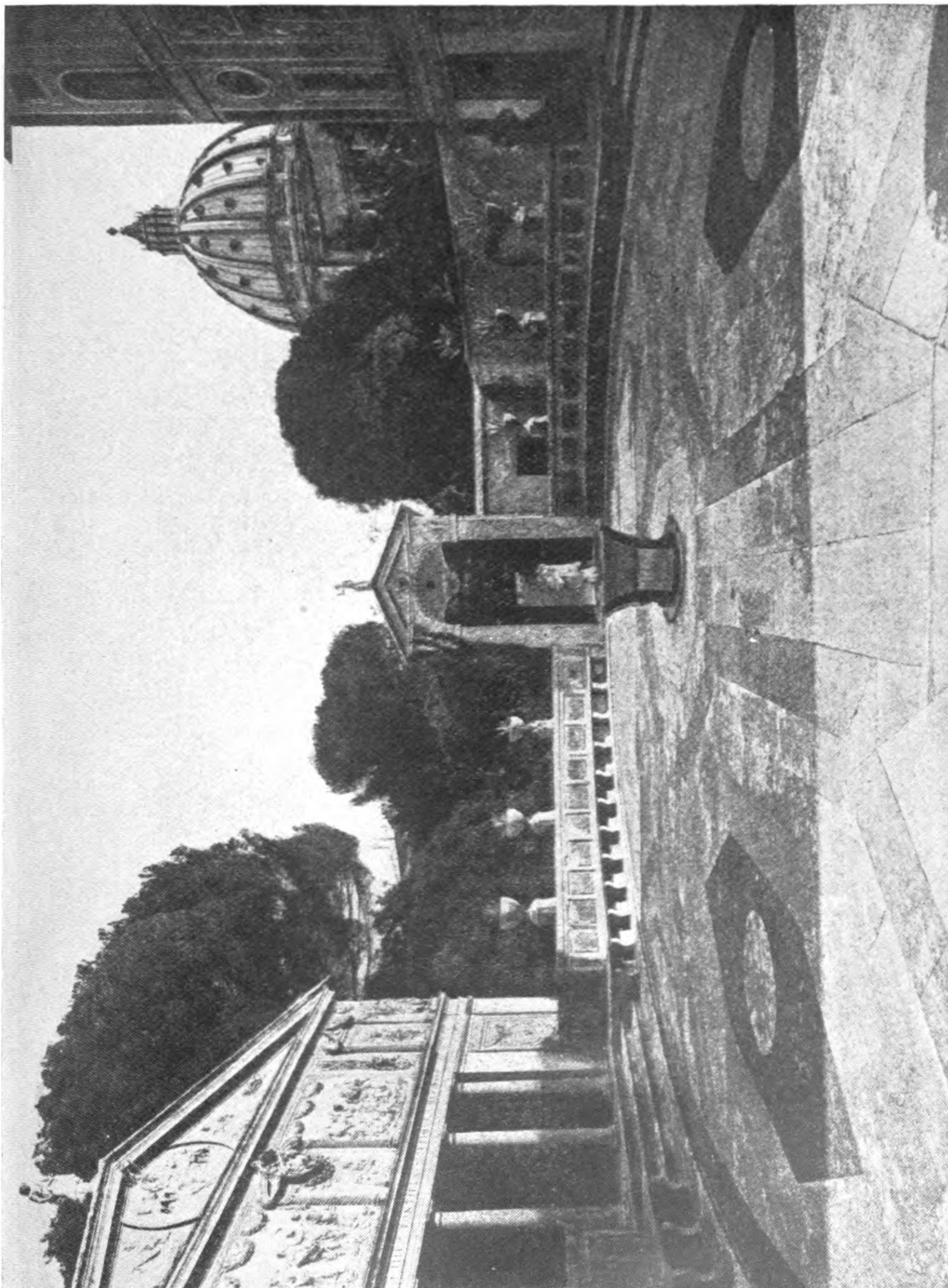
PLAN OF VILLA FARNESE

driveways and the gardens have been removed. The fine old garden of the Corsini Palace is no more. It is replaced by the fantastic paths of a modern botanic garden. Monte Gianicolo is changed and the labors of garden-builders are blotted out. This change has taken place also in other parts of the city, especially in the Ludovisi quarter, where buildings now cover the villa land. On Monte Palatino the Villa Farnesina has been removed to uncover the houses of the Cæsars. Palaces, as the Barberini, Borghese and Farnesina, have had their grounds encroached on, so that the Rome of the Renaissance is changed.

Progress has felt itself obliged to trample on and efface much of Rome's past beauty. The city has been modernized. Stone quays are building along the river banks. Streets are cutting through old quarters, vistas are opening up, and an attempt is made to give to Rome some of the beauty gained by a preconceived plan of a large city. These changes mean the destruction, here and there, of things of merit. However, there are still to be seen many fine gardens.

The Villa Medici is a good example. It is situated on Pincian hill, between the renowned Pincian garden and the convent Trinità del Monte. The villa, built in the sixteenth century, is now the seat of the French academy. Its plan, pleasing as it is, is the result of limitations of site. The typical villa plan, as exemplified by the Villa D'Este at Tivoli, Aldobrandini at Frascati, Farnese at Caprarola and the Villa Madama outside of Rome, is one in which the casino is reached after traversing the gardens — the casino, usually the final point, the focus of all interest, and on the main axis of the design. The Villa Madama, although now a ruin and never complete, would have been the finest example of all if the conception had been carried out.

The urban villa was usually limited in its site. Just so is the Villa Medici. The palace is reached directly from the street, as is also the garden, by a gradual ascent. The ascent leaves one at the cross-axis of the heavily shaded park divided by rectangular walks and drives. Here the sun scarcely penetrates on the brightest days. One may walk or sit in this half-mys-



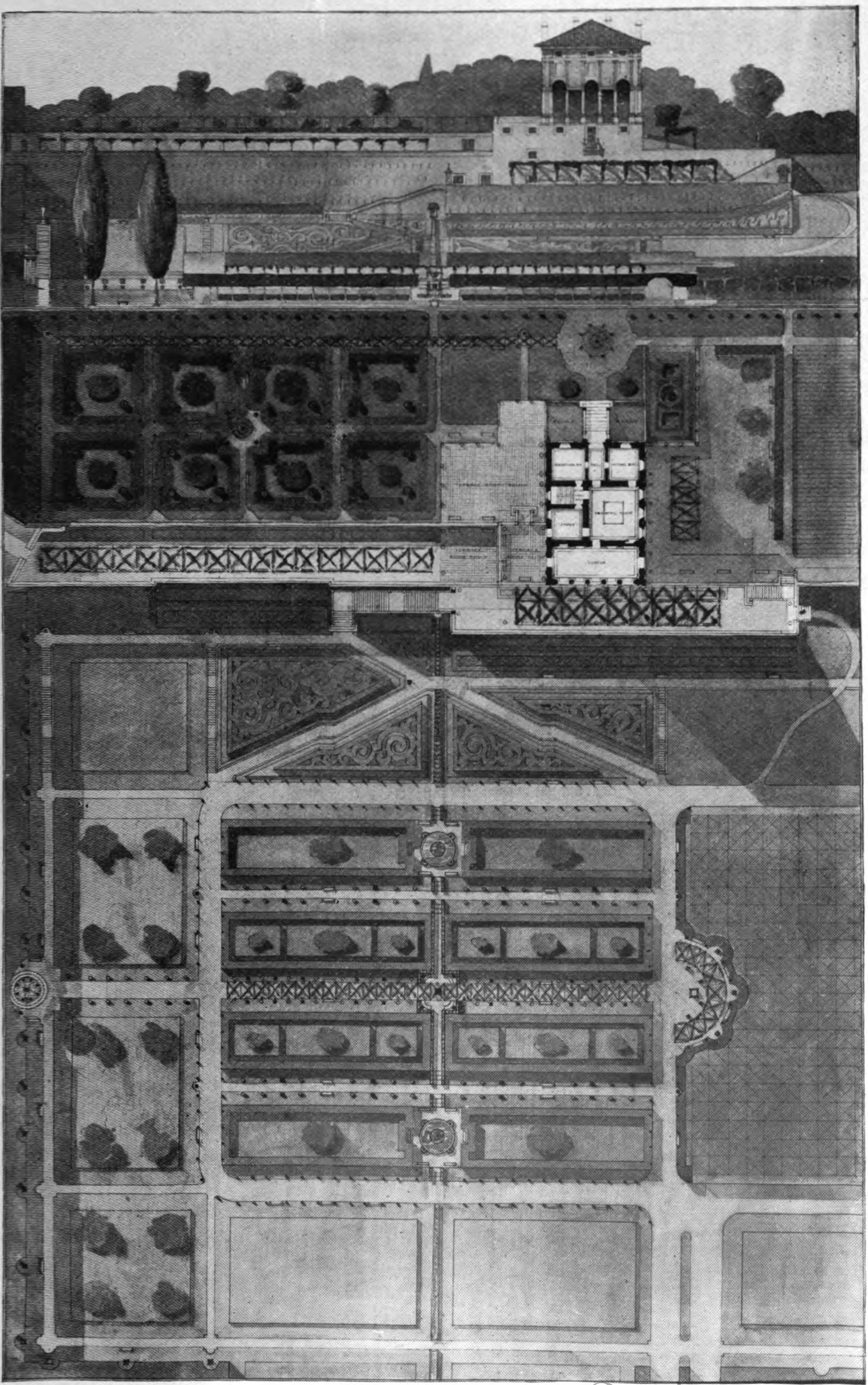
terious cooling shade and catch glimpses of the open garden beyond. There all is brightness. The warm plaster façade of the palace, the golden-yellow gravel under foot, the fresh green hedges, the bright-colored flowers and the mellow-toned marble statues, all lighted from a clear blue sky, are in telling contrast to the shade and the natural wooded track beyond.

The Villa Borghese, next reached in a circuit of the city, has a fine casino, now an art-gallery. This building is palatial in its proportions and has an excellent plan. A short walk brings one to the palazzo of Papa Giulio, now a dilapidated building. Close by is the Villa Giulio, by Vignola. Only the casino remains. It is an extremely interesting building, but suffers much from having lost entirely its ancient setting of grounds. A walk along the old Flaminian road, across Ponte Mole, leads to Monte Mario on the left. Here, scarcely discernible from a distance, nestling in against the hill, is all that remains of the magnificently conceived Villa Madama. This was designed by Raphael for Cardinal Giulio di Medici, afterwards Pope Clement VII. It was intended to have been the finest villa of Italy, to surpass those of antiquity which Raphael knew from pictorial remains. One wing only was completed. Its stucco work, done by Giulio Romano, is quite as good as that of Roman days, and, no doubt, was prompted by the antique decoration, then uncovered, known to Raphael's school.

On the summit of the hill is the Villa Melini, notable for a fine view. Below, Rome is spread out, and off to the right lies St. Peter's and the Vatican garden, in which stands one of the prettiest little casinos in the country, the Villa Pia. It is now neglected, the Pope having a later, less elegant, summer-house.

This little building is of the sixteenth century. It is placed in a less formal part of the garden, on steep sloping ground, and on the axis of an entrance to the garden. The façade of the casino is much like that of the Villa Medici, covered with a wealth of decoration. The oval court is geometrically paved and bordered by stone seats and a low parapet.

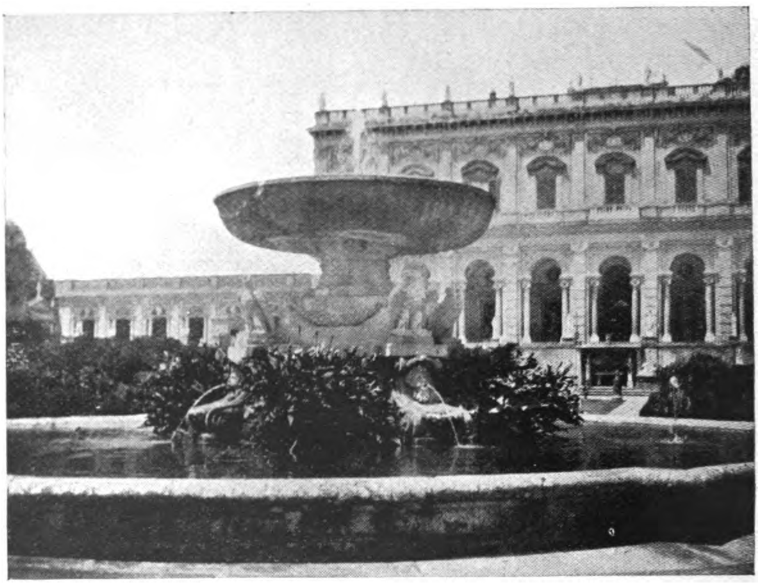
Continuing by the Via del Lungara one arrives at the Villa Farnesina, by Peruzzi. Little remains of the gardens, but the



VILLA LANTE, BY GRAHAM

casino is very fine. On Monte Gianicolo is the casino of the Villa Lante, by Giulio Romano. Here again the grounds have all but disappeared, the park of Garibaldi cutting off the fore court, and the slope of the hill, neglected for years, has about lost the outline of the garden which once beautified it.

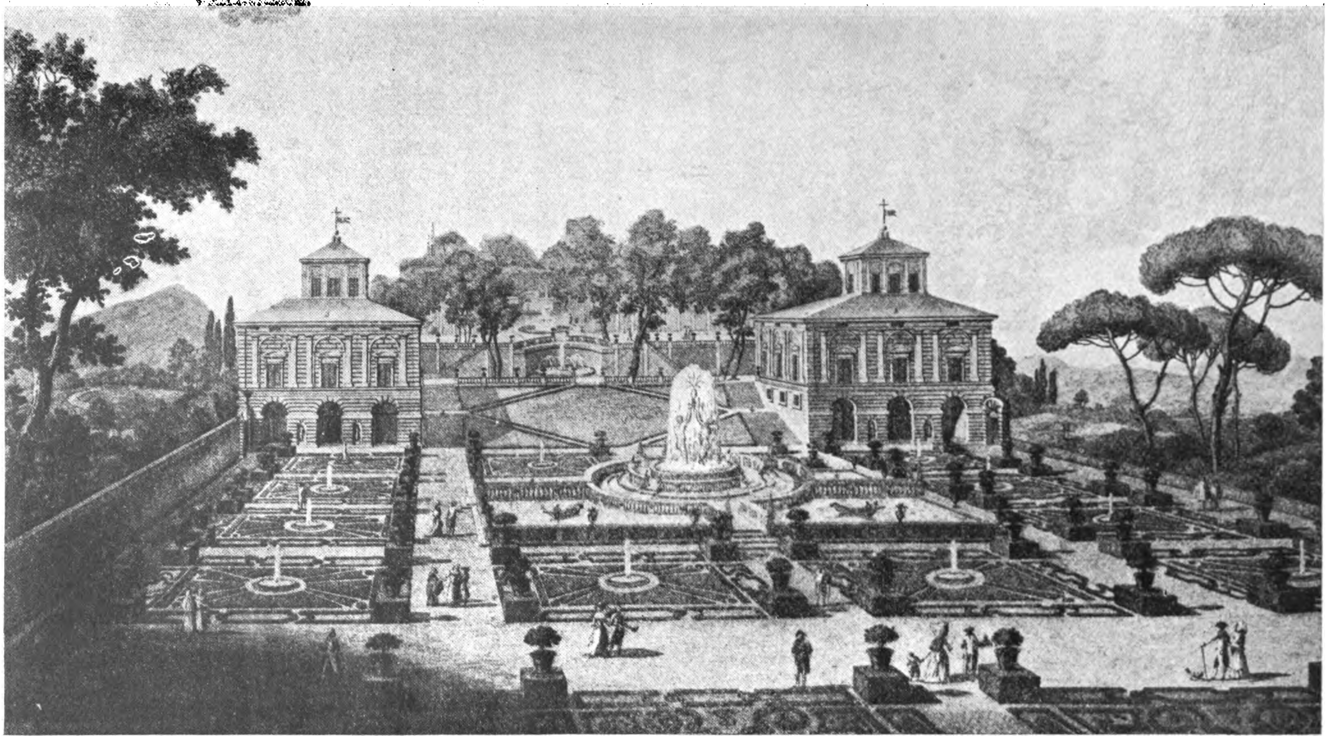
Farther on is the despoiled Villa Spada, the casino only remaining. This is on a steep slope with ground at different levels on three sides. A short walk leads to the Villa Pam-



VILLA ALBANI (PALACE AND FOUNTAIN)

philia Doria, one of the largest and most visited of those of Rome. The garden spreads out in wide terraces from the casino, a not pleasing building. From here a fine view of St. Peter's may be had.

Returning through the Porta Pancrazio and crossing the Tiber by Ponte Rotto, one reaches Monte Palatino, the home of the Cæsars. The Villa Farnesina was here, built on earth that had covered ancient Rome for centuries. Now, only the casino remains. On Monte Celio is the Villa Mattei, with an interesting garden.



VILLA LANTE AT BAGNAJA

Outside the Porta Salario is the Villa Albani, built in 1760 by the Cardinal Alessandro Albani. This is the latest great villa of Rome. Cardinal Albani was a lover of art, and built this immense estate to contain the works which his friend Winkleman, the archæologist, had collected for him. Within the city is the Villa Aldobrandini and the noteworthy and beautiful Palazzo Colonna garden.

At Frascati, the first high ground after crossing the Campagna, are many villas. Those of Conti, Aldobrandini, Piccolomini, Ruffinella, Madragone, Falconieri and Muti are the most important. The Villa Muti, possibly the least known, has a very fine garden.

At Albano is the Villa Barberini, of interest principally for the remains of Domitian's villa.

At Tivoli is the celebrated Villa D'Este, with the most beautiful garden in Italy. A short distance out of the town is the Villa Braschi, on a narrow strip of hillside. It affords a splendid view of Rome and the Campagna. Neither its garden nor casino are good. Below Tivoli, raised above the Campagna, is the Villa Adriana. It is only a ruin, but of extreme interest. North of Rome some thirty miles, at Caprarola, is the Palazzo Farnese, also the casino and gardens. Outside Viterbo at Bagnaja is the Villa Lante. This is a beautiful garden, thoroughly cared for. It gives one, perhaps, a better idea than any other of the charm of Italian gardens during the days of their makers, when cardinals and princes walked their paths and listened to the splashing of their fountains.

SOME DIFFICULTIES OF ROPE TRANSMISSION.

ARTHUR HOLMES MORSE, '01.

"And now remains
That we find out the cause of this effect
Or rather say the cause of this defect:
For this effect, defective, comes by cause."

THE use of ropes for the transmission of power is so general and of such great importance that a knowledge of the principles which govern the design of rope-drives is useful if not indispensable to everyone engaged in engineering work. A more or less adequate treatment of the subject is to be found in standard works on practical mechanics, and several manufacturers of rope or of rope-drive machinery publish pamphlets which contain much valuable information; but it is noticeable that all observe an eloquent silence in regard to the difficulties which are often encountered and the defects which sometimes develop even in the most carefully designed drives. The purpose of this article is to point out the difficulties which confront the engineer in designing rope-drives, and to discuss some of the defects which lead to unsatisfactory working. It will therefore be necessary to deal for the most part with what might be termed the adverse side of the subject, though it should be understood at the outset that the writer has no intention of implying anything derogatory to rope-drives in general, but that he is led by the exigencies of his subject to say more of their occasional faults than of their established merits.

No attempt will be made to introduce mathematics, and only the practical aspect of the subject will be considered.

Two distinct systems of rope-drives are in common use, known respectively, from the countries in which they originated, as the English and American systems. The English system is the simpler of the two. In its simplest form it comprises two sheaves or grooved pulleys, one being the driver and one the follower or driven sheave, each having a single groove, and one endless rope running on the sheaves. If it is required to trans-

mit more power than can be done with one rope, the number of grooves in each sheave is increased and a separate endless rope is placed in each groove, there being as many separate ropes as there are grooves.

Two prominent advantages are peculiar to the English system. *First:* it adapts itself readily to sudden fluctuations of load, each rope carrying an approximately equal share of the increased load; and no matter how many ropes are employed this sensitiveness remains undiminished. In situations where the amount of power transmitted is subject to wide and abrupt variations, as in rolling-mill practice, the English system is preëminently the one to use. *Second:* since it is highly improbable that all the ropes in a drive will break at once, a stoppage of machinery due to sudden failure of the drive is a very remote possibility. If one rope appears to be giving out it can be removed at a convenient opportunity and a new one substituted without great expense of time. In most cases a rope can be spliced and put on the sheaves in from two to four hours.

To offset these advantages the system has certain defects. All ropes are subject to a considerable amount of stretch, and unless the stretch is taken up in some way it accumulates and causes the rope to run slack. As the English system provides no means for removing this slack, it is necessary from time to time to shorten the ropes by resplicing, reducing the length sufficiently to make them run tight. A rope stretches most when new, and in some cases a rope shortened after two or three months' running will not require shortening again until worn out.

The American or continuous system in its simplest form comprises besides the driving and driven sheaves a so-called take-up or tension sheave, mounted in a movable carriage and located so that the slack part of the rope passes around it, embracing usually half its circumference. Weights are attached to the carriage in such a manner as to resist the pull of the rope, and by suitably proportioning the weight any desired tension may be produced and maintained in the rope. The tension is unaffected by the stretch of the rope, since the take-up recedes as fast as the rope stretches, keeping it always tight.

The sketch (Fig. 1) shows the simplest arrangement of the American system. B is the driving sheave, A and C are the two grooves of the driven sheave, and D is the take-up. An endless rope is wound on all three sheaves in the manner indicated. When a greater capacity is required than that of one rope, grooves are added to both driving and driven sheaves and one endless rope is wound in all the grooves, just as a rope is reeved over the sheaves of a pair of tackle-blocks, the two free ends being brought together around the take-up sheave, which is inclined so as to be in alignment with the two outside grooves. By this means only one endless rope is employed and only one splice introduced, whence the system is often called the Continuous system.

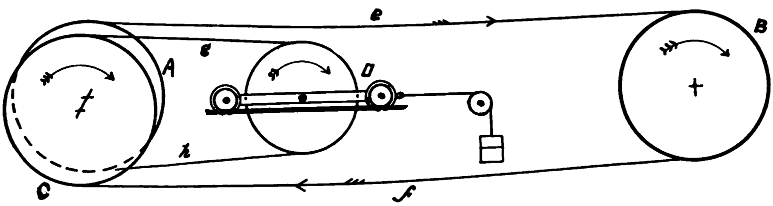


Fig. 1.

Whatever the number of wraps employed, it is evident that the sheave from which the rope passes to the take-up must have one more groove than the other. The diagram represents a single-wrap drive, which will suffice for the purpose of illustration. The take-up sheave D receives the rope from the groove C and delivers it to the groove A of the driven sheave. The portion *e* of the rope is the tight or driving part, *f* is the slack part, *g* and *h* are parts having equal tensions, the tension in each being equal to approximately half the pull applied to the take-up. As long as the load is unchanged, the tension in each part of the rope remains constant, consequently the sag of each part undergoes no alteration and the take-up remains stationary. Now suppose the load to increase: the part *e* is drawn tighter in consequence of the added load. In tightening it is made straighter and consequently shorter, and the slack which is

removed from e is carried around by the driving sheave and thrown into f, whose length is at once increased and its tension diminished as a result. The function of the take-up is to remove this slack from f, keeping its tension constant. Before this can be accomplished the rope must be made to *slip* through the groove C, since otherwise it is taken up by the groove A as fast as it is delivered by the groove C, and the take-up remains stationary. It is evident that, in order to effect this slipping, the pull applied to the take-up must be much greater than would be required were the take-up so placed that it could act *directly* on the slack part of the rope. Much of the sensitiveness of action is also lost, because with any reasonable weight on the take-up the slack part of the rope becomes very loose indeed before the take-up is able to overcome the friction of the rope in groove sufficiently to cause it to slip. This difficulty may be overcome by making the groove C a separate sheave, which is allowed to run loose on the shaft; it becomes in effect an idler, and it permits the take-up to adjust itself immediately to a change of load and to maintain in the slack part of the rope a practically constant tension.

In case the take-up is placed at the *driving* instead of the *driven* end there is the same necessity for the loose sheave Referring to the sketch: C would be the driving sheave, B the driven sheave, and A the loose sheave. Frequently the mistake is made of driving with the sheave A, leaving sheave C loose as in the first arrangement, which results virtually in putting the take-up in the tight or driving side of the rope, an arrangement which does not work well in practice. Some authorities claim that no loose sheave is required when the take-up is placed at the *driving* end, though all admit the necessity for it at the *driven* end, and many drives are constructed in accordance with this theory, but it is evident that theoretically the same necessity exists in both cases, and results obtained in practice lead to the same conclusion.

The American system is in theory nearly perfect, inasmuch as the tension weight may be adjusted with exactness to produce any desired tension in the rope, which tension is supposed to be kept

constant and equal in *all* the slack parts of the rope. In fact, however, this advantage is fully realized only with the drive of one wrap. With more than one wrap it is usually difficult or impossible to keep all equally tight; consequently the work is unevenly distributed, as is plainly to be seen by observing the varying amounts of sag in the different wraps of a drive in operation. The two outside wraps are almost always tightest, for the reason that they are nearest to the take-up and consequently most sensitive to its influence. The intervening wraps may be observed to sag considerably more than the others, and they often exhibit a see-saw action among themselves as a portion of the slack shifts itself from one wrap to another. This shifting involves slipping of the rope in the grooves, and this slip usually occurs spasmodically, because the difference in tension between two adjacent wraps increases gradually until the static friction of the rope in the groove is overcome, and then relieves itself suddenly. The slip is extensive enough to be heard under favorable conditions by listening in the vicinity of one of the sheaves. The obvious result of this slipping is to increase the wear of the rope, not often to a serious extent unless the grooves be very rough. It is on long out-door drives that the see-saw action above described is sometimes attended by serious consequences. A strong wind sets up considerable lateral swaying in the ropes, often blowing one past its neighbor, and if while thus crossed the higher of the two chances to sag enough to bring it below the other, it is highly probably that both will be thrown into the same groove, or perhaps thrown off the sheave entirely. The latter event may be and sometimes is guarded against by placing vertical rollers close to the outside ropes and near the sheave towards which the ropes move. It is not feasible to place rollers between the ropes, because the distance is not sufficient to admit a roller of suitable size; but guards, made either of round iron or of flat iron set with its long diameter parallel to the ropes, are often found on out-door drives, placed one in each interval between two ropes. Such guards are very effective in keeping each rope in its proper groove, and the wear of the rope caused by chafing against the

guards is very slight if they are properly set. The only objection to their use seems to be on the score of their appearance, which is certainly not pleasing. *Deep grooves* are highly effective in keeping the ropes where they belong, and are to be advocated under all conditions. As a rule, the greater the number of wraps, the more irregularity of action, and in drives of a large number of wraps, say fifteen to twenty, a progressive and nearly uniform decrease of tension may often be observed, the slackest ropes sagging two to three times as much as the tightest, which of course implies a more than proportionate difference of tension. Drives worked almost to the limit of their capacity are much more likely to work uniformly than when running with a comparatively light load, and for this reason it is not in general desirable to design a drive with a large surplus capacity, when this capacity must be obtained at the cost of additional wraps of rope.

When it is impossible to avoid the use of a large number of wraps, the expedient is sometimes adopted of using two or more endless ropes with a separate take-up for each, making virtually a subdivision into a number of smaller drives. In this manner some of the advantages of the English system are obtained, while those peculiar to the American system are not lost. In case one of the ropes should show signs of weakness and have to be discarded at short notice, the remaining ropes could, by adding weights to the take-up, be made to carry the entire load with safety until a convenient time for putting on a new rope. It would seem that, by carrying the process of subdivision to an extreme and using a take-up for every two or three wraps, the American system might supplant its rival in places where variations of load are violent and sudden; but such a scheme would involve expensive complications and multiplication of parts, and moreover in practice it is found extremely difficult to effect an even distribution of the load. One set of ropes is almost always inclined to do more than its share of the work, no matter how great care is used in making the grooves all of equal diameters, and incessant adjustment of tension weights is needed to maintain even approximate uniformity.

If, as is usually the case, all the sheaves of a drive revolve in vertical planes, lack of uniformity in the sag of the ropes is not likely to lead to disaster unless the ropes are exposed to the wind. When, however, the ropes are to be led around a corner it is frequently necessary to use guiding sheaves set approximately in a horizontal plane — "mule" sheaves as they are called. These are set as nearly as possible in a plane parallel to the tangent to the extremity of the curve in which the rope hangs. If this can be done with exactness, the ropes will run in nearly perfect alignment with the grooves, but if no two ropes sag the same amount the sheave cannot be set correctly for all, and as a result some must rub continually on the sides of the groove. There is also danger of the ropes becoming displaced.

Many rope-drives with the design of which no fault can be found are unsatisfactory in their working on account of imperfections in the machinery. Perhaps the most common of these imperfections is wabbling sheaves, caused either by their not being bored true, or by their being set on a shaft which is sprung or crooked. The effect of the wabbling is to produce lateral vibration of the rope, always annoying, but more violent and therefore more dangerous at high than at moderate speeds. A still worse defect is eccentricity of the sheaves, caused by making the bore of the hub not concentric with the groove, or by boring it too large for the shaft and keying it tight, which throws the center to one side. The result of eccentricity is alternately to jerk and to slacken the ropes with every revolution of the eccentric sheave. In horizontal drives the sag of the rope relieves the jerk to some extent, but when the drive is vertical there is no sag, and the incessant jerking is speedily destructive to the rope. This particular defect is by no means as rare as might be expected, even in machinery made by manufacturers of good reputation.

Inequality in the diameters of different grooves in the same sheave is another cause of short-lived ropes. When this inequality occurs, it is evident that the rope in one groove cannot move with the same velocity as the rope in another groove of different diameter unless slipping occurs, the *angular* velocity

being of course the same for both. The amount of this slip is greater than is apparent upon a superficial consideration of the conditions. Suppose two adjacent grooves in a 48-inch sheave to differ in diameter by $\frac{1}{16}$-inch. This corresponds to a difference in circumference of approximately $\frac{3}{16}$-inch. If the sheave runs, say, 300 revolutions per minute, the slip will amount to $\frac{900}{16}$ or about 56 inches per minute, more than half a mile a day — something to be avoided. The most serious consequence of this inequality is not, however, the increased wear which results from the continual slipping, but the unequal tensions produced in the different parts of the rope, owing to the tendency of one part of the rope to move faster or slower than another. This results in making some parts of the rope abnormally tight while others run correspondingly slack, and the work is unevenly distributed.

We have already noted several of the causes which tend to produce vibration in ropes, but there remains one more which, though comparatively rare in its occurrence, is of sufficient importance to merit our consideration. This cause is *synchronism*. Sometimes the ropes of a drive are observed to be subject, either continually or at intervals, to violent vibration or swaying from side to side, which cannot be accounted for by any imperfections of machinery or fluctuations of load. In such cases, if the number of vibrations which the rope makes per minute be counted, it may be found that its period (or time required to make a vibration) corresponds with that of some piece of machinery with which it is connected — most frequently the stroke of the engine driving it; but it may be any machine which has a periodic motion. It is a well-known fact that the period of vibration of a cord in tension (the string of a musical instrument, for example) depends on three things: (1) the weight of the string per linear unit, (2) the length of the vibrating portion, and (3) the tension. This is as true of a rope weighing a pound to the foot and a hundred feet long as it is for a banjo string, only in the case of the rope the period is several thousand times as long; and every rope which is in tension and free to vibrate possesses a fairly definite period, or, to borrow a term from the vocabulary of music, "*pitch*." This being the case, a very slight recurring

impulse, if its period coincides with that of the rope, is sufficient to produce oscillations apparently out of all proportion to the cause which occasions them.

The remedy for this difficulty is obviously to change the period of the rope — to put it, as it were, out of tune with the disturbing impulse. This might be done by changing any of the three quantities enumerated above. The weight of the rope is practically unchangeable unless a larger or smaller one be substituted, which is not often feasible. We can, however, change the length of the rope at will by placing an idler either over or under it at some point intermediate between its extremities. This is an effectual remedy, but somewhat expensive and not always applicable. With the American system we can always change the tension, and by doing this to a sufficient extent the synchronism can be destroyed. It is to be observed that reducing the tension is as effective in this respect as increasing it, and reduction should always be tried first if it can be done without too much decreasing the capacity of the drive. The prevailing idea among the mechanics to whose care rope-drives are intrusted seems to be that vibration can be stopped by adding sufficient tension weight, and so indeed it can; but the result is an overloaded and consequently short-lived rope, and the loss of much power in unnecessary friction. Probably a careful diagnosis of the drives which have to be overloaded "to keep the ropes on the sheaves" would show that synchronism is at the root of the difficulty in a large proportion of cases, and the rational remedy would doubtless effect many cures.

An important feature in the design of rope-drives is the location of idler sheaves, that is, sheaves through which no power is transmitted, but whose only function is to change the direction of the rope. Whenever possible they should be placed so as to avoid *reversing* the bend in the rope, or bending it first in one direction and then in the other. The effect of this reversed bend is seriously to shorten the life of the rope by increasing the internal wear. Convenience often requires that the take-up move at right angles to the general direction of the ropes, and in such a case two idlers are necessary, the common arrange-

ment being that shown in Fig. 2. It will be seen that in this case two reversed bends are introduced. The arrangement shown in Fig. 3 accomplishes the same thing and eliminates

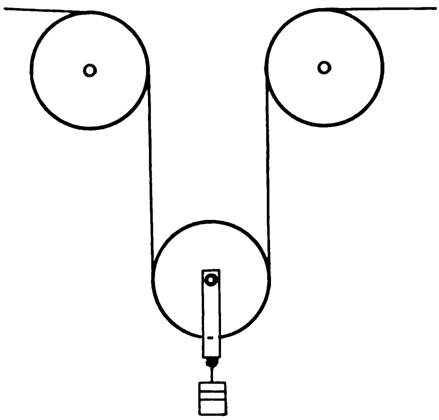


Fig. 2.

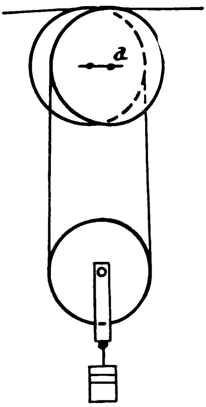


Fig. 3.

the reversed bends. The shaft *a* carries two sheaves, one of which is loose on the shaft so as to allow the take-up to act without having to slip the rope. This arrangement has also the advantage of requiring less space, and imposing less frictional resistance, since one shaft with its bearings is dispensed with. An arrangement similar to Fig. 2 is sometimes used when power is applied to or taken from the middle of a long span of rope, as is often the case with traveling cranes.

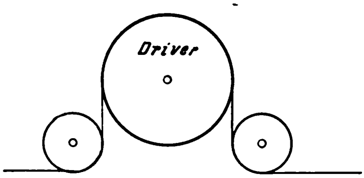


Fig. 4.

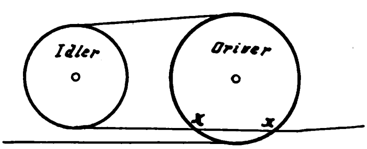


Fig. 5.

The usual arrangement, shown in Fig. 4, is particularly vicious, for the idlers are often small, and placed very close to the working sheave. A better arangement is shown in Fig. 5, which requires but one idler, and that of ample size, and intro-

duces no reversed bends. The idler is inclined sufficiently to permit the rope to clear the working sheave at x. In some instances reversed bends are unavoidable, but when they are used the sheaves should be made as large as circumstances allow.

A word remains to be said concerning the speed of rope, which in designing a rope-drive is one of the first things to be considered. It may be well to examine briefly the relation between speed, tension, and power transmitted. The "working load" or maximum tension in the tight or driving side of the rope is taken ordinarily at $\frac{1}{25}$ to $\frac{1}{30}$ of the rope's ultimate strength. The "effective tension," or tension which is utilized in doing work, is the difference between the tension in the tight side and the tension which it is necessary to maintain in the *slack* side of the rope in order to keep it from slipping. Under average conditions it is found that the tension in the slack side must be equal to about half the tension in the tight side when the drive is carrying its rated load.

As the speed of the rope is increased, the element of centrifugal force enters into the problem, and its value must also be subtracted from the working load before the true effective tension is found. Centrifugal force increases as the square of the velocity, and at a speed of about 8,500 feet per minute, with rope of average weight and strength, it equals the entire allowable tension of $\frac{1}{30}$ the ultimate strength, leaving no margin at all for doing useful work. Since, however, the work done is proportional to the product of speed and "effective tension," there must be a point somewhere between 0 and 8,500 where the speed is such as to give a maximum amount of work. This point is found at about 5,500 feet per minute, and such a speed, if adopted, would permit the smallest possible amount of rope to be used to transmit work at a given rate.

The final question of economy, however, includes considerations not only of the first cost of rope and machinery, but of wear and tear and liability to derangement and accident as well. Wear is regarded as nearly proportional, and when the rope runs steadily this assumption is probably not far from the truth.

Vibration of the rope and irregularities of motion in general are very much exaggerated at high speeds. Sheaves intended to run at high speed require to be balanced with great care, and given not only a static but a "running" balance in order to run free from tremor. For all these reasons the rope speed is usually kept below 5,500 feet, 4,500 or 5,000 being as high as is often met with, save under exceptional conditions. Still lower speed, 3,000 to 4,000 feet, is to be advocated as conducive to durability and steadiness. Individual requirements frequently reduce the practicable speed to a still lower figure. There is also a relation between the diameter of rope and diameter of sheaves which must not be overlooked. Authorities differ as to the smallest sheave suitable for a given size of rope, but a sheave whose diameter is 40 times that of the rope may be taken as the minimum where durability is expected, and this figure might advantageously be increased to 50 or 60. Sometimes sheaves as small as 30 or even 20 diameters are found, but in most cases the ropes used on them are short-lived.

In designing a drive (we will assume the American system to be used), one usually knows at the outset the power to be transmitted and the speed of the shafts on which the sheaves are to be placed; it is left to determine the size of the sheaves and of the rope and the number of wraps to be used. As these quantities are closely dependent, one on the other, it is necessary to effect a compromise, and the selection of the most advantageous combination requires care and good judgment. The worst difficulty is encountered with high-speed shafting. In order to keep the rope speed within reasonable limits the sheaves must be made small, and this in turn involves the use of small rope and many wraps, with their attendant disadvantages. For illustration, suppose that it is required to transmit 150 horsepower from a shaft running 200 revolutions per minute. Assuming 4,500 feet to be the highest speed we are willing to adopt, we find the largest sheave we can use is about 67 inches in diameter. Taking the diameter of the sheave as 40 times that of the rope, the largest rope practicable to use is found to be 1½-inch. A 1½-inch rope at the speed of 4,500 feet per minute

is good for about 30 horse-power, so we shall need 5 wraps. If 1¼-inch rope were used, 8 wraps would be needed, which of course would require sheaves with at least 8 grooves, and the total cost would be more than for the 1½-inch rope, requiring but 5. Moreover the increased number of wraps is a further objection to the use of the smaller rope. If we could use 2-inch rope, 3 wraps would suffice, reducing the cost still more; but few would consider using so large a rope over so small a sheave, especially at the speed in question. The conditions just cited are fairly favorable and permit a satisfactory solution, but even at this speed it is plain that if a large amount of power is to be transmitted the number of wraps becomes undesirably great. At higher rotative speeds the difficulties are increased, and it is in this direction that the limitations of the rope drive are most seriously felt. It may be added that some large drives on the English system are working excellently at speeds as high as 6,000 and 7,000 feet per minute. In one instance, at least, if the tension in the rope were calculated from its observed sag, it would be found to be less than the computed tension due to centrifugal force alone! Yet this drive transmits several hundred horse-power and works well; whence it may be inferred that even at this late day some things still remain to be learned about rope transmission.

HARVARD ENGINEERING LABORATORY INVESTIGATIONS.

THE HEAT OF COMBUSTION OF SOLID FUELS.

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THE heat of combustion of a fuel can be ascertained either by calculation based upon its chemical composition, or by actually burning a sample of the fuel and determining the amount of heat given up.

The Determination of Heat of Combustion by Calculation.

The method of calculation requires a preliminary chemical analysis. For all the ordinary fuels, solid, liquid or gaseous, this analysis is one of extreme difficulty owing to the presence of complex hydrocarbons. The difficulty is so great that the complete analysis is but seldom attempted. The calculation of heat value is ordinarily based upon a determination of the ultimate composition of the fuel, that is, upon the weights of each of the elements present. This necessarily neglects any consideration of how the elements are combined with one another in the fuel,— an important omission, since heat is absorbed in breaking up the hydrocarbons before they are burned to carbon dioxide and water, and since also some of the hydrogen found by the ultimate analysis to be present may be already combined with oxygen. Consequently, the calculation of the heat value of a fuel from its ultimate composition has to be made by the use of some empirical formula, the only sanction for which is the agreement of the results obtained by its use with the direct experimental results. The method of calculation is more complicated, requires a much longer time, and is less reliable than the direct determination. The results of many tests indicate that the calorific value of bituminous coal can be determined from its ultimate analysis with a probable error of not more than two per cent; the probable error for anthracite coal is not so great.

The Experimental Determination of Heat of Combustion.

The experimental determination of heat values by actual combustion is more direct, expeditious and accurate than the determination by calculation. The combustion has to be carried on in a calorimeter, so arranged that the total heat evolved can be measured. The fuel must be supplied with oxygen in sufficient quantity to permit complete combustion. The oxygen is obtained from some substance (such as potassium chlorate or lead oxide) giving up oxygen on heating, or from air, or pure oxygen may be supplied.

Combustion at Constant Pressure and at Constant Volume.

Fuel calorimeters may be divided into two classes, according as the combustion occurs in a vessel communicating with the atmosphere (or some equivalent receiver), or in a closed vessel. In the former case the combustion will occur at constant pressure; in the latter case at constant volume.

With the constant-pressure calorimeter, oxygen is supplied continuously throughout the combustion, and the products of combustion escape as soon as formed; with the constant-volume calorimeter, an excess of oxygen is imprisoned with the fuel in a closed vessel, and the products of combustion remain thoroughly mixed with the excess of oxygen throughout the whole period of combustion.

The heats of complete combustion of a fuel burned in each of these ways are practically the same. There will be a slight difference in the case where the volume of products of combustion escaping from a constant-pressure calorimeter is not the same as the volume of air entering.

Suppose the fuel burned in a constant-pressure calorimeter to be rich in hydrogen. The hydrogen combines with oxygen, forming water vapor, which is nearly all condensed in the calorimeter and becomes of negligible volume. The volume of gas escaping is therefore less than that entering, so that less work is necessary to force it into the atmosphere than was done in introducing the air. On the whole, the calorimeter will gain energy

from this cause. On the other hand, the escaping gas is practically always at a higher temperature than the entering gas, and this makes its volume greater, which increases the external work done in forcing the products into the atmosphere, at the expense of some of the heat of combustion, which otherwise would have been given to the calorimeter. With a constant-volume calorimeter there is no communication with the exterior, and consequently no external work is done. Similar considerations hold for all other chemical actions (such as the combustion of carbon monoxide) in which a change of total volume results from the combination.

Another source of difference between the heats of combustion obtained with the two types of calorimeter is the variation in the completeness of the condensation of the water vapor formed by the combustion. The weight of water vapor that can exist uncondensed depends only on its volume and temperature. The temperatures of the products of combustion are usually the same in both forms of calorimeter, but the volume of the products in the constant-pressure calorimeter is very many times greater than in the other form of calorimeter. The weight of water vapor escaping uncondensed is consequently many times greater in the constant-pressure calorimeter, and it takes with it some latent heat which would be given up in a constant-volume calorimeter. This difference may be considerable, and is one of the causes of the higher calorific results obtained with the constant-volume calorimeter. The other causes will be noted later.

The Constant-pressure Calorimeter in the Harvard Engineering Laboratory.

Calorimeters of both the constant-pressure and constant-volume types have been devised, constructed and tested in the Engineering Laboratories of Harvard University. The constant-pressure calorimeter was designed with the intention of burning a comparatively large weight of fuel so as to ensure a more representative sample, and also to reduce the importance of the minor corrections, and consequently to eliminate some of the

refinements of manipulation and observation that are necessary when dealing with a small weight of fuel. The weight chosen was twenty grams,—the weight used by most investigators is only one gram.

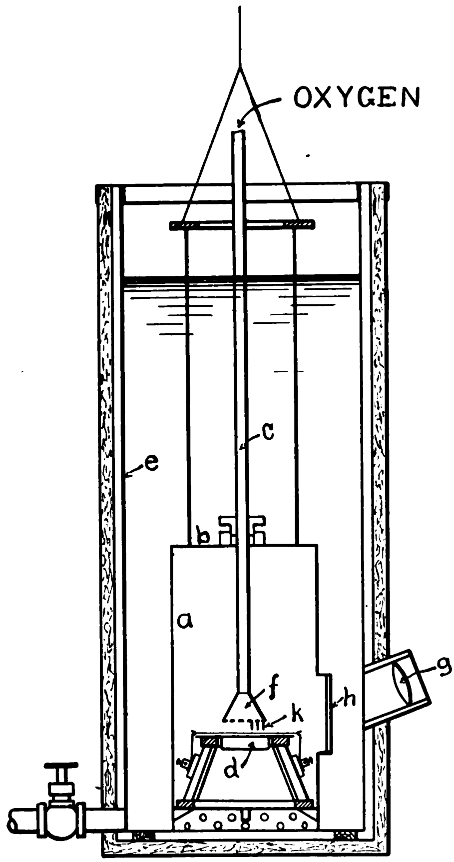

Fig. 1.

CONSTANT-PRESSURE CALORIMETER.

The calorimeter, Fig. 1, consists of an inverted copper cylinder, or combustion-chamber, a, closed at the top by a plate, b, through a stuffing-box in which enters the oxygen-pipe, c; the cylinder is open at the bottom and is perforated with numerous small holes near its lower rim for the escape of the products of combustion.

The fuel to be burned is placed in a flat platinum dish, d, inside the inverted cylinder. The whole is placed in a copper vessel, e, which is suitably protected from external radiation losses. The ignition of the fuel is started by electrical means, and is supported by a regulable stream of oxygen sprayed from the adjustable rose, f, on to the fuel. The progress of the combustion can be seen through the lens, g, and the glass side, h, of the combustion-chamber. The pokers, k, fastened to the rose, permit stirring of the fuel when necessary. An agitator keeps the water surrounding the combustion-chamber at uniform temperature.

This calorimeter can be used only for solid fuels, and has all the defects of the class to which it belongs,— defects which are the more pronounced the greater the content of volatile components in the fuel. At the beginning of the combustion the volatile components are liberated so rapidly that it is generally impracticable to ensure their complete combustion. Also, the fuel has to be burned in small pieces, for if it is powdered, an explosive combustion occurs, scattering the fuel. It is difficult to ensure the complete combustion of these small pieces. They are rapidly coked during the first few minutes of the combustion and then become gradually covered with a coating of ash, which prevents the free access of oxygen to the unburned part, so that combustion becomes continuously more and more sluggish and may cease before it is complete. These two possible sources of error — the incompleteness of combustion of the gases and of the solid matter — make this method of combustion unreliable. A correction can be made for the unburned coal remaining after combustion is ended. The total residue is put in a combustion-tube and heated, pure dry oxygen is passed over it, the resulting gases go over red-hot copper oxide, and the carbon dioxide and water resulting from the combustion are absorbed and weighed. The total weights of carbon and hydrogen in the residue can then be determined and their heats of combustion calculated and added to the heat given up in the calorimeter. This correction, applied (when necessary) to an anthracite coal, will give a satisfactory result; the correction is, however, troublesome to make and consumes considerable time. With bituminous coal the result is generally untrustworthy even after this cor-

rection has been made, on account of uncertainty as to the completeness of the combustion of the volatile components of the fuel.

Constant-volume Combustion.

The uncertainties as to completeness of combustion can be removed entirely and the whole process can be greatly simplified if combustion is carried out in a closed vessel in which are imprisoned the fuel and a sufficient excess of oxygen. This method of combustion is rapidly displacing all others for the determination of the calorific value of solid fuels. It is also applicable to liquid and gaseous fuels, and is the best for such fuels when they are available in small quantity only; if obtainable in larger quantity, Junker's continuous calorimeter method (a constant-pressure method) is preferable. The results obtained with the constant-volume calorimeter are consistently higher than those deduced from constant-pressure combustion.

In the constant-volume method, the fuel and oxygen under great pressure are confined in a small chamber during the whole of combustion, so that there is abundant opportunity for thorough admixture of the oxygen with the gases given off during the combustion. As a result of the high pressure, the intensity and rapidity of combustion are greater, and because of the better admixture, the necessary amount of excess oxygen is decreased. Both these causes make the temperature of combustion greater, and this in turn helps to make combustion more complete. Repeated investigations have shown that the combustion of solid fuels can be made quite perfect in such a calorimeter. The duration of combustion is seldom more than six minutes, as against twenty minutes generally necessary with combustion at constant pressure. The amount of oxygen used is only from one-third to one-fourth of that which the experience of the writer has found necessary with the other form of calorimeter.

The Constant-volume Calorimeter in the Harvard Engineering Laboratory.

On account of the high internal pressures to which they are subjected, the constant-volume fuel calorimeters have to be

made of considerable strength and thickness, and are commonly known as "bomb calorimeters." Various forms have been devised and used by Berthelot, Mahler, Hempel and others.* The bomb designed, constructed and investigated in the Harvard Engineering Laboratory is a modification of the Hempel bomb, differing from it in several important details. It consists, Fig. 2, of a cylindrical steel vessel, a, hermetically sealed by the solid steel plug, b, which is screwed down hard on to a lead washer. The fire-clay crucible, c, in which the fuel is placed, hangs from platinum wires attached to the brass rod, d, which is driven into the plug, b, and the rod, e, which passes through the plug, and is electrically insulated from it. A valve, f, closes the passage, g, through which oxygen is admitted to the bomb. The bomb is lined with a layer of porcelain, which prevents the oxidization of the steel, but is so thin as to offer but little resistance to the passage of heat.† The bomb is placed centrally in a flanged copper vessel, h, which is supported by a thick wooden vessel, k, with an intervening air-space, l. The wooden vessel is provided with a cover through which the agitator, m, is worked, and the thermometer is inserted.

The first investigations with this apparatus were to determine the most satisfactory form in which to burn the fuel. Powdered coal was found to be unsatisfactory, because the combustion became explosive and scattered the powder onto the cold sides and bottom of the bomb, where it remained unburned. Hempel's method of compressing powdered coal into briquettes was found satisfactory for bituminous coal, which formed strong briquettes, but when coke or anthracite were burned some adhesive substance had to be added. This could be either a non-combustible substance, such as silicate of soda solution, or a combustible of known heat value, such as naphthaline. In the latter case there is some chance for error in that the weight of the briquette is not exactly the sum of the weights of the fuel and naphthaline used, owing to some loss of substance in making the briquettes, and it is not possible to apportion the

*See Poole: "The Calorific Power of Fuels," Chap. V.

†It may be noted here that this lining remains practically intact after several hundred combustions.

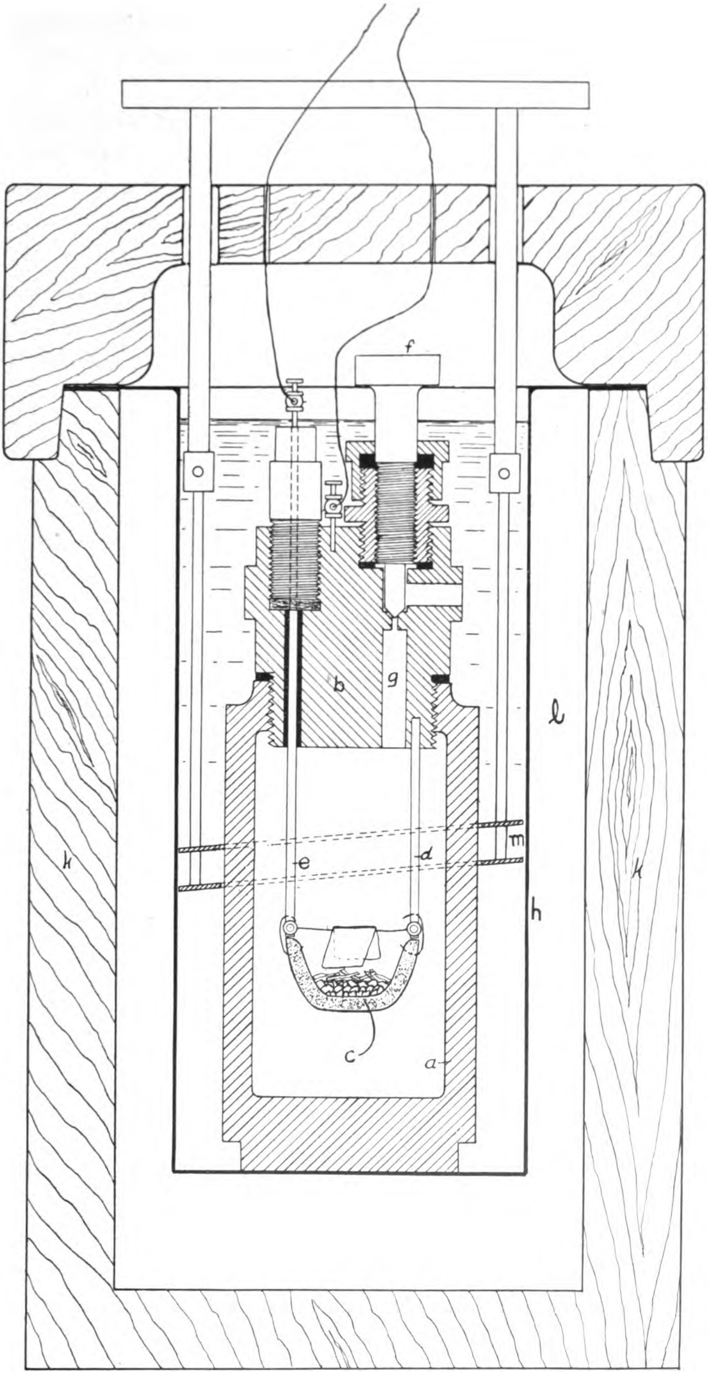

Fig. 2.

BOMB CALORIMETER.

loss of weight correctly to the two constituent substances. Another method, which eliminates this trouble, is to put the powdered coal in the crucible and to fuse a known weight of naphthaline on top of it. The writer found, however, that there was occasional trouble due to the rapid vaporization of the naphthaline and its condensation on the cold containing walls before it could burn, and consequent incomplete combustion.

The method finally adopted has the merit of greater simplicity than those mentioned above. Several pounds of the fuel is taken, crushed to a small size, and that portion of it which passes through a $\frac{1}{12}$-inch sieve but remains on a $\frac{1}{20}$-inch sieve is retained. A known weight of these lumps, averaging about $\frac{1}{16}$-inch, is placed in the crucible and burned. This size was found to be so large as not to scatter, but small enough to ensure complete combustion.

The ignition of the fuel can be brought about in several ways, all electrical. Berthelot sent a current through a known weight of iron wire so arranged that on its ignition it fell on the fuel and started combustion. Hempel imbeds a platinum wire in his briquette, and sends a known current through it for a known time. In the calorimeter under discussion, a definite current is passed through the platinum wire connecting the brass rods, d and e. This wire is of such size that it becomes red hot in one second, and the current is continued for two seconds. This is sufficient to ignite the touch-paper which is placed across the wire, and this in turn falls upon some wood which has been placed on top of the fuel, which in its turn is ignited and starts the combustion of the coal. The size and kind of the touch-paper are standardized, and the heat given up by it and by the electric current was determined by a series of preliminary experiments, so that the correction for the heat so generated involves only simple subtraction at the end of the calculation of total heat given up during the combustion.

The oxygen pressure necessary for complete combustion was found by experiment to be about 300 pounds per square inch when one gram of fuel is burned. This gives about four times the amount of oxygen chemically necessary when the fuel is

pure carbon; the excess of oxygen is considerably less when the fuel is rich in hydrocarbons.

The procedure in the use of this apparatus is as follows: The fire-clay crucible is dried and weighed. The coal is broken up, sifted to the proper size, and about one gram of it is weighed out accurately. The electrical ignition arrangement is tested, $\frac{1}{10}$ gram of wood is weighed out, the coal and wood are put in the crucible, the crucible is hung on the brass rods, the touch-paper

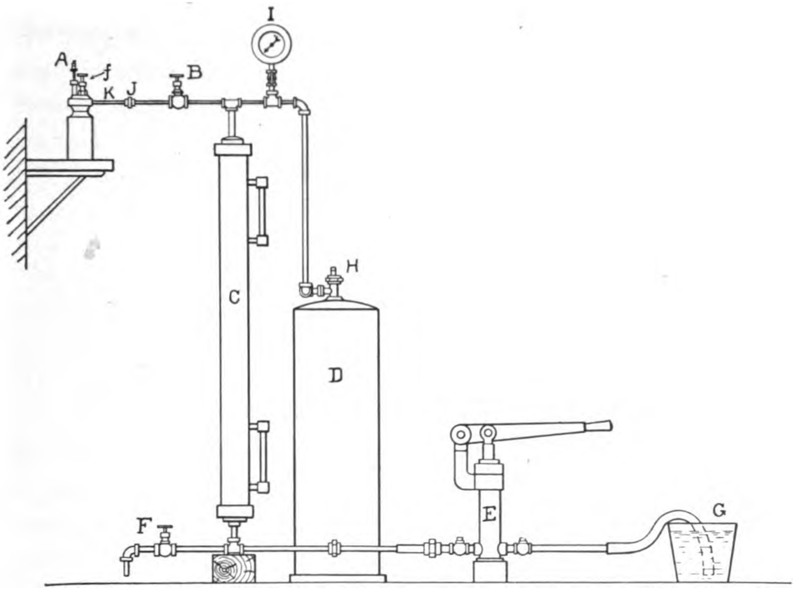

Fig. 3.

rider is set on the platinum wire, and the plug is then inserted and screwed down with great care. The short length of pipe, K, is screwed into the bomb, A, Fig. 3, and the valve, f, screwed back one turn. The bomb is then taken to the oxygen compression apparatus and connected to it by the union, J. The compression cylinder, C, has previously been filled with water by pumping at E, while the valve, B, is open. After the bomb is connected, the valve, H, in the oxygen holder is opened, and by opening valve F the water runs out of C, its place being occupied by oxygen under pressure from D. When C is nearly

full of oxygen the valves F and H are closed, and water is pumped into C till the gauge, I, shows 300 pounds per square inch pressure. The valve, f, on the bomb is then screwed down, and after closing B, the union, J, can be disconnected, the length of pipe, K, taken out, and the bomb is ready for insertion in the calorimeter vessel. The weight of the bomb and the dry calorimeter without its cover is now accurately ascertained. Water, at about the temperature of the room, is then poured into the vessel h, Fig. 2, till it is full to within an inch of the top, and the weight is again taken. The wires going through the cover are fastened to the binding-screws on the plug, the cover is put on, a thermometer graduated to $\frac{1}{10}$°F. is hung in the water, and the agitator is moved gently. The thermometer is now read at intervals of one minute with great precision (to $\frac{1}{100}$°F.) for five minutes, or until the change of temperature per minute becomes constant. The switch in the electric circuit is now closed for two seconds and the thermometer starts rising rapidly, the gentle agitation of the water being continued and the temperatures still read at the same intervals. After five or six minutes the thermomeer generally reaches a maximum, but the observations are still continued for another five or six minutes till the change in temperature becomes regular again. The bomb is now taken out, the valve, f, opened gradually to let out the products of combustion, the plug unscrewed with great care, and the crucible taken out and weighed, to find the weight of residue.

The Calculation of the Heat of Combustion.

The heat value of the fuel is determined from the observations described above and from certain constants. The total heat evolved is the sum of the heat given up by the fuel, the wood, the touch-paper and the electric current.

If W_f is the weight of fuel burned, grams,
W_w is the weight of wood burned, grams,
H is the total heat evolved, B. T. U.,
Q_f is the heat of combustion per gram of the fuel, B. T. U.,
Q_w is the heat of combustion per gram of wood, B. T. U.,
h is the heat given up by touch-paper and current, B. T. U.

Then $$H = W_f Q_f + W_w Q_w + h. \quad (1)$$

If W is the weight of water in the calorimeter, lbs.,
K is the water equivalent of the calorimeter, lbs.,
Δ T is the rise of temperature of the water corrected for radiation, °F.
Then

$$H = (W + K) \Delta T. \quad (2)$$

In the two equations given above, Q_f is the only unknown. Of the other quantities, h, Q_w and K are constants determined by previous investigations, and Δ T is calculated from the temperature observations.

The determination of the water equivalent, K, was made in three ways: —

(1) By calculation from the known weight and specific heat of each part of the calorimeter.

(2) By the method of mixture; that is, by half filling the calorimeter with a known weight of cold water, observing the temperature, adding a known weight of warm water at an observed temperature, and finding the temperature of the mixture.

(3) By the combustion of different weights of wood in the calorimeter.

The value of Δ T is obtained from the temperature observations. The temperature observed at the moment of starting combustion is the actual temperature at that time. The temperature at the end of combustion must, however, be corrected for the heat exchange to the atmosphere during the period of combustion. This correction is made by finding, from the temperature observations before combustion and after its completion, the radiation rates at each of these periods and deducing from them the radiation rate during each minute of the combustion. The sum of the radiation corrections throughout the combustion is the total radiation correction.

The method may be illustrated from the following set of observations: —

Preliminary Period.		*Combustion Period.*		*Final Period.*	
0 minutes . .	67°.20	6 minutes . .	73°.80	12 minutes . .	75°.89
1 " . .	67°.22	7 " . .	75°.30	13 " . .	75°.88
2 " . .	67°.25	8 " . .	75°.73	14 " . .	75°.87
3 " . .	67°.27	9 " . .	75°.86	15 " . .	75°.85
4 " . .	67°.28	10 " . .	75°.90	16 " . .	75°.84
5 " . .	67°.30	11 " . .	75°.90		

Average variation per minute before combustion, $\frac{67^\circ.30 - 67^\circ.20}{5} = + {}^\circ.02$

Average variation per minute after combustion, $\frac{75^\circ.90 - 75^\circ.84}{5} = - {}^\circ.012$

The combustion is complete at the eleventh minute.

The exchange of heat between the calorimeter and the atmosphere being proportional to the difference of temperature between them, the above figures show that there will be no heat exchange when the calorimeter is at 72°.7, and that the temperature variation of the calorimeter due to radiation is $\pm{}^\circ.0037$ F. for each degree the calorimeter differs from 72°.7 F.

Then from the average temperature during each minute in the combustion period we find the temperature change of the calorimeter due to radiation to be as follows : —

In minute	5 – 6 . . .	°.008 rise of temperature.
"	6 – 7 . . .	°.007 fall "
"	7 – 8 . . .	°.011 " "
"	8 – 9 . . .	°.012 " "
"	9 – 10 . . .	°.012 " "
"	10 – 11 . . .	°.012 " "
Total change	. . . . .	°.046 fall of temperature.

Therefore the total rise in temperature during combustion, without radiation loss, would be $75^\circ.90 - 67^\circ.30 + {}^\circ.046 = 8^\circ.646 = \Delta$ T.

Bunton's Investigations.

The apparatus described above was tested very thoroughly by George H. Bunton, L. S. S., 1900, with the object of determining the degree of accuracy obtainable in its use. As the method is itself free from error so long as the combustion is complete, any errors in the results obtained will be due either to errors in the constants used or in the observed quantities. The consistency of results obtained is a satisfactory measure of the accuracy of the apparatus and of the observations. The constants were all determined several times with consistent results.

Bunton's investigations, after the preliminary determinations of the water equivalent and of the heat given up by the touch-paper and the current, consisted of a series of combustions of various fuels, repeating the combustion of each fuel about six times, to find the variation in result obtained. The fuels chosen were wood, coke, bituminous coal and anthracite.

The wood burned was soft white wood from the same block that was used throughout the tests for starting ignition. Five combustions were made, with the results tabulated below:

Calorific Value of White Wood.

Weight of wood burned. lbs.	Weight of water in calorimeter. lbs.	Temperature at beginning of combustion. °F.	Corrected temperature at end of combustion. °F.	Heat of combustion per pound of wood. B. T. U.
.004422	2.78	67.30	75.94	7933
.003301	2.77	67.80	74.27	7940
.004479	2.95	77.52	85.91	7924
.004437	2.77	77.47	86.16	7929
.004449	2.83	77.63	86.21	7931
			Average	7931

Maximum variation of calorific value from the average result $= \frac{9 \times 100}{7931} = .11\%$

The tests of coke were made on pieces broken from a large lump of Otto gas coke. Five tests were made, with the following results: —

Calorific Value of Gas Coke.

Weight of coke burned. lbs.	Weight of ash. lbs.	% ash.	Weight of wood used. lbs.	Weight of water in calorimeter. lbs.	Initial temperature of water. °F.	Final corrected temperature of water. °F.	Calorific value per lb. of coke. B. T. U.	Calorific value per lb. of combustible. B. T. U.
.0022913	.0001254	5.47	.0001100	2.94	72.92	80.55	13671	14462
.0023408	.0001276	5.23	.0001573	2.94	75.00	82.91	13727	14492
.0022814	.0000836	3.66	.0001650	2.93	68.43	76.27	13971	14501
.0023012	.0000748	3.25	.0001474	2.83	71.94	80.11	14084	14621
.0023540	.0000836	3.55	.0001650	2.86	73.71	81.90	13848	14357
							Average	14487

Maximum variation of calorific value per lb. of combustible from the average result $= \frac{134 \times 100}{14487} = .9\%$

The bituminous coal tested was from George's Creek, Maryland. It was not necessary to use wood for the ignition of this

coal. The paper alone sufficed. Seven tests were made, with results as under: —

Calorific Value of George's Creek Coal.

Weight of coal burned. lbs.	Weight of ash. lbs.	% ash.	Weight of water in calorimeter. lbs.	Initial temperature of water. °F.	Final corrected temperature of water. °F.	Calorific value per lb. of coal. B. T. U.	Calorific value per lb. of combustible. B. T. U.
.002138	.0001022	4.78	2.58	68.32	76.41	14272	14973
.0023175	.0001025	4.90	2.86	63.10	71.21	14260	14965
.002033	.00007176	3.53	2.80	66.02	73.30	14395	14922
.0021175	.0000693	3.27	2.90	64.52	72.01	14554	14978
.0020948	.0000970	4.63	2.77	67.30	74.79	14285	14977
.0021237	.0000841	3.96	2.83	64.73	72.27	14376	14969
.0022109	.0001010	4.57	2.87	66.47	74.20	14299	14984
						Average	14977

Maximum variation of calorific value per lb. of combustible from the average result $= \frac{55 \times 100}{14977} = .35\%$

Of the anthracite, five tests were made, with the following results: —

Calorific Value of Anthracite.

Weight of fuel burned. lbs.	Weight of ash lbs.	% ash.	Weight of wood used. lbs.	Weight of water in calorimeter. lbs.	Initial temperature of water. °F.	Final corrected temperature of water. °F.	Calorific value per lb. of fuel. B. T. U.	Calorific value per lb. of combustible. B. T. U.
.0023628	.0001398	5.92	.0001408	2.86	71.79	80.17	13842	14713
.0030338	.0002398	7.90	.0002002	2.88	70.01	80.25	13553	14717
.0025872	.0001364	5.27	.0001276	2.86	70.83	79.78	13931	14706
.0027456	.0002288	8.33	.0001738	2.87	66.32	75.56	13464	14687
.0024345	.0001293	5.31	.0001503	2.88	68.12	76.65	14086	14698
							Average	14704

Maximum variation of calorific value per lb. of combustible from the average result $= \frac{17 \times 100}{14704} = .11\%$

An examination of the four preceding tables shows a remarkable consistency of results when expressed in terms of weight of combustible used. The maximum variation from the average result occurred in the case of the coke and did not amount to one per cent, whereas with the wood and anthracite the variation barely exceeded one-tenth of one per cent. The weighing

scales of the calorimeter could be read with an error not exceeding .005 lbs., or about one-sixth of one per cent of the total weight. The thermometer could be read to $\frac{1}{100}°$ F., or about one-eighth of one per cent of the total rise of temperature. Consequently it is only in the case of the coke that the maximum variation from the average result exceeded the sum of the probable errors of reading of the instruments, so that it is only in that case that it is necessary to seek an explanation of the variability of result obtained in the variability of the heat of combustion of the samples burned.

The variation in heat of combustion per pound of coal is shown by these tests to be considerable, but it is also shown to be due almost entirely to the variation in the amount of incombustible matter in the sample. The tests, then, support the conclusion of other investigators that in a given seam of coal the heat per pound of combustible is constant, though the amount of ash present may vary considerably. There is no reason to expect such remarkable consistency of result in the case of a product such as coke, and consequently the greater variation that was found in that case might have been anticipated.*

It may be safely concluded that the bomb calorimeter may be used with complete confidence for the determination of the heat of combustion of any of the ordinary solid fuels, and that it will give results with a probable error of less than one per cent — a degree of accuracy entirely satisfactory in ordinary engineering practice.

* Favre and Silbermann's result for gas coke is 14485 B. T. U. per pound of combustible — a result practically identical with Bunton's.

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Editorial.

We regret to announce the resignation of Gilbert S. Meem, Jr., L. S. S., 1904, from the Board of Editors. He has decided not to return to the Lawrence Scientific School, as he is at present working with the Columbia Improvement Company in Tacoma, Washington.

At the last meeting of the Board, Donald W. Howes, first-year graduate, was elected to take the place of Gilbert S. Meem, Jr.

The following Harvard men have been working on the Stadium during the summer: H. F. Tucker, '01; R. W. Greenlaw, '02; K. E. Adams, '03; H. H. Fox, '03; A. S. Proudfoot, '03; J. A. Wilson, '03; C. E. Tirrell, '04; C. H. Fisher, '04; C. Gilman, '04; H. M. Hale, '04; W. Tyng, '05; C. M. Harrington, Sp.; W. H. H. White, '05; G. C. Townsend, '06; H. M. Turner, '06; C. Morse, '07.

Graduate Notes.

C. W. Stark, '03, is working for the Baltimore & Ohio Railroad, and is located at Chicago.

J. P. H. Perry, '03, is working for the C., B. & Q. R. R., and is at present located at Plano, Ill.

J. A. Wilson, '03, is inspector of concrete slabs at the Stadium.

H. H. Fox, '03, is general inspector of construction on the Stadium.

José Camprubi, '02, is with the Terre Haute Electric Co., Terre Haute, Ind.

G. S. Meem, Jr., '04, is assistant to the Electrical Engineer of the Columbia Improvement Co., Tacoma, Washington.

E. N. Willis, '03, and T. B. Coffin, '03, are both with the General Electric Co. at Schenectady, N. Y.

A. G. Chapin is working with the General Electric Co. at Lynn.

H. W. Locke, '03, is working for Stone & Webster at Houston, Texas.

A. L. Snyder, '02, is also with Stone & Webster.

J. J. Eaton, '96, who has been teaching in the Philippines for two years, was recently appointed manager of the Government Timber Testing Laboratory at Manila.

R. A. White, '99, is electrical engineer for Fort, Bacon & Davis at Birmingham, Alabama.

S. S. Montague, '98, is working for the Warren Bros. Co., Boston.

K. Sherburne, '03, is in the Steam Turbine department of the General Electric Company at Schenectady, N. Y.

A. Durant, '03, is in the Power and Speed Controller Co. at 147 Milk Street, Boston.

A. G. McAvity is at present working for the Buffalo Forge Co.

Architectural Notes.

THE growing interests of Architecture in the University may here in this journal find a very fitting place for their expression. It is the aim of the JOURNAL, with regard both to engineering and architecture, to promote a general knowledge and discussion of subjects common to each of these important fields of work and study, and sometimes subjects common to both, as the building of the Stadium or the heating and ventilating of an important building.

From the architectural point of view alone it is confidently hoped that articles, diagrams and drawings may be published which will be helpful to the student and draughtsman, or at least of interest to them both. It is expected that there will be opportunities for recording results of investigation and really observant travel and study at home and abroad, a chance to record the work of Harvard men as practising architects and designers, an opportunity to keep in touch with aims and methods of different localities. Above all it is hoped that as its highest ideal the JOURNAL may assist in some measure, however slight, in developing an appreciation for the best ideals in the profession, may call attention wherever possible to that vital idea of "quality before quantity" which is certainly what Harvard stands for. In addition to this, it is hoped that past students of architecture in the University may be kept in touch with each other and with the life and work of the Department of Architecture.

This Department, described so fully in the June number of this journal, in 1902, with regard to its home and its aims, commences the new college year with its largest enrollment, having also an increase in the number of college men beginning their professional studies. The University is most fortunate in obtaining as additions to its teaching staff the services of Mr. J. L. Smith as instructor in Freehand Drawing, and Mr. W. L. Mowll, '99, as an instructor in Architecture. Mr. Smith, who has a wide reputation as a painter of architectural subjects, was formerly an instructor in the Museum of Fine Arts in Boston. A large collection of his water colors, many of them loaned to the Department by Dr. Denman W. Ross, has hung in the Freehand Drawing Room in Robinson hall since its opening.

Mr. Mowll, a graduate of the department, has just returned from a two years' stay in Europe, where he was a holder of the Rotch Travelling Scholarship in Architecture, an honor competed for annually by the best draughtsmen in Boston offices. Previous to holding this scholarship Mr. Mowll was in the office of Messrs. Peabody and Stearns in Boston.

The article in this number by E. T. P. Graham, '00, is, it is hoped, one of a series of articles by travelling scholars of the University which may prove helpful to future students of architecture or landscape architecture in arranging their plans for work while abroad.

Of the graduates of the Department, G. S. Parker, '00, and E. B. Lee, '99, are now studying in Paris, the latter in the École des Beaux Arts. Mr. Lee is also the second holder of the Austin Travelling Fellowship in Architecture, open only to graduates in architecture in the University. Of the other graduates, J. A. Gade, '96, and H. D. Whitfield, '98, in New York; J. R. Harris, Jr., '96, and T. M. Hastings, '98, in Philadelphia; and A. H. Blevins, '98, and G. H. Breed, '99, in Boston, are practising architects in partnership with others, and of the rest of the total number of thirty-one graduates the others, with three exceptions, are in architects' offices in different parts of the country.

L. P. Burnham, '02, the third holder of a travelling fellowship in architecture from the University, is at present in the office of Messrs. Wheelwright and Haven in Boston, and is shortly to sail for Europe to begin his studies.

JANUARY, 1904

HARVARD ENGINEERING JOURNAL

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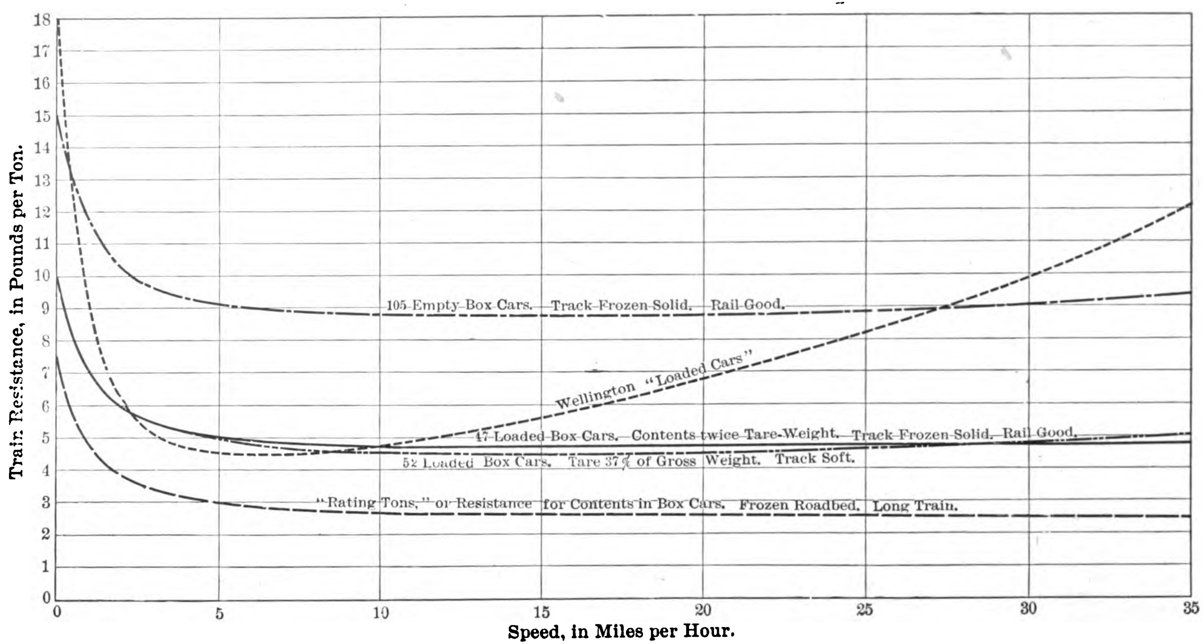

(*Fig. 4.*) TRAIN RESISTANCE CURVES FOR HEAVY FREIGHTS.

See "Train Resistance."

HARVARD ENGINEERING JOURNAL

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and Architecture at Harvard University

VOL. II JANUARY, 1904 NO. 4

TRAIN RESISTANCE.

C. O. MAILLOUX.

(*Continued from the November issue.*)

ALL authorities agree that train resistance is a composite quantity, made up of diverse elemental or component frictional resistances. Numerous attempts have been made by investigators to trace the influence and to estimate the effect of variations of speed, load, temperature, size and form of car, length of train, track construction, and various other conditions, on these component resistances, either individually or collectively. Our knowledge is far from complete, however, and there is still much divergence of opinion in regard to the interpretation, enumeration and classification of the results thus far obtained. Indeed, a comparison of these results often reveals serious discrepancies, inconsistencies, and even contradictions. In the comprehensive paper of Mr. J. A. F. Aspinall on "Train Resistance" (Proceedings Inst. C. E., vol. 147), the different frictional effects constituting train resistance are classified under three heads, namely, axle, atmospheric, and miscellaneous resistances. A similar classification is adopted or assumed by several recent investigators. Such a classification, while perhaps suitable enough for practical purposes, in calculating and comparing train-resistance values, is not the most suitable or convenient one for the study and analysis of train resistance, or

the determination of its component parts, and of the role and the effect of each of these parts. We will obtain a much more clear and comprehensive idea of these frictional effects if we study them by reference to the fundamental laws of friction. The physicist distinguishes three principal kinds of friction, namely, sliding friction, rolling friction, and fluid friction. All three of these kinds are involved in train resistance. If we can group the different resistance components by reference to the particular kind of friction on which they depend, we will have the following classification:

A. Sliding Friction.

(Including two varieties, both involved in train resistance.)

I. *Lubricated Sliding Friction.*
  (1) Rotational friction of axle or journal.
  (2) End-play friction of axle or journal.

II. *Unlubricated Sliding Friction.*
  (3) Slipping or skidding friction.
  (4) Wheel-flange friction.

B. Rolling Friction.

(1) Friction due to mangling or crushing effects.
(2) Friction due to non-yielding inequalities of surface.
(3) Track hysteresis.

A. B. Composite Friction.

(Combining sliding and rolling friction.)

(1) Effects of oscillation and concussion.
(2) Effects of curves.

C. Fluid Friction.

(Including two varieties, both involved in train resistance.)

(*a*) *Semi-fluid friction:*—
  (1) Friction of ties, ballast, embankment, earthwork, etc.

(*b*) *True Fluid Friction:*—
  (2) Air friction at head of train.
  (3) " " " rear " "
  (4) " " " sides " "
  (5) Wind friction.

We will first consider these items separately, as physical phenomena. We will thus obtain an idea, much clearer than would be possible otherwise, of the characteristics and of the relative effects and importance of each; and we will, in consequence, be better prepared to appreciate the influence of each on the total result when the items are assembled. We will then review briefly the formulæ, or rather the types of formulæ, which have been proposed or used for expressing the total result,— the train resistance in pounds per ton,— and we will indicate those types of formulæ which appear most satisfactory and useful, in the present state of our knowledge of the subject.

Car-journal Friction. This is the principal element of train resistance at the time of starting and at very low speeds. Its importance diminishes as the speed increases, and it becomes of minor importance, as a factor of the total train resistance, at high speeds.

The entire weight of a car body and of its contents rests upon the "journals" at the ends of the car-axles, a saddle-shaped metal piece of special alloy, called the "bearing," being placed over the journal to receive the weight and to distribute it over the upper surface of the journal. When the car moves, the journal turns on its axis, and the portion of its upper surface which is in contact with the bearing is caused to slip or glide against the contact surface of the bearing, thus causing friction. The lower surface of the journal is kept lubricated by contact with cotton waste soaked with oil.

The dimensions of car-axle journals and the contact area of the bearings vary according to the maximum weight intended to be carried per axle. For ordinary electric cars, intended for maximum weights not exceeding 15,000 lbs. per axle, the journals have a standard length of 7 inches and a diameter of 3.75 inches; the bearing is about 6.25 inches long (allowing an end play of 0.25 inch), and its actual contact area is a fraction over 25 square inches, or about 33 per cent of the total cylindrical surface of the journal.

This journal, which is the smallest used in steam railroad practice, is known as "type A," "Master Car Builders' stand-

ard," there being in all four "M. C. B." standards (types A, B, C, D).

Table I gives certain data concerning these types and also concerning five types of car journals adopted as standards by the American Street Railway Association at the Detroit Convention, in October, 1902. Types Nos. 3 and 4 are the same as M. C. B. types B and C, respectively. The others are different.

TABLE I. STANDARD CAR JOURNALS.

	"M. C. B." STANDARD.				"AM. ST. RY. ASS'N" STANDARD.				
TYPE.	A	B	C	D	1	2	3	4	5
Maximum Weight per car (in tons)					15	20 to 28	30	40	50
Maximum Weight in 1000 lbs. per axle	15	22	31	38					
Length of Journal, in inches.	7	8	9	10	6	7	8	9	9
" Bearing, "	6.25	6.88	7.88	8.88	5.63	6.25	6.88	7.88	8.31
Diam. of Journal, "	3.75	4.25	5.00	5.50	3.25	4.00	4.50	5.00	5.50
Mean Length of Arc of Bearing Contact, in inches	4.07	4.30	4.94	5.47	2.81	4.19	4.30	4.94	
(Calculated) Contact Area of Bearing, when worn (in square inches)	25.5	29.6	38.9	48.5	15.8	26.2	29.6	38.9	

Journal friction is itself a complicated physical phenomenon, influenced by several conditions, including: (*a*) Rubbing surfaces; (*b*) Lubrication; (*c*) Temperature; (*d*) Pressure; (*e*) Rubbing velocity. We will briefly consider these influences.

(*a*) The friction of a journal is, in general, found to depend upon the material, character, and finish of the rubbing surfaces. In this case, since all car-axles are now made of steel, and all car-journal bearings are made of some kind of bearing metal, the influence of the materials is substantially the same in all cases.* The character and finish of the rubbing surfaces, however, play a very important part as a factor of difference in friction.

* [The latest and most authoritative information regarding the materials used in car-axle bearings will be found in a very able and comprehensive paper — "A Study of Alloys Suitable for Bearing Purposes" — read by G. H. Clamer before the Mining and Metallurgical section of the Franklin Institute, March 28, 1903, and published, together with a very valuable discussion, in the *Journal* of the Franklin Institute, vol. 96, pp. 49–77 and pp. 297–300.]

Sliding friction is presumed to cause loss of power by reason of collisions between the salient or protruding particles of the two rubbing surfaces and the consequent abrasion of these particles, or their attrition, when relative motion occurs between the rubbing surfaces. The work done, *i.e.* the energy expended, in overcoming the mechanical resistance due to sliding friction is converted into heat. The smoother the surfaces in rubbing contact, and the more perfect their finish, the less numerous and the smaller the colliding particles will be, and, consequently, the smaller will be the frictional effect produced, and the less the energy absorbed, by relative motion between the rubbing surfaces. It is precisely for this reason that a well-worn journal, having a mirror-like polish, produces so little friction and runs so cool, no matter whether it be of steel, cast iron, or wrought iron, all of which metals produce very nearly the same friction when in this condition. It has been said, and not without reason, that "every scratch counts" on a railroad car journal or bearing, and that it has a money value, adding to the cost of transportation of every ton carried over that journal.

(*b*) Lubrication is the interposition of a film of oil, grease, or other "lubricant" between the rubbing surfaces, or, in reality, between the protruding particles thereof which cause the friction. The thicker the film of lubricant the farther apart the rubbing surfaces will be, and the fewer will be the particles which can scrape against each other. If the film were sufficiently thick, there would be no direct contact between any protruding particles; and the latter would then act merely as paddles moving in the lubricant and causing minute waves and eddies in it. In such a case there would be no sliding friction, the only friction produced being fluid friction. In the majority of cases, the rubbing surfaces still come in contact to some extent, owing to the film of lubricant being squeezed out at some point of the bearing. Practically, therefore, the effect of lubrication is to convert a part only of the sliding into fluid friction friction. The relative proportions of sliding and fluid friction produced may differ greatly in certain cases. When the rubbing surfaces are comparatively rough, the proportion of sliding fric-

tion is likely to be greater. With a very smooth, highly polished journal, the contrary would be the case. The amount and the quality of the lubricant composing the separating film evidently influence the result greatly. It is known that, in any given case, the frictional effect produced is influenced by the thickness of the film and by the "viscosity" of the lubricant. (The term viscosity expresses a physical property or quality which is opposite to or reciprocal of "fluidity," and it designates the relative "consistency" or "body" of a liquid.) A more "viscous" lubricant, like a thick oil or a grease, will produce, between the rubbing surfaces, a thicker and less yielding film than can be produced with a "thin" oil. But while the *sliding* friction may thereby be decreased, the *fluid* friction is increased, because the protruding particles encounter more resistance in "paddling" a viscous liquid than a non-viscous one, the difference becoming more marked as the "rubbing velocity" is increased, for the reason that fluid friction increases as the square of the rubbing velocity. Viscosity is not, then, of itself, a quality which is primarily desirable in any lubricant, excepting in the cases where the "rubbing velocity" is very low. It is found, as a matter of practical experience, that the heavier lubricants, such as the greases or the more viscous oils, are wholly unsuited for lubricating journals which are run at high speeds. The higher the rubbing velocity the less viscous or the more "fluid" the lubricant should be. The viscosity of all lubricants is greatly influenced by temperature, however, and the temperature at which the bearing is allowed to run normally is, therefore, of importance. Indeed, in the case of a lubricated journal, temperature affects the frictional resistance produced, mainly, if not wholly, by its modifying influence on the lubricant.

The extent to which lubrication is effective in reducing journal friction, in any given case, depends not only upon the quality and fitness of the lubricant, but also, as experience has abundantly proven, upon the mode of applying the lubricant to the journal. This was pointed out many years ago by Prof. R. H. Thurston, whose own investigations and whose writings have made him a leading authority on friction and on lubrica-

tion. According to this authority, the coefficient of friction obtained with a flooded journal may be as low as one-tenth of the value obtained with the old oil cup and wick method of lubrication. Later investigators have found similar results, and it is generally admitted that the variation in values of the coefficient of friction obtained with the same lubricant, applied in different ways, may amount to several hundred per cent.

In the ordinary car journal-box, the oil-soaked cotton waste touching the lower surface of the journal forms an oil pad, from which the journal wipes off the oil as it revolves. The lubricant generally used is some grade of mineral oil. The lubrication obtained is not nearly as effective as it would be, and the coefficient of friction is undoubtedly higher than it would be, if the journal could be flooded instead of being merely smeared with the lubricant. Owing to the fact that the journal-boxes are never absolutely dust-proof, the lubricant is apt to become contaminated with dust particles which materially diminish the lubricating power of the oil.

There is reason to believe that the up and down motion of the car body, resulting from the action of the car springs, facilitates, to some extent, the lubrication of the car journals. This up and down motion causes alternations in the pressure acting upon the journal bearings, the effect of which may be to change the thickness of the lubricating film. Whatever may be the real explanation, it is accepted as a fact that the lubricant is more effective, and that the limiting bearing pressure allowable, or the total weight per axle, is much higher than would be the case if car springs were not used.

The peculiar fact has been noted that the journal friction is usually found greater in cars which have been standing for a certain time. The accepted theory is that the lubricating film between the bearing and the journal becomes more or less squeezed out after the car has been at rest a certain time. In a recent paper presented by Mr. A. C. Dennis, before the American Society of Civil Engineers, it is stated that the tractive effort required, as measured by dynamometer, was found to be about 2 lbs. greater per ton when the train has stood for some

time; and it is stated that this effect persists until the train has been run long enough to allow the journals to again become well lubricated and warmed.

(c) Since the energy expended in overcoming the various frictional resistances of a journal revolving in a bearing is always converted into heat, it follows that the temperature of the journal, of the bearing, and of the lubricant, will tend to rise. The rise of temperature of a bearing is, indeed, an indication of the fact that the rate of generation of heat in the

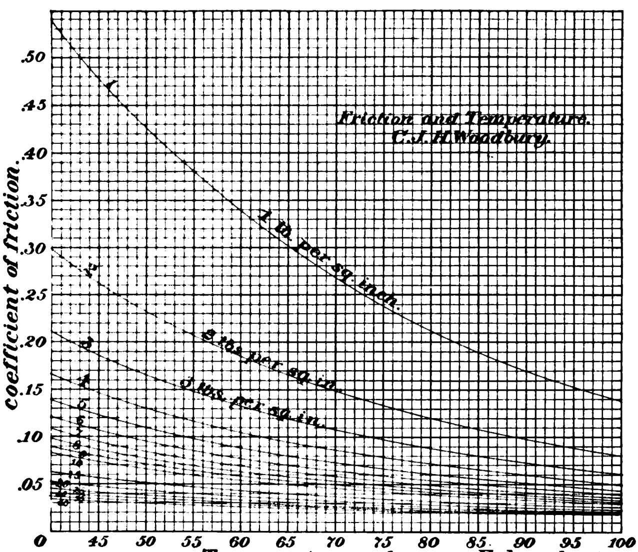

Fig. 1.

bearing, by sliding friction or fluid friction, is in excess of the rate of dissipation of heat from the journal and bearing by conduction, convection or radiation.

It is a popular but erroneous belief that rise of temperature is objectionable in a bearing. The truth is that a certain rise of temperature may be actually desirable as the means of causing the frictional resistance in the bearing to be materially reduced. Railroad practice furnishes a striking demonstration of this truth in the well established and recognized fact that the

maximum freight load of a steam locomotive is from 10 to 15 per cent less in winter than in summer, the traction force required to pull a freight train being 40 to 50 per cent more, or from 1.5 to 2 lbs. per ton greater in winter than in summer.

The influence of temperature on lubricated sliding friction is now well understood, owing largely to the important investiga-

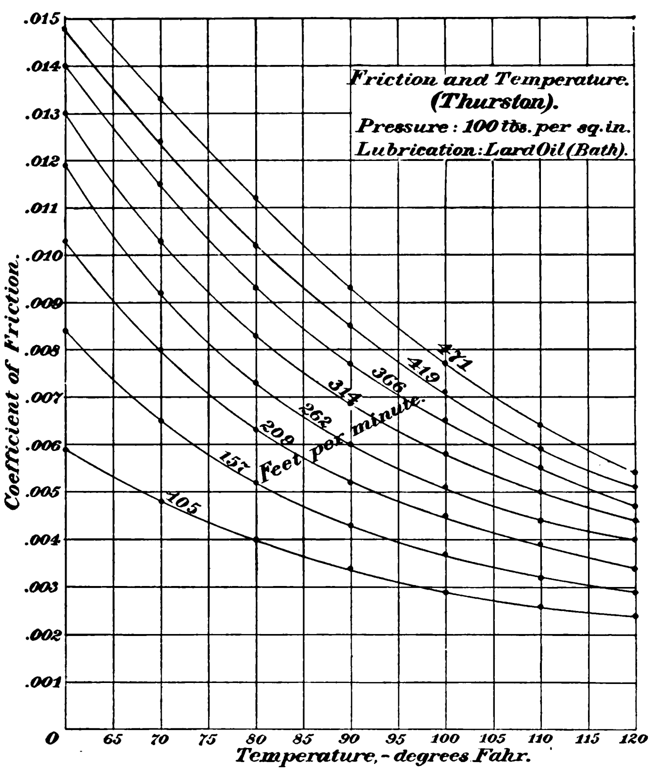

Fig. 2.

tions of Mr. C. J. H. Woodbury and of Prof. R. H. Thurston. Mr. Woodbury's results may be found in two papers on "Measurements of the Friction of Lubricating Oils," read before the American Society of Mechanical Engineers, the first one in 1880, the second one in 1884. Prof. Thurston's results are given in his work on "Friction and Lost Work."

The curves in Fig. 1 show graphically some of the results obtained by Mr. Woodbury, as given in his second paper. The lubricant used was paraffine oil and the velocity of rubbing was 300 ft. per minute, for all the curves. Each curve indicates the values of the coefficient of friction for different temperatures, between 40° F. and 100° F., when the pressure on the bearing remains constant. The highest curve corresponds to a pressure of 1 lb. per square inch, and the lowest curve corresponds to a pressure of 40 lbs. per square inch. For light bearing pressures (as the upper curves show), the effect of any given increase in temperature in reducing the coefficient of friction is very marked. The same difference of temperature produces less and less effect as the bearing pressure is increased; but the effect is still perceptible, even with the higher pressures.

The curves in Fig. 2 show some of the results obtained by Prof. Thurston, with lard oil as the lubricant, and with a constant bearing pressure of 100 lbs. per square inch. The velocity of rubbing was varied in these tests, the highest curve being that which corresponds to a rubbing velocity of 471 ft. per minute, and the lowest curve being that corresponding to a rubbing velocity of 105 ft. per minute.

It is well known that heat decreases the viscosity and increases the fluidity of all lubricants. Prof. Thurston has shown that the decrease in viscosity follows a law which is closely analogous to that of the decrease in the coefficient of friction as a function of the temperature, and that, consequently, the curves representing the relation between the viscosity and the temperature have substantially the same form as those (Figs. 1 and 2) exhibiting the relation between the coefficient of friction and the temperature. This circumstance warrants, to some extent, the presumption that a rise of temperature occasions a decrease in the frictional resistance precisely because it decreases the viscosity of the lubricant. It would not necessarily follow that the viscosity could be decreased indefinitely with benefit. It is found, practically, that there is a critical temperature at which the coefficient of friction is a minimum. The curve in Fig. 3, which was plotted from

Table III of Mr. Woodbury's second paper, shows a minimum value of the coefficient of friction corresponding to a temperature of 82° F., the lubricant being paraffine oil, the bearing pressure being 33 lbs. per square inch, and the velocity of rubbing being 296 ft. per minute. Mr. Woodbury's view is that beyond this temperature the oil became so fluid that the pressure reduced the thickness of the film of oil until certain portions of the rubbing surfaces met in actual contact. As the temperature rises above the critical point, the viscosity of the

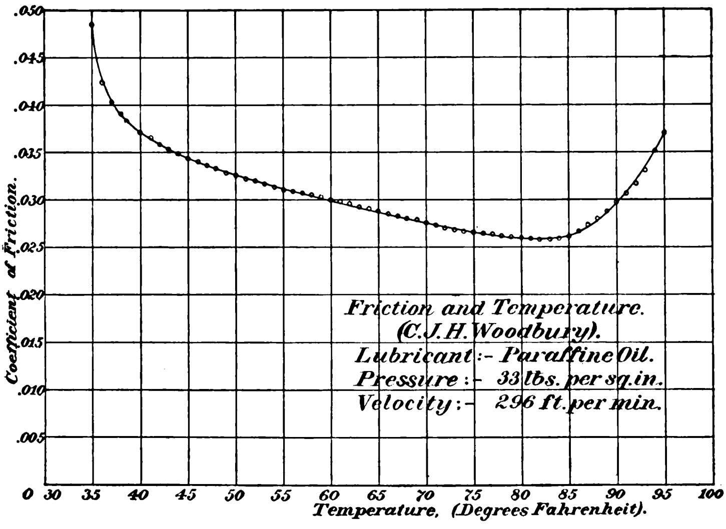

Fig. 3.

lubricant further decreases; the thickness of the film of lubricant interposed between the rubbing surfaces also decreases, and a greater number of particles of the two rubbing surfaces come into actual contact; hence the amount of sliding friction is increased, while the amount of fluid friction is decreased. The right-hand end of the diagram is, therefore, to quote Mr. Woodbury's language, "a graphical representation of the beginning of a hot bearing."

It might be expected that this critical temperature would vary for the same lubricant inversely as the mean pressure on

the bearings. The heavier the pressure the greater must be the minimum viscosity, in order that the film may not be squeezed out, and, consequently, the lower should be the temperature corresponding to minimum friction. It is not unlikely that there is some such relation between critical temperature and mean pressure. Experimental evidence is still wanting on this point.

The temperature at which chemical changes begin to occur in the lubricant is of interest in this connection. The question

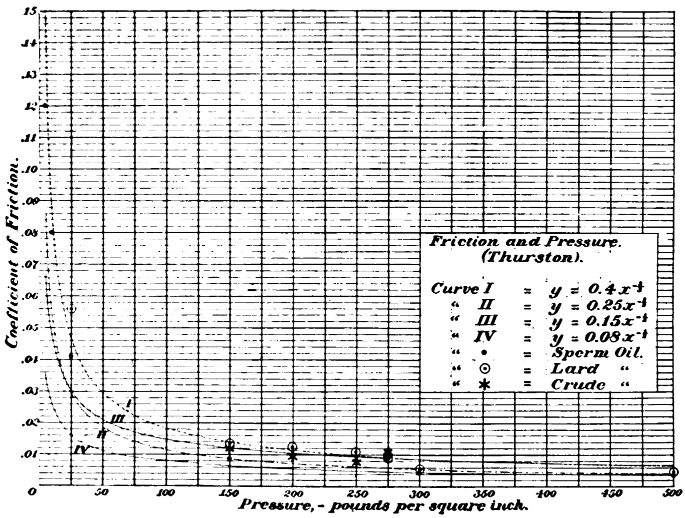

Fig. 5.

suggests itself, whether the increase of friction observed when the critical temperature is exceeded may not be due to chemical changes in the lubricant. The temperature corresponding to minimum friction is generally under, and probably seldom over, 100° F. Some of the greases and certain oils used as lubricants may undergo chemical changes at this temperature; but it is probable that this temperature is far below the temperature at which permanent chemical changes take place in the greater number of oils used as lubricants.

(*d*) The fact that journal friction is affected by the pressure

on the bearings is demonstrated by the difference found in the power required to haul freight-cars when empty and when loaded. This difference is proved to exist by numerous tests Mr. J. A. F. Aspinall, in his comprehensive paper on "Train Resistance" (Proceedings Institution of Civil Engineers, 1901), reports the following results of dynamometer tests with English freight-cars: The train resistance of a train of unloaded wagons 1,045 ft. long was found to be 18.33 lbs. per ton at a speed of 26 miles per hour, while the same train, with the wagons

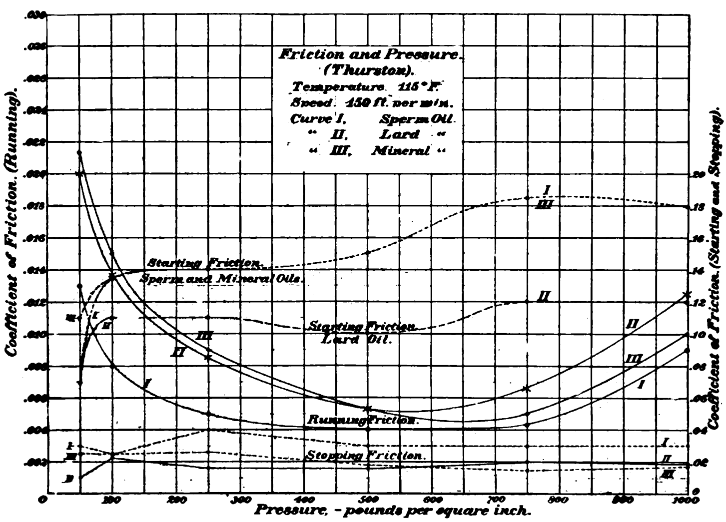

Fig. 6.

loaded, running at the same speed, had a resistance of only 6.21 lbs. per ton. From this it would seem that the work done in hauling a train of empty cars may be almost as great as that of hauling a train of loaded cars. With American freight trains the results for both loaded cars and empty cars are different, as will be seen from Fig. 4, which is reproduced, by permission, from the interesting paper of Mr. A. C. Dennis, on "Virtual Grades for Freight Trains," read before the American Society of Civil Engineers, December, 1902. These curves are based upon data obtained by Mr. Dennis from dynamometer-car tests

aggregating 3,000 miles of run, and they may, therefore, be considered representative of American practice and results. For trains of 2,000 tons or more, having a tare of about one-third of the gross weight, and running on a solid frozen road-bed, the resistance, according to Mr. Dennis (Fig. 4, middle curve), is practically constant at about 4.7 lbs. per ton, for speeds above 7 and under 35 miles per hour. For empty cars running at the same speed, the resistance (as shown by the upper curve in Fig. 4) is from 80 to 90 per cent greater. According to Mr. Dennis, warm weather and a less solid road-bed modify these results but slightly, the resistance values being relatively lower at low speeds and relatively higher at high speeds. According to Mr. A. M. Wellington ("Economic Theory of the Location of Railways," 6th edition, 1902, p. 506), the average results in American practice, representing the effects of load and weather on the resistance of freight-trains, are as follows:

	Train Resistance. (lbs. per ton.)	
Season of Year.	Loaded Cars.	Empty Cars.
Summer	4 lbs.	6 lbs.
Winter	5.5 to 6 lbs.	8 to 9 lbs.

The investigations of Mr. Woodbury and of Prof. Thurston clearly demonstrated the fact that the coefficient of friction decreases as the pressure applied to the bearings is increased. It is readily seen from Fig. 1, for instance, that the increase of pressure from 1 to 2 lbs. makes a great difference (reduction) in the coefficient of friction. It is evident from the spaces between the various lines, representing various pressures, that less effect is produced as the pressure becomes greater, and that for the highest pressure (40 lbs.), exhibited in Fig. 1, the effect of a variation of one pound in the pressure is very small. Figs. 5 and 6 show some of the results of Prof. Thurston's investigations of the relation between the bearing pressure and the coefficient of friction. Fig. 5 shows that, for pressures under 500 lbs. per square inch, this relation can be represented with a fair degree of approximation by some form of hyperbolic curve, such as the curves (I to IV) for which formulæ are given on the diagram. Fig. 6 shows

the influence of bearing pressures ranging from 0 to 1000 lbs. per square inch, not only on the "running" friction, but also on the "starting" and the "stopping" friction. The friction coefficient values for "running friction" are to be read from the scale of ordinates given at the left-hand end of the diagram; the values for starting and stopping friction are to be read from the scale

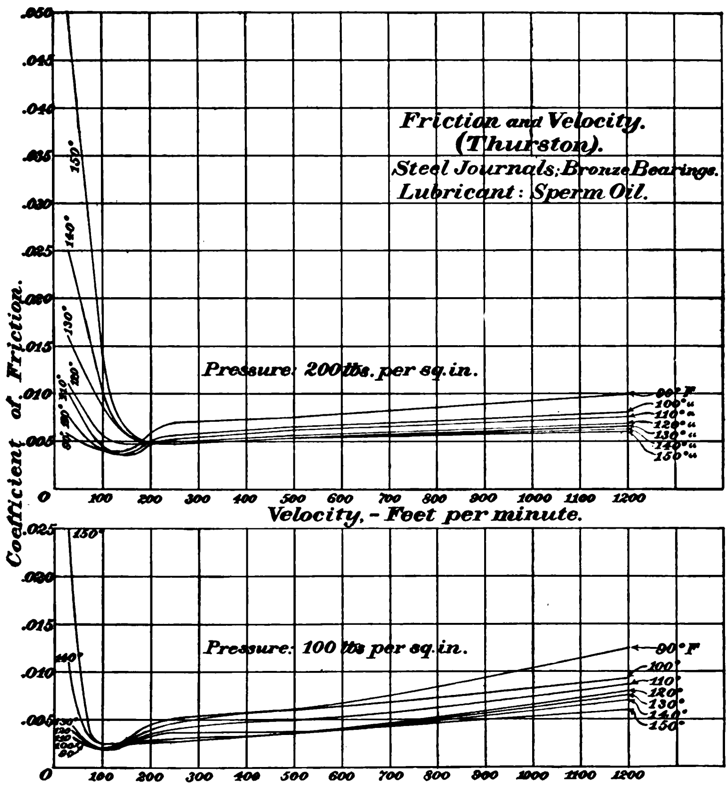

Fig. 7.

of ordinates given at the right-hand end of the diagram (this scale, it should be carefully noted, being ten times greater). These curves show that the coefficient of friction does not decrease indefinitely as the pressure increases, as Fig. 5 would indicate, but that there is a critical pressure, between 500 and 700 lbs. per square inch, at which the coefficient of friction is a minimum,

and beyond which it increases more and more rapidly as the pressure increases further. Evidently, at this critical pressure, the lubricant is being compressed so strongly that the lubricating film threatens or begins to give way at certain points, and to allow the two rubbing surfaces to come in contact and scrape against each other. The further increase of pressure causes the film to yield more and more until it has been wholly squeezed out. When this point has been reached, there is no more lubricating action, and we have true sliding friction. This pressure point is doubtless not far removed from the point at which "seizing" and "cutting" occur, either on the journal or bearing, or on both of them.

The curves of "starting" friction in Fig. 6 show that this friction increases, instead of decreasing, as the pressure becomes greater. The increase is very rapid at the lower pressures, or between 0 and 100 lbs. per square inch. It remains nearly constant for pressures ranging between 100 and 400 lbs. per square inch. It again rises at pressures ranging between 500 lbs. and 700 lbs. per square inch, attaining a maximum at 750 lbs. to 800 lbs. per square inch.

The "stopping" friction, or the friction obtained at the instant of stopping, remains very nearly constant at all pressures, as indicated by the curves at the lower portion of Fig. 6.

(To be continued.)

ARCHITECTURAL LETTERS.

WALTER DANA SWAN,

Instructor in Architecture.

PLATE DRAWN BY A. E. HOYLE, '04.

IT is proposed to publish from time to time in this magazine plates which may be of service to those who are beginning the study of architectural form. The publication of these plates will follow, in a measure, the sequence which would be adopted in a scheme of instruction in elementary architectural drawing, but this can of course be but a mere outline.

As a transitional step from the general delineation of school days to the more specific and technical knowledge of the architectural student, there are few things more appropriate or interesting to draw or study than a good Roman alphabet, with such modifications and additions as have come down to us. The letters themselves involve the questions of proportion of mass, of detail and, above all, of observation. Lowell says that the "divine faculty is to see what everybody can look at," and how many of us who have read well printed books and papers for many years, and some of the very good advertisements recently published, have failed to notice which lines of the A, N, M, and W are wide and which are narrow, and perhaps have not remembered it because we did not know why it was so.

Until he knows thoroughly the best forms and their significance, the draughtsman has not that standard of taste which enables him to experiment with and adapt and design the letters themselves, which, in addition to the grouping, spacing and composing of his words and sentences, make him the master of the art of lettering. To be sure, there is not a great scope for originality, but each prominent designer in the country expresses (as he does in his handwriting) his own individuality by his lettering, based as this is on such a standard model as is published here. In the valuable book recently written by Mr. F. C.

Brown, "Letters and Lettering," the whole matter is treated exhaustively, and students will find this work of the greatest help and inspiration.

The improved standards of public taste, and the excellence required in competitive drawings, have without doubt greatly influenced the quality of draughtsmanship in all architectural work, including this matter of lettering. In its monumental use, of which we have good examples in the University on Robinson and Austin Halls, and on many of the memorial gateways, it differs somewhat from its use on office drawings, but in both aspects there is more interest taken in it now than there has been since it became a matter of choice and not of tradition. In the case of office drawings, the clients and the workmen must be able to read the titles and directions with ease and — it is not too much to say with regard to the clients — with pleasure. These must be forms appropriate to the pencil, the pen or the brush, and the words must be kept in the proper relation for the clearest reading. In monumental lettering, inscriptions and the like, rules of composition are somewhat relaxed, punctuation becomes less imperative, and a dignified inscription is the only real essential. "The utmost the reader has a right to ask is that there shall be no mistake about it when he comes to study it," which rightly implies that an inscription should be studied rather than glanced at. But architectural dignity is best preserved by the adoption of the simplest character, and by distributing the lettering in the severest and most formal way. Here also the forms of the letters must be appropriate to the material in which they are made, and that point brings us again to the consideration of this particular plate of alphabets, which represent the best standard forms as they are used today. If the first alphabet were cut in stone, the narrower portions of the letter would be made a trifle wider than if the same letters were to be drawn with pen or brush. The history of the alphabet, and in fact of all lettering, is given in a very thorough way in a book by Mr. Lewis F. Day, "Alphabets Old and New," as it is also in Mr. Brown's book above mentioned; but it will suffice to say here that the narrow and wide portions

A PLATE OF LETTERS

ABCDEFGHIJKL
MNOPQRSTUV
WXYZ

a b c d e f g h ı ȷ k l m n o p q r

s t u v w x y z 1 2 3 4 5 6 7 8 9 0

a b c d e f g h i j k l m n o p q r

s t u v w x y z 1 2 3 4 5 6 7 8 9 0

A B C D E F G H I J K L M N O P
Q R S T U V W X Y Z

A B C D E F G H I J K L M N O P
Q R S T U V W X Y Z

Drawn by A. E. Hoyle, 1904

of the letter resulted from the influence of the upward and downward stroke of the pen in the early days of lettering, before the use of the chisel to express the same forms in stone. It will be seen that the natural up-stroke is narrow while the down-stroke is wide, and Mr. Brown has formulated rules covering these points and some others with reference to the Roman alphabet. The third line between the outer ones is of course the result of the chisel-cut section of the letter. The small letters on this plate are not derived historically to the extent that the Roman capitals are, but are in reality adapted from the movable type of the printers of our time, which originated in the handwriting, probably, of the Gothic period. It is to be observed that the sloping letters have more the angular appearance of handwriting then the upright ones, so that it is appropriate to use them in a more informal way, as in preliminary studies at a small scale, or in hasty sketches, as the hand moves more rapidly at such an angle. The two lower alphabets are simply modifications of the first.

Like many other historical forms, those of the Roman letters have been analyzed and formulated, and geometrical diagrams for their construction worked out, by masters of the Renaissance in Italy and by Albrecht Dürer in Germany. Mr. Brown in his book shows modern applications of these diagrams to both such capitals and small letters as are shown on this plate, but it is the form itself which is vital and not the formula, and by means of observation and practice the beginner can best acquire proficiency, working freehand as soon as possible, but in architectural lettering depending always on T square or triangle for boundary lines at top and bottom. Consistency of line and tone are of course also necessary. Although this plate itself is composed with a distinct and well-felt sense of design, that most important question of composition will not be considered in this brief article. Methods of composition and many fine examples, which are perhaps more helpful than precepts, are shown in a second book by Mr. Lewis F. Day, called "Lettering in Ornament." It is of course the putting together of letters and words in proper and even beautiful relations of space and mass that constitute the

art of lettering well, but cultivated taste with regard to the elementary forms of the letters themselves is the first requirement, and it is hoped that this plate may be of service in providing a model of simplicity and refinement.

A SINGLE-PHASE LIGHTING SYSTEM IN ST. LOUIS.

L. A. DE BLOIS, S. B., '99.

THE Missouri Edison Electric Co. of St. Louis, Mo. (now a branch of the Union Electric Light and Power Co.) is unique among large lighting plants of this country in its use of a single-phase alternating system with distribution for light and power from two- and three-wire alternating current low-tension networks — a system radically different from the three-phase rotary three-wire direct-current system now accepted as the standard for heavy lighting work. Many details of the plant, such as the use of single-phase motors, the arrangement of the underground system, the operation of the alternators (not in parallel, but on a common and grounded bus), are departures from the usual engineering practice. The maintenance and extension of a system so far different from the accepted standards requires unusual engineering treatment.

An examination into the lighting situation in St. Louis, both past and present, shows clearly that this system is not the result of free growth, but rather of growth deformed and retarded by local conditions. Contrary to what might be expected by a casual observer, competition carried to extremes often results in exhaustion rather than stimulation — in enforced economy of expenditure to offset excessive reduction in rates. Add to this cause another, due to the consolidation of several totally different systems, and we have the combination of conditions which has directly determined the growth of this St. Louis company. In fact, there is good reason to believe that under less compelling conditions the system would have undergone considerable alteration or, perhaps, have been replaced by something less special in type.

There are now two stations, formerly belonging to independent companies. "Station A," the Westinghouse station, is situated on the outer edge of the underground district, at some distance from both railroad and water-front. It had been equipped for

quarter-phase distribution at 1,100 volts, the intention of the designing engineers being to run motor and lighting circuits on separate phases, a plan, however, that was never carried out. "Station B," the General Electric station, on the other hand, had been built directly at the railroad tracks, a mile from Station A. It had been equipped for city arc lighting, and for years had operated belt-driven arc machines. These were later replaced by some of the present alternators to supply what was the first series alternating arc system in the country. This equipment was for 1,200 volts, 9 per cent higher than the other station, but was monocyclic. Both stations had the same frequency, however, — 60 cycles — though Station A was still operating some of its old equipment of 133 cycle belted machines. The resulting combination was an 1,150 volt, 60-cycle, single-phase system supplied from either or both stations, one mainly for the underground district and part of the overhead, the other mainly for arc lighting. When the contract for the municipal lighting was lost, this latter station took on the rapidly increasing overhead load. The stations were operated in conjunction through a heavy "tie-line," an arrangement not entirely satisfactory from an economical point of view, since both, besides being badly situated for coal supply, were at some distance from their respective centers of load — a location that time and growth did not improve.

Station A, as we noted, was equipped for 1,100 volts, quarter-phase. There are three 800 kw. Westinghouse alternators, direct connected to high speed, three-cylinder compound vertical engines, built by the Lake Erie Engine Co., and one 300 kw. Westinghouse alternator, direct connected to a Westinghouse compound engine. These four alternators are run single-phase, only one phase of the winding being in use and that supplying full load — a maximum of 1,200 kw. on the 800 kw. machines during the peak of the load. The exciter equipment consists of three 125-volt, 100 kw. engine-driven generators, one of which is sufficient to supply excitation for all the alternators.

At Station B there were two 800 kw., 1,200-volt, monocyclic General Electric machines of the stationary-field type, direct

connected, one to a simple and the other to a cross-compound Hamilton-Corliss engine. To these was added a 1,500 kw. revolving field alternator, also General Electric, and driven by a Hamilton-Corliss cross-compound. This is classed as an "A.M. B. 80-1500-90, 1,200 volts," with an inherent regulation of 44 per cent and full load efficiency of 94 per cent. All of these alternators are monocyclic, a system originally devised to ensure greater steadiness in parallel running and to enable the manufacturer to circumvent the patents of other companies on single-phase alternators of this type. In addition to the three 75 kw. engine-driven exciters for the above machines there are also one 500 kw. and two 250 kw. 500-volt direct-current generators; the former, a General Electric multipolar machine, direct connected to a simple Hamilton-Corliss engine, and the latter two belted "relics." These three 500-volt generators feed a two-wire, 500-volt power system, supplying motor customers that have not as yet been taken on to the alternating system, and are of no particular interest to us.

To sum up: We have, then, at "A" four quarter-phase alternators run single-phase, and at "B" three monocyclic alternators run single-phase. At A, lack of fly-wheel capacity (a fault irremediable with this type of engine) and high speed (180 r. p. m.) rendered parallel running extremely unreliable and, therefore, out of the question for commercial service; while at B the changes of angular velocity of the simple engine as compared with the two compounds rendered it, though possible, at best unsatisfactory, It is true that the parallel operation of the two compounds would have been possible, but the advantages gained would have been slight; it was, in fact, a case of all in parallel or none in parallel. These considerations, then, decided the case in favor of independent operation, but other points in its favor may be noted.

Where competition is keen, an even greater necessity than close regulation is absolute continuity and reliability of service. To an untrained eye a sudden variation of 2 per cent in pressure on incandescent lamps is rarely noticed; 3 or 4 per cent variation becomes annoying only when it takes place as a continued

flicker, although the effect on the life of the lamps is disastrous. An absolute cessation of light, however, even for so short a time as five seconds, cannot escape detection. The customer growls, and, if the interruption is often repeated, the growls take form in a written protest. This may be followed by orders for disconnection. The matter is more than annoying. To permit the lights to go out in crowded theaters or exhibition halls is absolutely dangerous. Herein, perhaps, lies the greatest worth of the storage battery in direct-current service — an insurance against interruption of service. Absolutely invariable continuity in any system is, of course, impossible, for accidents will happen. Engineers acknowledge the possibility of this when they provide excess generator capacity, install automatic or reserve switches and breakers, "sectionalize" bus-bars, run feeders in separate groups, etc., all arrangements to meet these accidents and render their results less serious.

It is here that we see the greater advantage of the separate circuit system as compared with the great interconnected network fed by numerous feeders. In the first, a serious catastrophe results in disturbance to only one comparatively small section; in the latter it may involve a whole system in trouble. On the other hand, in the first, the loss of a feeder is fatal; in the second the system may not suffer at all from such loss. As with the feeders so with the generators: it is a question of independence versus interdependence. To gain the advantages of parallel operation, the sudden loss of a generator must not momentarily cripple the service; that is, the generators must not be loaded to their full capacity. This matter, of course, has no bearing on accidents to the main bus-bars themselves, or to the exciter plant, which would cause a complete shut-down, contingencies wherein the non-paralleling station, each unit with its own bus and exciter, has an evident advantage. The question of the storage battery also enters here; but, in general, given a certain generator equipment, it is possible to run it as fully loaded, with the same reliability of operation, on independent bus-bars as in parallel, and, moreover, to enjoy an immunity from certain accidents brought about by the policy of putting

"all the eggs in one basket." Also, when imperative to shut down an engine while under load, a fresh machine can be put to work in an appreciably shorter time if it is not necessary to synchronize it. Another point bearing on the matter is the question of complexity in switches and bus-bars, a point not so greatly in favor of parallel running as would appear at first, when we consider the modern arrangements of emergency

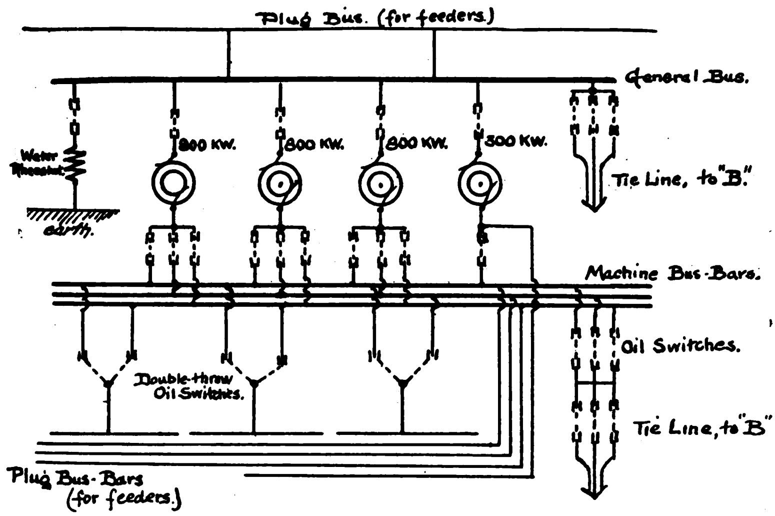

Diagram of Bus-Bars, "Station A" (simplified.)

bus-bars and sectionalized bus-bars. Again, independent operation admits of "high" and "low" busses without the medium of boosters or potential regulators — a manifest advantage.

In each of the two St. Louis stations there is a so-called "general bus" to which one leg of each alternator is connected through switches. These two general bus-bars are connected by the tie-line, forming, in fact, one continuous bus. In addition to this, there are a number of "machine" bus-bars to which the other legs of the alternators are connected individually; in other words, each alternator has one bus exclusively to itself

and one in common with all the alternators. From the machine busses are fed, in some cases through double-throw oil switches, the feeder or "plug" bus-bars, from which the feeders themselves are supplied. The generator switches are simple knife switches and are never opened under load, since all switching is done on either the feeder transfer switches, the tie-line switches, or the double-throw oil switches just mentioned. No circuit-breakers or automatic switches are installed, neither are there any fuses but those on the feeders, the switchboard attendants being relied upon in all circumstances to avert serious trouble by opening the field switches of the alternators — "killing" them — and immediately opening the machine switches. Of course, it is rarely necessary to do this suddenly, but the alternators are built to withstand the breaking of full field current without risk of puncturing the insulation. At B the arrangement of the switchboard is similar, all transfer of load from bus to bus being done on the feeder transfer switches. From the diagram it will be seen that the tie-line at A has the same arrangement of switches as the alternators. These, however, are hand-operated oil switches of the type called "remote control," placed in compartments below the switchboard, and are regularly used in switching full load.

In operation, the stations are run in conjunction; that is, the tie-line is operated in either direction, according to the amount and distribution of the load, and for a part of the night one station is entirely shut down. With the tie-line in reserve during the peak of the load, the reserve capacity in alternators is reducible to one-half, since one alternator can act as reserve for either station. The tie-line itself consists of three 1,000,000 cm. concentric cables in multiple. Although two cables would be ample to carry the maximum output of any machine now installed, the importance of this service warrants the use of three. Strictly speaking, no one of these is held as reserve, although at any time they can be thrown out of service separately and tested. In fact, it has been the Company's experience that reserve capacity in feeder or tie-line cable is most efficient as reserve when kept in continuous service.

The underground district of St. Louis covers perhaps one-fifteenth of the city, but represents about one-half of the total load. In it the Missouri Edison Electric Co. has a heavy 115–230 volt three-wire alternating network of cable and Edison tube, from which the customers are supplied. This is divided up into five sections, entirely separate from one another save that the neutral wires are solidly connected throughout. Each block length of cable or tube is fused in the manholes at the street intersections, and in event of a heavy short circuit will blow itself clear of the rest of the system. In these manholes are placed the transformers that step down the 1,150 volt primary current to 115–230 volts, three-wire. They are 110 kw., oil-cooled, a special type with poor regulation (4 per cent) and low core loss, — the advantages of which will be seen later, — and are fused on the secondary side only. The primary feeders, one to each transformer, are uniformly 150,000 cm. duplex, "twin-conductor," lead-covered cable, and vary in length from a few hundred to seven thousand feet. At the switchboard they pass through a polarity-changing switch to an ammeter, a hand-regulated booster, a non-arcing fuse, and a transfer switch and plug-panel, the latter for transferring the circuit from bus to bus — on one side only, the other remaining on the "general bus." The boosters are of the transformer type with secondary, having a number of steps, in series with the feeder. They are designed to raise or lower the voltage 18 per cent in steps of approximately 1 per cent, and are provided with a reversing switch.

Such is the general construction of the underground system — a number of distinct, secondary networks, each fed by several transformers, each of which is on a separate feeder and booster. It is evident that boosting up the voltage on any particular feeder will raise the terminal voltage on the transformer, so that it will take more load than its neighbors. In this way, having a secondary system of mains with comparatively low impedance, it is a simple matter to distribute the load evenly by means of the boosters, and with very little variation of pressure on the secondaries. The loss of a feeder, too, is of comparatively little

consequence, since its load may be easily divided among its neighbors. Moreover here lies the advantage of poor inherent regulation in the transformers, in their consequent tendency to shirk an undue share of any concentrated load. In other words, the terminal voltage of the overloaded transformer nearest the unusual demand drops slightly and the load is thrown more on the other feeders, thus helping to accomplish automatically what the boosters are made to do to a greater degree by hand regulation.

The pressure is controlled as follows: From the secondary terminals of each transformer pressure wires in separate cables (or, according to the latest plans, incorporated in the feeder cables) are brought back to the voltmeters at the station. A feeder whose load corresponds most nearly to the average conditions of all the feeders in that particular district is selected as "standard." Its booster is kept at the point of no boost and its voltmeter maintained on the mark by regulating the alternator or exciter field rheostat. Then the other feeder boosters are manipulated, some up, some down, to obtain a uniform secondary pressure and an even distribution of load. In fact, after the boosters are once set, they require very little attention, the work of maintaining the pressure falling almost entirely on the field rheostats. The result is a closeness of regulation and reliability of service not surpassed by the direct-current, storage-battery system or by any of the automatic regulators. In efficiency, also, the system ranks very high, since the poor regulation of the transformers permits them to be built with especially low core losses, a condition that results in unusually high light-load and all-day efficiency. The power factor, also, is high, since the main reactance — that of the transformers — is partially balanced by the capacity effect of the cables.

This, then, is the arrangement of each division of the underground district, the number of feeders per division varying from four to twenty, depending on the amount of load. The number is, of course, somewhat arbitrary, especially since the districts themselves are changed from time to time. This change is a matter merely of removing and replacing fuses; but there is the

restriction that, since the alternators are not in parallel, all feeders of a district must be supplied from the same alternator. For this same reason it is necessary in operating the system to switch an entire division at a time and not feeder by feeder. This is the function of the double-throw switches between the machine and the plug bus-bars—switches designed to throw the load so quickly that the switching is almost imperceptible on the lights.

These underground divisions make up the bulk of the load at A, but there are, in addition, three overhead circuits feeding the overhead district. These are similar to the feeders at B, where the load is entirely overhead. The switchboard equipment of these circuits is similar to that of the underground, but the circuits themselves, unlike the underground circuits, are independent. They are of 125 kw. capacity. The feeders, varying in length from a few hundred feet to four miles, run overhead on pole-lines, along several different routes, to their points of feed into the primary mains, and from these points return-pressure wires are brought back to the voltmeters at the station. In every case, the massing of feeders to one district, either overhead or underground, is carefully avoided — a precaution to guard against the possible crippling of an entire district by a "burn-out" or an accident to a pole-line. The primary mains take, as near as possible, the form of a heavy cross-main, feeding the centers of lighter mains, the feeder itself supplying current at the load-center of the cross-main. The distribution from these primary mains varies with the amount and nature of the load. It begins, where customers are few, with scattered transformers of the regular outdoor, oil-filled, three-wire secondary type. These are replaced by larger sizes as the load grows heavier, or more are added as the customers become more numerous. In turn, these give way to the three-wire secondary circuits of heavy mains, one or two blocks in length, fed by from two to five large transformers, placed some little distance apart — a thoroughly economical and satisfactory form of distribution. In one part of the city, an experimental section of fifteen of these block lengths, interconnected throughout by cross-mains, much

as in the underground district, has been in satisfactory operation for over a year. This is a more complicated arrangement, but allows a still further reduction in transformer capacity.

An interesting comparison of the relative transformer economies of the underground, three-wire secondary overhead and individual-transformer overhead forms of distribution is given by the ratio of transformer capacity installed to total connected load. It is:

(*a*)	Underground	.816
(*b*)	Three-wire secondary overhead	.406
(*c*)	Individual-transformer overhead	.871

Since the demand for light in the business portion of the city (underground) is so much greater than in the residential portion (overhead), the figure (*a*) can hardly be compared with (*b*) or (*c*); but (*b*) and (*c*), because they cover practically the same conditions of demand, offer an opportunity for transformer comparison. From them we see that the three-wire secondary method affords a reduction of 50 per cent in transformer capacity over the individual transformer method. The figures vary somewhat, but the minimum demand for current falls as low as 15 per cent of the total connected load.

The use of single-phase motors on this system has already been noted. They are of the Wagner automatic self-starting type, 208 volts, and range from ½ to 35 horse-power. In sizes of 5 horse-power and over they are started on 115 volts (to reduce starting current) and then, by means of a double-throw switch, thrown over on to the 230 volt connections. This, of course, requires a three-wire service, which is taken directly from the underground secondary mains or from separate transformers in the overhead district. A more recent device for large motors is a special oil-cooled, non-inductive starting-box placed in series with the motor on 230 volts — simply an arrangement to reduce the starting current by introducing resistance in the circuit. It is, in most cases, this starting current far more than full-load current, or intermittent loads, that tends to limit the use of large motors on lighting mains, but of course the question

is one of copper and distance, and where the mains and feeders are properly proportioned, very little voltage disturbance results. It has been found necessary on the overhead circuits, however, to place the motors on separate transformers from the lights, otherwise the excessive drop that results when the motors are started up would be very noticeable. The starting current of these motors varies from 150 to 175 per cent of full-load current, full-load power-factor from .73 in the small sizes to .88 on the large, and full-load efficiency from 61 per cent to 89 per cent — figures that compare very well with three-phase or direct-current motors. The motor load represents about one-tenth of the connected load, and of the remaining nine-tenths the constant potential enclosed arc lamps form about one-seventh. There are also a number of Nernst lamps, electric heaters and other miscellaneous devices.

The resulting power-factor at the generator terminals varies from .70 at periods of light load to .95 during the peak. This lower limit is due mainly to the low no-load power-factor of the transformers. The capacity of the underground cables tends, at all times, to improve the general situation, while the arc lamps and lightly loaded motors tend to aggravate it. In the summer, the great number of fan motors on the circuits lower the power-factor, particularly during the day. As the load increases towards the peak the power-factor increases, and it is not unusual to find an alternator almost self-regulating, the excitation remaining constant and the rheostat untouched as the kilowatt load increases.

As yet nothing has been said in regard to "grounding," a policy that this Company pursues in full. The "general bus" is connected to the ground through a water rheostat, recording ammeter and switch, and the resistance so adjusted that the bus is about 350 volts from the ground, as compared with 800 volts between the other bus-bars and the ground. With a perfectly insulated system, under normal conditions, no current but charging current would flow through this connection (part of the charging current of the condenser formed by the wires and the earth). No system of any extent, however, can be perfectly in-

sulated; a certain amount of surface leakage and leakage directly due to defective insulation is inevitable, and an interchange of current between different parts of the system by way of the earth is the result. This leakage is, of course, largely dependent on the amount of moisture in the atmosphere. On a system such as this, not grounded permanently, a ground of greater or less magnitude always exists, and "floats" from one side of the system to the other. It is always there and always a source of danger to linemen, and yet may not be great enough to burn off an opposing ground, such as the falling of a wire to earth or the break-down of a defective transformer bushing. In other words, it is useless as well as dangerous. Ground the system permanently, however, in the way described, and the ground "floats" no longer. If a wire on the "outside," or non-general bus side, of a circuit should break and fall across the car-tracks, the full 850 volts potential would be at the contact, and, except for the presence of the water rheostat, the rush of current would instantly open the circuit fuse. This is its function — to limit the flow of current to an amount sufficient to burn off the trouble and yet not open the circuit fuse. Designed to clear the lines of outside grounds and fix the ground permanently, this device fulfils other important functions. It relieves the system of static accumulation and induced charges of lightning — a very considerable source of trouble in this region of frequent thunderstorms. Wires that become crossed with those of other grounded systems — street railways, for instance — are burned off without interruption to service. The ground connection, moreover, reduces the actual risk to linemen, etc., since they use more care in handling a grounded system than one that is only occasionally or partially grounded. Lowering, as the connection does, the potential between one side of the system and ground, it reduces the chances of trouble on that side, while actually raising the chances on the other side but very little. In fact, it has been found by experience that trouble very seldom comes to the general bus side and is very speedily removed on the other.

Another practice of the Company having a distinct bearing on the safety of the customer is the grounding of the neutrals of

the underground system and of the three-wire secondary circuits. Here a break-down of insulation between primary and secondary, which would otherwise place dangerous potential in the customer's premises, is followed by a rush of primary current to ground sufficient to open the transformer fuses. These secondary neutral grounds also remove static accumulation, indicate the presence of grounds on the customer's wiring, and cause the "blowing clear" of defective cables and Edison tubes. This policy of grounding is even carried to the exciter bus-bars and the 500-volt d. c. system.

Such is the single-phase system of the Missouri Edison Electric Co. at the present time. It is working efficiently and giving regulation fully on par with that of other systems, but it has some serious faults and disadvantages. Its future expansion in order to handle great and concentrated loads would be very problematical. In order to, literally, get at the load great changes would be necessary — higher voltage, larger transformer units and heavier feeders. These changes are rendered almost out of the question by the present station equipment, the crowded condition of the conduits and pole-lines and the size of the manholes. Besides these, the economical situation of the stations is to be considered. As it happens, however, the processes of consolidation are again going on, and, in the future, rather than extension along present lines will come the gradual disuse of this system, when it shall give way to a greater one, conforming more to modern ideas and modern engineering practice.

HARVARD ENGINEERING LABORATORY INVESTIGATIONS.

II. TEST OF A VARIABLE-SPEED POWER TRANSMITTER.

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THERE has arisen in recent years a great demand for mechanisms by means of which the speed of a machine may be varied so as to suit exactly the work which the machine is doing. The speed of a machine tool should be varied according to the metal being cut and the depth of cut; the speed of an automobile must be capable of variation, with constant speed of the motor; the speed of a printing-press has to be adapted to the kind of work being done in order to give the maximum output. In these cases it is generally satisfactory if the speed variation is by steps, that is, it is satisfactory to approximate roughly to the required speed, so that if the mechanism will give a series of, say, eight speeds within the limits of use of the machine the result will be all that is practically desirable. The variation of speed in this manner is usually obtained by stepped cone pulleys, which have the disadvantage that the machine must be stopped in order to change the speed, and which become of great length if much power is to be transmitted or if many speeds are required. These objections can be overcome, and a compact speed-changing device can be obtained, varying the speed by steps without stopping the machine, by using a nest of gears, any pair of which can be engaged while all the rest are unemployed.* A combination of gearing and stepped cone pulleys is also commonly used.

It is often necessary to have a continuous variation in speed of the driven machine, especially in cases where some material is being treated by passage through successive rolls, as in paper making, or through successive dies, as in certain wire-drawing processes. In such cases it is essential that the speeds of the

* See *American Machinist*, Aug. 21, 1902.

different parts of the apparatus should be capable of exact adjustment relative to one another, so as to take the same amount of the material through each of the processes in a given time. It is also desirable in such a process as facing in a lathe that the speed of the lathe should increase uniformly as the tool moves uniformly in towards the center.

To satisfy the conditions suggested above a large number of mechanisms have been devised. The use of a pair of cone pulleys is not uncommon, but it does not give good results, because a belt will not run well on a conical surface. A very ingenious method has been devised for using simple cone pulleys and yet having the belt run on an ordinary crowned surface,* and this is apparently successful in paper-mill service, where small total speed-variation is required.

Other belt-transmission devices use expansible pulleys,† or other arrangements which practically bring about the same result.‡

Besides the mechanisms indicated above there are many variable-speed power transmitters in which friction-wheels are used. To this class belongs the machine which has recently been presented to the Engineering Laboratory, and the tests on which are here recorded.

The machine, Fig. 1, consists of two driving disks, a, b, with deep annular grooves, mounted on a sleeve, f, which is supported by the bearings m, m, and is driven by a belt on the pulley g. This part of the mechanism runs at a constant speed, which in this particular case is about 350 revolutions per minute. The driven disks, c, d, which are opposite and similar to the disks a, b, are driven by them through four friction-rollers. These friction-rollers have their surfaces spherical and of the same radius of curvature as the annular grooves in the disks. The rollers are so mounted that the center of their spherical surface is also the center of curvature of the annular grooves at the plane of contact, so as to give contact between the rollers and grooves over the whole of the width of the surface of the roller. The four

* See *American Machinist*, July 3, 1902.
† See *Scientific American*, April 19, 1902.
‡ See *Engineering News*, March 1, 1900.

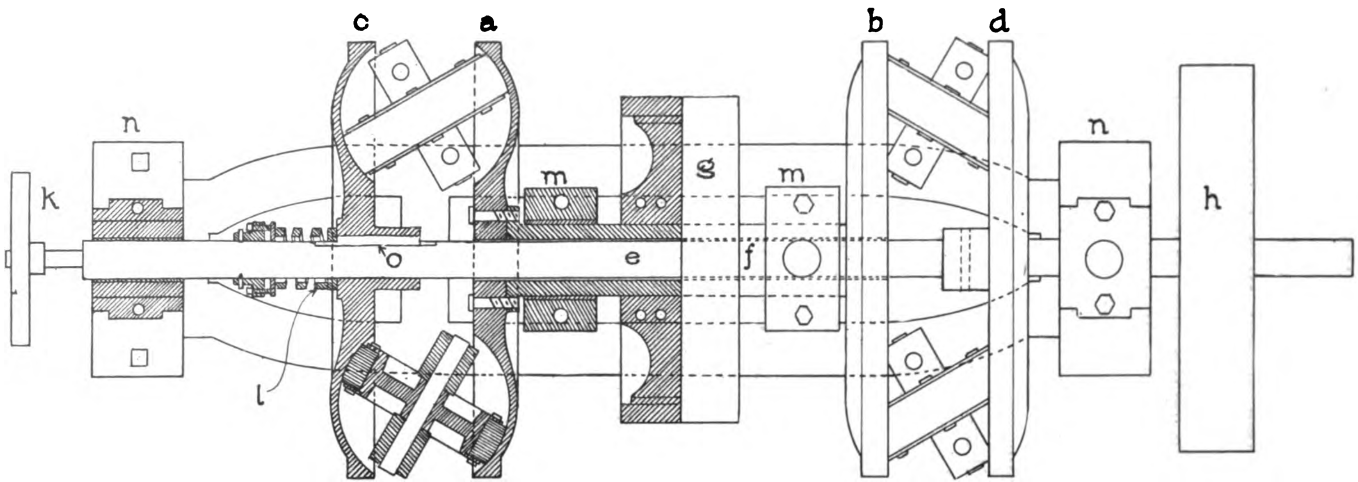


Fig. 1.

PLAN OF VARIABLE-SPEED POWER TRANSMITTER.

rollers are symmetrically placed, and are mounted in such a way that they can be swung simultaneously about a vertical axis so as to make other angles with the shaft, *e*.

In the positions shown the rollers have contact with the driving disks near the largest diameter, and touch the driven disks at a small diameter, so that the speed of the driven disks exceeds that of the driving disks. If the rollers are all swung parallel to the shaft the speed of driving and driven disks will be the same, assuming no slip. If the rollers are swung round still further, till they touch a larger diameter on the driven than on the driving disks, the speed of the driven disks will be less than that of the drivers. By varying the position of the rollers, any speed of the driven shaft, from one-half to twice the speed of the driving shaft, can be obtained.

The disk *d* is fastened to the shaft, *e*, by a taper pin; the disk *c* slides on the feather *o*. A spiral spring, *l*, has one end pressing against the disk *c* and the other end pressing against a collar fastened to the shaft, and the resulting end thrust is taken up by the pressure of the disk *d* against the friction-rolls. By this device the pressure necessary for the driving friction is obtained without any unbalanced end thrust, and consequently without any special thrust-bearing. The rollers are mounted on rocking supports, so that whatever pressures are put upon them by the disks *c*, *d* are transmitted to the opposite disks. The compression of the spring can be varied by set screws in the fixed collar. The disks are of cast iron, the rollers of compressed paper which has had its surface treated with heavy oil and resin to increase its friction properties. The driven shaft, *e*, revolves in a direction opposite to that of the driving shaft, and power is taken from the pulley *h*.

A series of investigations was made on this machine for the purpose of determining its efficiency under the full range of working conditions. The conditions varied were as follows: —

(1) The position of the rollers was varied, giving different speed-ratios (the ratio of the speed of the driven shaft, *e*, to that of the driver, *f*).

(2) The power transmitted at each speed was varied by

changing the resisting moment or the torque of the driven shaft, *e*.

(3) The compression of the spring *l* was altered to find the effect of its pressure on the efficiency and capacity of the machine.

For the purposes of the test, the machine was driven by a

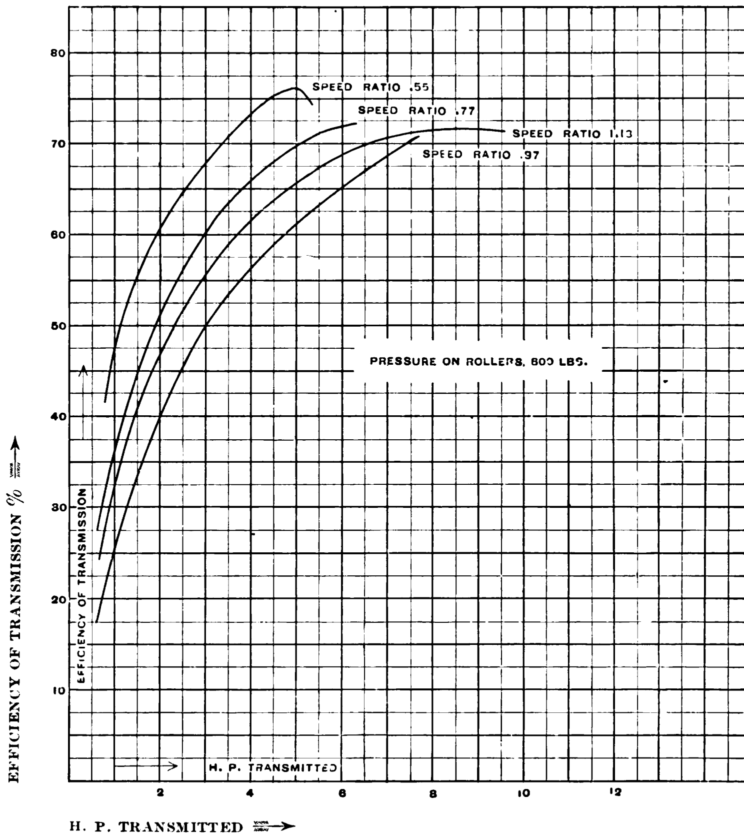


Fig. 2.

belt from an electric motor which had previously been tested, so that its efficiency was known. The power was absorbed by a rope brake on the pulley *h*.

With a known compression of the spring *l*, and a fixed position of the rollers, a series of runs was made with different powers absorbed by the rope brake, taking the observations

necessary to determine the efficiency of transmission. Then similar series were repeated, with the same compression but with other positions of the rollers. When the action of the machine had been thus investigated for a known compression, a similar series of tests was carried out for each of a series of other com-

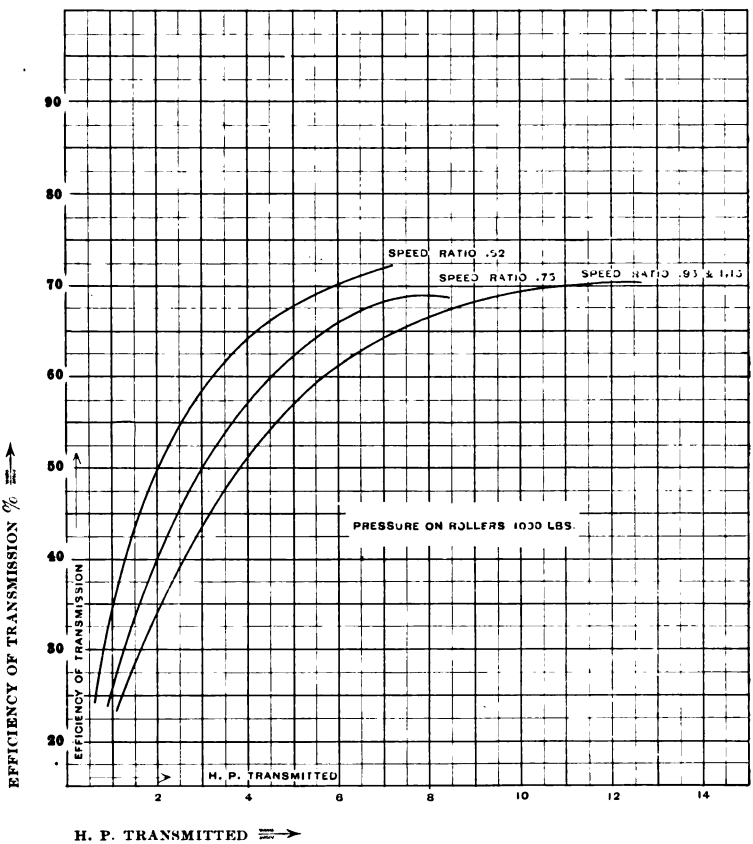


Fig. 3.

pressions. Incidentally, the capacity of the machine was determined under each condition.

The tests were carried out by L. A. Hackett, L.S.S., '04, and W. M. Stone, L.S.S., '04, under the direction of the writer.

The principal results of the tests are shown graphically in Figs. 2, 3, 4 and 5, where the efficiency of transmission is shown

for different horse-powers transmitted. These results are given in the different figures for four different pressures of the spring, viz.: 600, 1,000, 1,400 and 1,600 lbs. respectively. At each spring pressure a series of curves is drawn, each giving the results found by a series of tests with a fixed position of the fric-

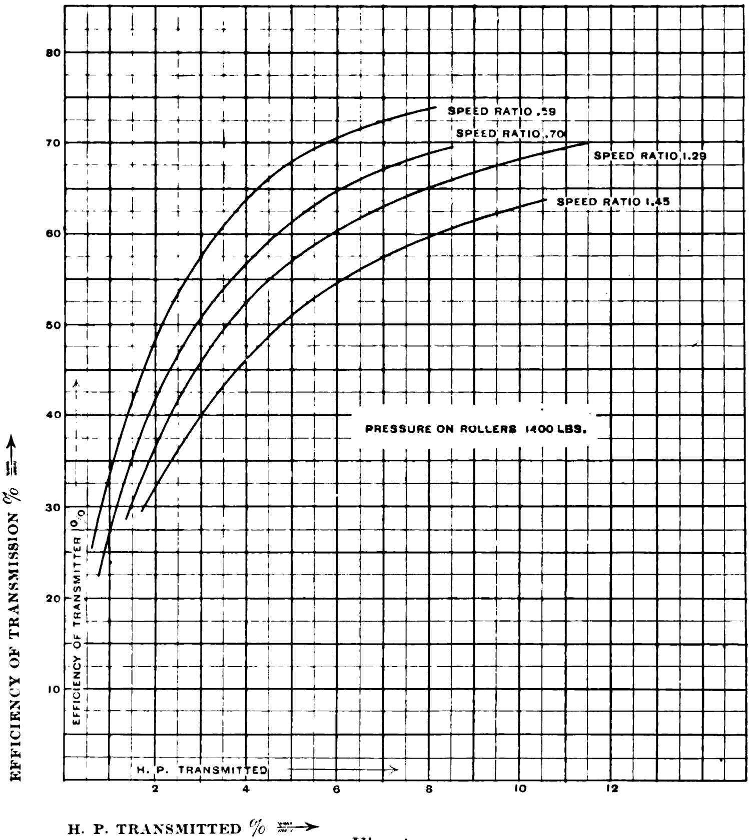


Fig. 4.

tion-rollers and consequently with an approximately constant speed-ratio. The mean speed-ratio is shown on each curve.

The curves drawn are representative curves selected from a larger number, the omissions being made to secure greater clearness in the drawings. An examination of the four figures leads to the following conclusions:

(1) The efficiency of transmission is higher with a small pressure on the rollers than with a greater pressure. The maximum efficiency with 600 lbs. pressure was 76.5 per cent; with 1,600 lbs. pressure it had fallen to 66.5 per cent; and with intermediate pressures it had intermediate values. The probable

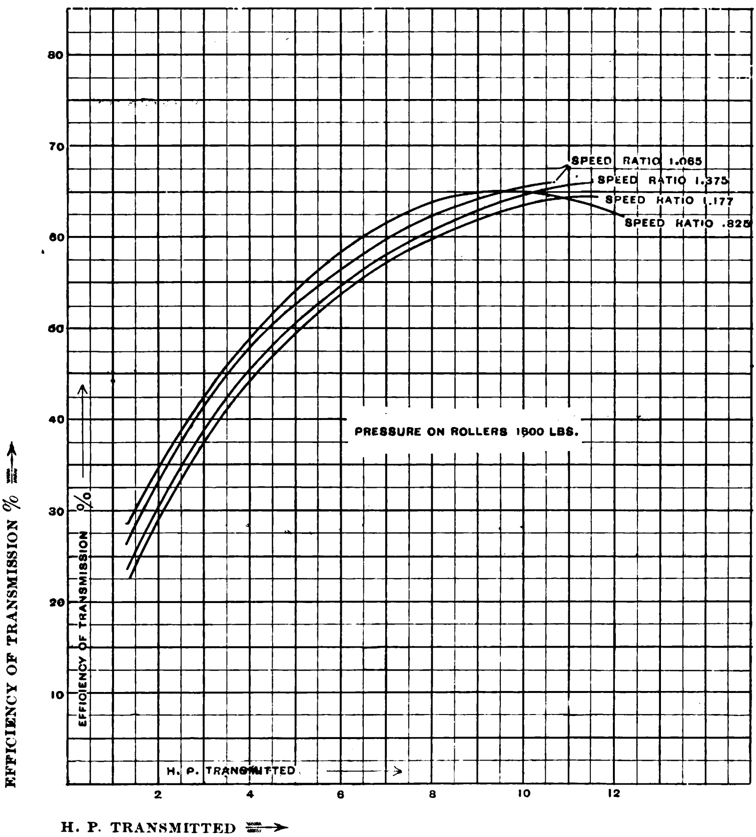


Fig. 5.

reason for this result is that most of the friction loss is due to what may be called necessary slip. All parts of the surface of the friction-rollers move at (practically) the same speed. The surface of each roller is, however, in contact with the disk along a diametral line, the points in which are all moving at different velocities on account of their different distances from the center of

the disk. Hence, if any one point on the roller is moving at the same velocity as the point on the disk with which it is in contact, all the other pairs of contact points will have relative velocities — that is, will have a sliding motion past one another. The amount of this necessary slip is a function of the width of the rollers, and is independent of the pressure between the rollers and disks. The frictional resistance to the sliding is a direct function of the pressure, and consequently the work done in causing this slip increases as the pressure increases. The advantage of the low pressure is more marked at low loads than at higher loads. At 600 lbs. pressure and with speed-ratio of about .8 the efficiency is 51 per cent when transmitting 2 horse-power; with 1,600 lbs. pressure the efficiency at the same speed-ratio and load is 35 per cent.

(2) The capacity of the machine increases both with increase of speed-ratio and with increase of the pressure on the rollers. The power available for driving the transmitter was not sufficient to carry it to its full capacity for the high pressures and large speed-ratios, but whereas the maximum power transmissible with 600 lbs. pressure did not exceed 10 horse-power, it was above 14 horse-power with 1,600 lbs. pressure, and probably was considerably greater than that amount.

(3) The efficiency of transmission is greatest with the smallest speed-ratios; it decreases till the speed-ratio becomes a little greater than unity, and then increases with further increase of speed-ratio.

Each series of runs was made with a constant position of the friction-rollers and with varying resisting torque. A constant position of the friction-rollers would give a constant speed-ratio if the slipping were constant, but as the slip increases naturally with the resisting torque, the speed-ratio decreases at the same time. To show the increase in the slip and the decrease in the speed-ratio the Fig. 6 has been plotted for a fixed position of the rollers; the other tests show precisely similar results. Here it will be seen that the speed-ratio decreases uniformly as the resisting torque increases until the torque reaches about 190 foot-pounds, when the machine becomes overloaded and the slipping

increases rapidly. In other words, the increase of slip is a simple function of the resisting torque and is directly proportional to it within the proper capacity limits of the machine. The limit varies, of course, with the compression of the spring.

The power loss in transmission by this machine is due to loss in the driving belt, to friction at the bearings m, m (Fig. 1), to friction at the bearings of the rollers, to friction at the bearings

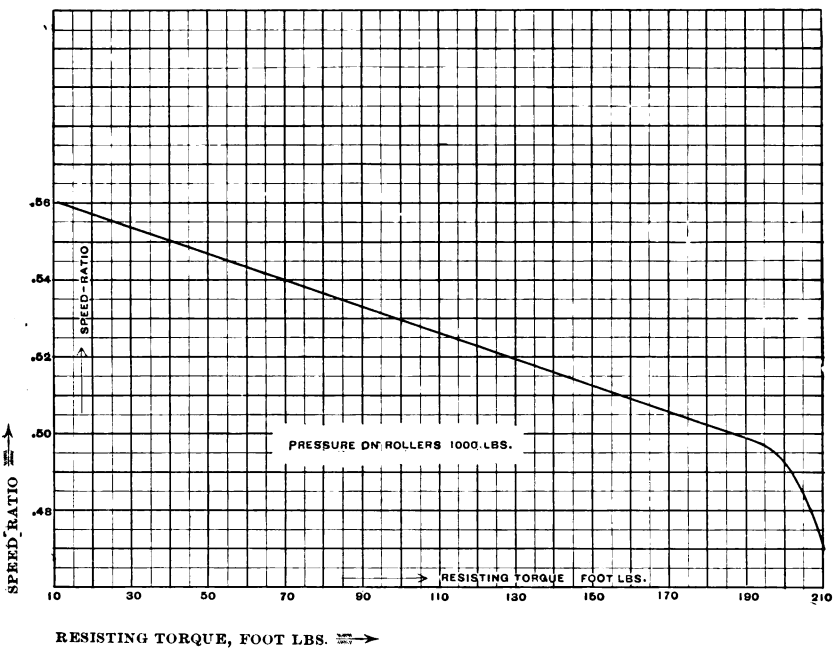


Fig. 6.

n, n of the driven shaft, and to slip between the friction-rollers and the disks.

The loss in the driving belt and at the bearings m, m was found by slacking away the spring and running the driving disks alone; the power required to do this is one-half horse-power, and is independent of the other conditions. The power required to drive the machine with no power transmitted (that is, with zero resisting torque) was next investigated for different speed-ratios and for different spring compressions.

The two curves in Fig. 7 show the results obtained with 600 lbs. and with 1,400 lbs. compression respectively. They show that the frictional losses increase both with increase of speed-ratio and also with increase of the pressure on the rollers. This result was to be anticipated, because the increase in speed of rotation of the friction-rollers and of the driven shaft means increased journal friction, and because also the greater pressure on

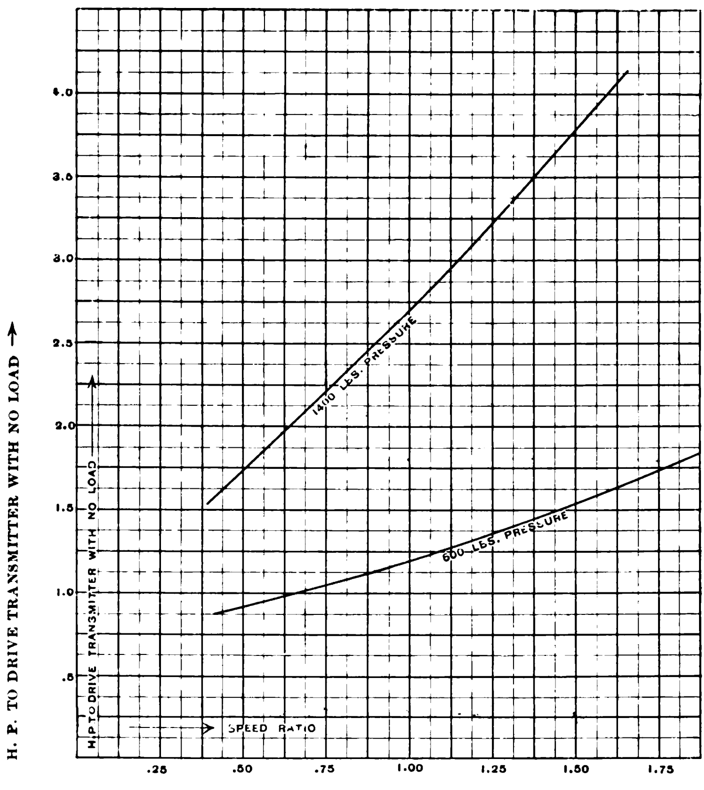
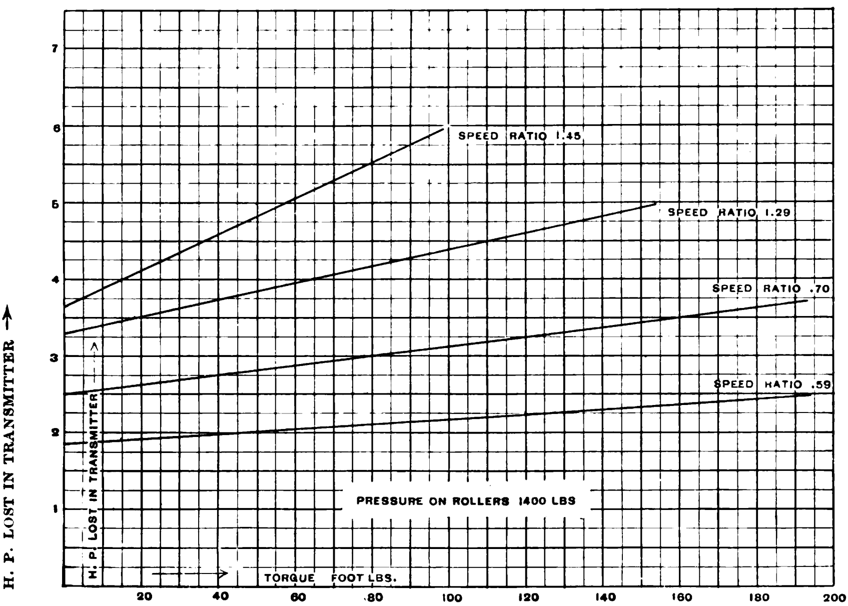


Fig. 7.

the rollers does not alter the necessary slip resulting from the width of the rollers, and only makes this occur against a greater resistance, and consequently demands more power to produce it.

The frictional resistances or losses when power is being transmitted are naturally greater than when the machine is running

without load. To show the increase of these losses with the load a series of curves has been plotted, Fig. 8 showing, for a spring pressure of 1,400 lbs., the losses in the transmitter at each of four roller positions. It will be seen at once that the power necessary to overcome the frictional resistances increases with the torque, and that the increase is directly proportional to the torque for any fixed position of the friction-rollers. Since the



TORQUE, FOOT LBS. →

Fig. 8.

slip is also proportioned to the torque, the increase in the frictional resistance is probably due to the increased slip alone.

A further examination of Fig. 8 shows that the rate of change of friction losses with the torque increases rapidly as the speed-ratio increases — in fact, increases almost as the square of the speed-ratio.

One of the important factors in determining the capacity and efficiency of the machine is the condition of the surface of the friction-rollers. The two curves in Fig. 9 were obtained with the

same pressure on the rollers and the same speed-ratio, but with the difference that the curve *A* gives the results with the rollers clean, and the curve *B* shows the efficiency when the rollers have been treated with oil and resin. It will be observed that this treatment, by increasing the coefficient of friction at the contact surface, increases the capacity from 7 to 9 horse-power, but at the cost of a decreased efficiency. This decrease in efficiency

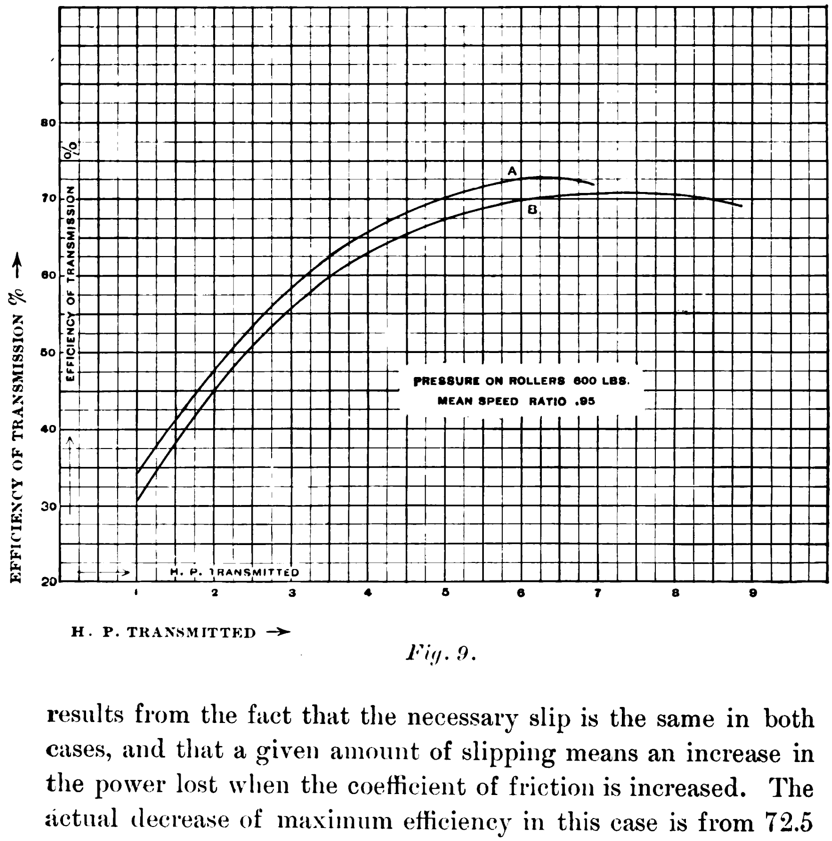


Fig. 9.

results from the fact that the necessary slip is the same in both cases, and that a given amount of slipping means an increase in the power lost when the coefficient of friction is increased. The actual decrease of maximum efficiency in this case is from 72.5 to 70 per cent.

From the foregoing results it is obvious that the principal cause of power loss in this transmitter (and the same is true of all transmitters of this general type) is the necessary slip resulting from the width of the friction-rollers. If the width of the rollers

is reduced the slip decreases. The width is determined by the compressive strength of the material of the rollers and by the coefficient of friction of this material on cast iron. The power transmissible is a function of the frictional resistance between the rollers and the disks, and is equal to the product of the pressure by the coefficient of friction. If the coefficient of friction can be increased by treatment of the rolls, the total pressure can be reduced and consequently the width of the rolls can be diminished. If a material of greater compressive strength can be substituted with the same friction qualities, the same total pressure can be supported on a narrower roller without crushing the material. The necessary slip can be reduced to a negligible quantity only in the case where the rollers are very thin — a condition which cannot be attained at present when any considerable amount of power is to be transmitted.

To reduce the slip losses to a minimum it is necessary to substitute a positive drive for a friction drive. At the present day no such mechanism exists, though much thought and effort have been expended in the endeavor to obtain it. Such a transmitter, permitting a gradual change throughout its whole range of speed, with the compactness which a positive gearing would permit, and maintaining a constant speed, when required, throughout its whole range of power, is greatly to be desired. Unfortunately, it appears also to be unattainable.

HARVARD ENGINEERING LABORATORY INVESTIGATIONS.

III. MEASUREMENT OF THE AVERAGE ELECTRICAL RESISTANCE OF SLEEVE-JOINTS IN ALUMINUM CONDUCTORS.

A. E. KENNELLY,
Professor of Electrical Engineering,
AND
F. P. COFFIN, 1903.

Object of the Test.

IN electrical conductors of copper, the joints are, or should be, soldered, in order to effect and maintain metallic continuity between the jointed ends. The electrical resistance of a soldered joint in copper wires, or in iron wires, is too small to be practically considered under ordinary circumstances. On the other hand, if the joint is not soldered, its resistance may be seriously large. In telegraphy, it is not uncommon to find that the total resistance of unsoldered joints occurring along a line exceeds the resistance of the line wire, so that the length of the line becomes more than doubled, from the standpoint of conductor-resistance, owing to the imperfect contact at unsoldered joints.

Aluminum wire cannot be readily soldered, owing to the swiftness with which the freshly cleansed surfaces of the ends become oxidized by the air. Consequently, a joint has been devised in which a sleeve of aluminum is slipped over the ends, and the joint is then twisted and compressed in a machine that is carried about for the purpose. It is a matter of interest and importance to test the joints thus made.

Seven samples of aluminum wire were received from the Pittsburg Reduction Company, in six-foot lengths, and in different sizes between No. 8 and No. 00 A. W. G., inclusive. These samples contained no joints, and were used as standards of reference for jointed wires.

Seven similar samples were also received, each having six joints. Each joint was from 8 to 9 inches long (20 to 23 cms.),

and was a regular sleeve-joint, made in a machine for that purpose. Each joint contained from four to eight complete twists.

The purpose of the tests was to determine the average resistance of the joints in each of these jointed conductors.

Method.

The method of measurement was by fall of potential. Fig. 1 illustrates the method.

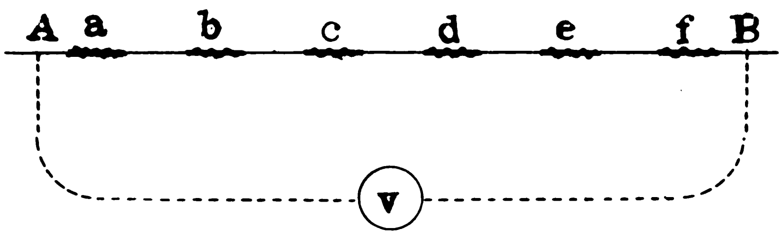


Fig. 1.

A B is the jointed conductor, with joints at *a*, *b*, *c*, *d*, *e* and *f*. A steady current was forced through the conductor — from a storage battery with the smaller conductors, and from a special motor-dynamo with the larger conductors. The strength of this current was read from a duly corrected ammeter in circuit with the wire. The voltmeter *V* was simultaneously observed, and its reading, duly corrected for scale error, gave the drop of pressure in the jointed conductor with the particular current-strength employed. The quotient of the pressure-drop in volts by the current-strength in amperes gave the total resistance of the jointed conductor in ohms, between the points *A*, *B*, to which the pressure wires were attached. This total resistance may be denoted by R. The distance between the points *A B* was also recorded.

By similar measurements on the corresponding length of unjointed conductor, the resistance of the length *A B* of unjointed conductor became known. Let this resistance be denoted by r.

Then the excess of the jointed-length resistance over the unjointed-length resistance, or $R - r$, is the total resistance of six joints. Consequently, the average resistance of each joint was

$$\dots\dots\dots \frac{R - r}{6} \dots\dots\dots \text{ ohms.}$$

Knowing the resistance of the conductor per foot or meter, from the measurements on the unjointed length, the average resistance of each joint is readily converted into equivalent feet or meters of unjointed conductor. An ideal joint would have an electrical resistance of not more than its own length of conductor, *i.e.* less than one foot.

The measurement was made on each jointed conductor, not merely with a single current-strength, but with a series of current-strengths ascending and descending, the maximum current-strength being such as sensibly warmed the conductor. In no case was the temperature of the conductor raised above 50° C.

The following is a sample series of observations May 7.

TABLE I.

Stranded jointed conductor equivalent to No. 0, A. W. G.
Length of conductor 10.625 ft. (323.9 cms.) over all.
Length of conductor 10.250 ft. (312.4 cms.) between pressure wires.
Observers A. E. K. and F. P. C.

No. Obs.	Corrected Current Amperes.	Corrected P. D. Volts.	R Total Resistance ohms.	r Length Resistance ohms.	R − r Total Joint Resistance ohms.	$\frac{R-r}{6}$ Average Res. per Joint ohms.	Remarks.
1	31.0	0.166	0.005355	0.001696	0.003659	0.000610	–
2	41.2	0.214	0.005195	–	0.003499	0.0005832	Minimum
3	51.4	0.267	0.005195	–	0.003499	0.000583	–
4	61.0	0.324	0.005311	–	0.003615	0.000603	–
5	81.0	0.433	0.005346	–	0.003650	0.000608	–
6	100.8	0.543	0.005387	–	0.003691	0.000615	–
7	122.8	0.671	0.005464	–	0.003768	0.000628	–
8	153.2	0.872	0.005693	–	0.003997	0.000666	–
9	183.2	1.096	0.005983	–	0.004287	0.000714	–
10	203.3	1.287	0.006330	–	0.004634	0.0007723	Maximum
11	253.5	1.598	0.006303	–	0.004607	0.000768	–
12	202.4	1.266	0.006255	–	0.004559	0.000760	–
13	151.7	0.935	0.006163	–	0.004467	0.000738	–
14	100.8	0.595	0.005904	–	0.004208	0.000701	–
15	90.6	0.530	0.005850	–	0.004154	0.000692	–
16	81.0	0.464	0.005727	–	0.004031	0.000672	–
17	70.8	0.407	0.005748	–	0.004052	0.000675	–
18	61.0	0.347	0.005689	–	0.003993	0.000666	–
19	51.4	0.283	0.005506	–	0.003810	0.000635	–
20	41.2	0.229	0.005558	–	0.003862	0.000644	–
21	31.0	0.172	0.005549	–	0.003853	0.000642	–
Average,		–	0.005691	–	0.003995	0.0006658	Mean

Since the resistance per foot of the unjointed conductor of this size was 0.0001655 ohms, the minimum resistance joint was that of

$$\frac{0.0005832}{0.0001655} = 3.524 \text{ feet } (1.074 \text{ meters}),$$

the maximum resistance per joint

$$\frac{0.0007723}{0.0001655} = 4.666 \text{ feet } (1.422 \text{ meters}),$$

and the mean resistance per joint

$$\frac{0.0006658}{0.0001655} = 4.023 \text{ feet } (1.226 \text{ meters}).$$

The mean result of the whole series of observations showed, therefore, that a joint had a resistance of 4 feet or 1.23 meters of wire; while the highest reading was $4\frac{2}{3}$ feet or 1.42 meters; and the lowest reading was $3\frac{1}{2}$ feet or 1.07 meters.

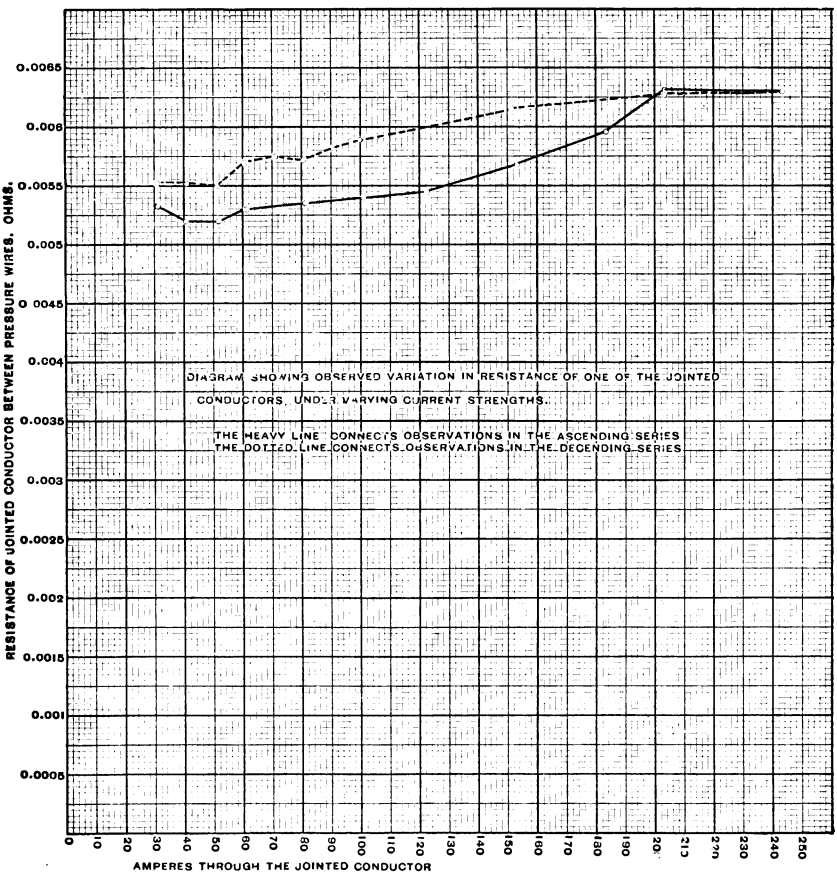


Fig. 2.

The graph of the observations in table I is given in Fig. 2. It will be seen that the resistance of the conductor at the maximum current of 253 amperes was about 18 per cent greater than at the minimum current of 31 amperes. Only a small portion of this rise is, however, attributable to increase in the resistance of the conductor with temperature, considered as an unjointed wire. The greater portion of the rise is attributable to change in the resistance of the joints with temperature. This change did not appear to follow any simple law. Not only was the resistance of the conductor at any particular current-strength slightly unsteady, but in some of the conductors the total resistance *diminished* when the current-strength increased. The temperature effect on the resistance of the joints seemed to involve a plurality of actions that may have been in mutual opposition.

Results.

Table II gives a summary of the results obtained on solid, or unstranded, jointed conductors; while Table III gives a corresponding summary of the results on multiple or stranded conductors.

TABLE II. SOLID WIRES.

No. A. W. G.	Equivalent Length per Joint.					
	Feet.			Meters.		
	Maximum.	Minimum.	Mean.	Maximum.	Minimum.	Mean.
8	6.9	4.6	6.0	2.1	1.4	1.8
6	11.2	3.8	5.6	3.4	1.2	1.7
4	25.0	10.4	12.6	7.6	3.2	3.8

TABLE III. STRANDED CONDUCTORS.

No. A. W. G.	Equivalent Length of Conductor per Joint.					
	Feet.			Meters.		
	Maximum.	Minimum.	Mean.	Maximum.	Minimum.	Mean.
2	1.1	0.9	1.0	0.34	0.28	0.30
1	1.9	1.6	1.7	0.58	0.49	0.52
0	4.7	3.5	4.1	1.44	1.07	1.25
00	4.7	3.2	4.3	1.44	0.96	1.31

The results in these tables are given both in feet and in meters. The general mean of the results in Table II is 8.1 feet or 2.4 meters; so that the average reisistance of a joint in the solid aluminum wires, from all the 18 such joints tested, is equivalent to 8.1 feet or 2.4 meters of the corresponding size of wire. The general mean of the results in Table III is 2.8 feet or 0.84 meter; so that the average resistance of a joint in the stranded conductors, from all the 24 such joints tested, is equivalent to 2.8 feet or 0.84 meter of the corresponding size of an unjointed conductor.

Summary.

Consequently, for rough purposes, the results of the tests may be summed up by the statement that the average joint in solid wires had the resistance of about 2½ meters, while the average joint in stranded conductors had a resistance of less than 1 meter.

Mechanical Particulars.

The mechanical particulars of the conductors tested are given in Table IV.

TABLE IV.

No. A. W. G.	Type.	Diam.		Area.		Length of Lay.		Joint Length.		No. of Twists.
		Inches.	Cms.	Circ. Mls.	Sq. Mm.	Ins.	Cms.	Ins.	Cms.	
8	Solid	0.1288	0.3271	16,590	8.404	–	–	8.85	22.4	9.0
6	"	0.1615	0.4102	26,080	13.21	–	–	8.80	22.3	7.0
4	"	0.2031	0.5158	41,250	20.90	–	–	8.70	22.	7.0
2	Strand of 7	0.0975	0.2476	66,540	33.71	4.7	11.9	8.28	21.	6.9
1	"	0.1100	0.2794	84,700	42.91	5.66	14.4	8.55	21.7	4.2
0	"	0.1230	0.3124	105,900	53.64	6.73	17.1	8.53	21.6	3.9
00	"	0.1375	0.3494	132,340	67.04	6.6	16.7	8.25	21.	4.0

The mean specific gravity of the wire was found to be 2.71.

Electrical Particulars.

The mean resistivity of the solid wire was found to be 2,775 absohm-cms. at 25° C. This corresponds to a mile-ohm of 430 lbs., to a kilometer-ohm of 75.3 kilogrammes, to a meter-gramme of 0.0752 ohms, and to a circular mil-foot of 16.69 ohms, all at 25° C. The linear resistances are given in Table V, with

these values for the solid wires. The linear resistances of the stranded wires are taken from potentiometer measurements on the stranded samples, the resistances per foot of the stranded conductors being apparently slightly greater than that of the solid conductors referred to equal cross-sections, according to the usual rule.

TABLE V. RESISTANCE OF CONDUCTORS.

No. A. W. G.	Resistance at 25° C. per			
	Cm. - absohms.	Foot - ohms.	Kilometer - ohms.	Mile - ohms.
8	33,030	0.001007	3.303	5.315
6	21,010	0.0006404	2.101	3.381
4	13,290	0.0004051	1.329	2.139
2	8,770	0.0002673	0.877	1.411
1	6,834	0.0002083	0.6834	1.099
0	5,430	0.0001655	0.543	0.8738
00	4,311	0.0001314	0.4311	0.6938

It is proposed to repeat the above tests after the lapse of some months, in order to observe whether the resistance of the joints tends to vary with age; also to ascertain the effect of tensile stress upon the joints.

We desire to express our thanks to the Pittsburg Reduction Co. for their courtesy and assistance in preparing the joints for testing.

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Editorial.

WE take pleasure in announcing the election of the following active members to the Board of Editors, from the class of 1905: Stephen Henley Noyes, *Civil;* Chester Joseph Cutting, *Mechanical;* Philip Merrill Patterson, *Electrical*; Harry Edward Warren, *Architectural.*

Graduate Notes.

R. J. Wright, '99, is head draughtsman of the Electric Controller and Supply Co. of Cleveland, Ohio.

Cabot Stevens, '95, is electrical engineer of the Brooklyn Edison Illuminating Co.

Nettleton Neff, '91, is Engineer Maintenance of Way, Pennsylvania Lines, Chicago.

Howard Elliott, '81, is president of the Northern Pacific R. R.

H. F. Wardwell, '98, is a special apprentice Motive Power dept. C. B. & Q. R. R.

L. J. Eddy, '03, is a special apprentice Motive Power dept. C. B. & Q. R. R.

W. D. Hawkes, ex '03, is with the Chicago Edison Co.

L. P. White, '03, is in the Auditor's Office of the C. B. & Q. R. R., Chicago, Ill.

R. G. Scott, '02, is in the Western Wheeled Scraper works at Aurora, Ill.

H. E. Huntington, ex '04, a former editor of the JOURNAL, has been elected general manager of the Los Angeles Railway Co., California.

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